Dynamic Fisheye Grids for Accreting Black Hole Binaries Simulations

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Contents

1 Introduction

2 Setup

3 Warped coordinate transformation
   • Warped Cartesian coordinates
   • Warped Spherical coordinates

4 Final Remarks
Outline

1. Introduction

2. Setup

3. Warped coordinate transformation
   - Warped Cartesian coordinates
   - Warped Spherical coordinates

4. Final Remarks
Context

General Relativity + Magnetohydrodynamics (GRMHD):

- quasars
- gamma-ray bursts
- active galactic nuclei
- accretion disks
Circumbinary accretion disks should form around massive binary black holes systems.

Understanding circumbinary accretion flows is essential for identification of binary black holes.
Gravitational Waves (GW) and light (EM) originate in different mechanisms, independently constraining models.

Either GW or EM observations of close supermassive BH binaries would be the first of its kind!

Follow up observations can often be made via coordinated alert systems.
Binary Black Hole Accretion Disks

- time-dependent gravity moves matter around
- gas is heated and becomes luminous
- light emitted reaches observer
- we can make predictions for this emitted light

Goals

- simulate EM waves coming from these objects
- focusing on inspiral, merger, and ringdown phase
- GW observations of these events in tandem with EM observations
- explore dynamics of high-energy plasma and strong field regime of gravity
- make predictions so that people now what to look for in the data
Outline

1. Introduction

2. Setup

3. Warped coordinate transformation
   - Warped Cartesian coordinates
   - Warped Spherical coordinates

4. Final Remarks
General Relativity + Magnetohydrodynamics (GRMHD):

\[ R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = 8\pi (T_{\mu\nu}^H + T_{\mu\nu}^{EM}) \]

\[ T_{\mu\nu}^H = \rho h u_\mu u_\nu + P g_{\mu\nu} \]

\[ T_{\mu\nu}^{EM} = F_{\mu\lambda} F_{\nu}^{\lambda} - \frac{g_{\mu\nu}}{4} F^2 \]

where \( \rho, \epsilon, P, u_\mu \) and \( h \equiv 1 + \epsilon + P/\rho \) are the fluid rest mass density, specific internal energy, gas pressure, 4-velocity, and specific enthalpy.
Interlude

Challenges

- Equations of the type

\[ q_t + qq_x = 0 \] truly non-linear (hydrodynamics) (1)

\[ q_t + vq_x = 0 \] linearly degenerate (pure GR) (2)

characteristic speed of the former depends on the solution itself while this is not the case for the latter;

- Courant condition greatly limits timestep of coupled system (the timestep is determined by c, even when MHD speeds are significantly smaller)
The Harm3d code

- Ideal-MHD on curved spacetimes (does not evolve Einstein’s Equations)
- Spacetime described through a vacuum post-Newtonian (PN) approximation
- 8 coupled nonlinear 1st-order hyperbolic PDEs ; 1 constraint eq.
- Finite Volume conservative scheme; Method of Lines with 2nd-order Runge-Kutta
- Mesh refinement via coordinate transformation: Eqs. solved on uniform “numerical” coordinates related to “physical” coordinates via nonlinear algebraic expressions
- Parallelization via uniform domain decomposition; 1 subdomain per process
Metric

- **Inner Zones (IZ):** close to BHs;
- **Near Zone (NZ):** “intermediate” region;
- **Far Zone (FZ):** gravitational wave region;
- **Buffer Zones (BZ):** transition regions.
MHD equations are solved in a uniformly discretized space of spatial coordinates \( \{ x^{(i)} \} \) isomorphic to spherical coordinates \( \{ r, \theta, \phi \} \):

\[
\begin{align*}
    r(x^{(1)}) &= M e^{x^{(1)}} \\
    \theta(x^{(2)}) &= \frac{\pi}{2} \left[ 1 + (1 - \xi) \left( 2x^{(2)} - 1 \right) \right. \\
    &\quad + \left. \left( \xi - \frac{2\theta_c}{\pi} \right) \left( 2x^{(2)} - 1 \right)^n \right] \\
    \phi(x^{(3)}) &= x^{(3)}
\end{align*}
\]
“Diagonal” gridding scheme

- Radial cell extents are smaller at smaller radii in order to resolve smaller scale features of the accretion flow there.
- More cells are concentrated near the plane of the disk and the binary’s orbit.
Example: circumbinary disk

$\log_{10}$ of density integrated in $\theta$ (surface density)
Outline

1. Introduction

2. Setup

3. Warped coordinate transformation
   - Warped Cartesian coordinates
   - Warped Spherical coordinates

4. Final Remarks
Outline

1. Introduction

2. Setup

3. Warped coordinate transformation
   - Warped Cartesian coordinates
   - Warped Spherical coordinates

4. Final Remarks
Warped Cartesian coordinates

\[
\frac{1}{X_{\text{max}} - X_{\text{min}}} \frac{\partial X}{\partial x} = 1 - a_x \tilde{\tau}(y, y_1, \delta y_3) \left[ \tilde{\tau}(x, x_1, \delta x_1) - 2\delta x_1 \right] - a_x \tilde{\tau}(y, y_2, \delta y_4) \left[ \tilde{\tau}(x, x_2, \delta x_2) - 2\delta x_2 \right]
\]

\[
\frac{1}{Y_{\text{max}} - Y_{\text{min}}} \frac{\partial Y}{\partial y} = 1 - a_y \tilde{\tau}(x, x_1, \delta x_3) \left[ \tilde{\tau}(y, y_1, \delta y_1) - 2\delta y_1 \right] - a_y \tilde{\tau}(x, x_1, \delta x_4) \left[ \tilde{\tau}(y, y_2, \delta y_2) - 2\delta y_2 \right]
\]
Example
Field loop evolution

Advected magnetic field loop

\[ B_x, B_y = \begin{cases} 
-A_{\text{loop}} y / r_c, & A_{\text{loop}} x / r_c; \quad r < R_{\text{loop}}, \\
0, & r \geq R_{\text{loop}} \end{cases} \]
Field loop evolution
Outline

1. Introduction

2. Setup

3. Warped coordinate transformation
   - Warped Cartesian coordinates
   - Warped Spherical coordinates

4. Final Remarks
Warped Spherical Coordinates

\[
 r(y) = R_{\text{in}} + (b_r - s a_r) y + a_r \left[ \sinh (s(y - y_b)) + \sinh (s y_b) \right]
\]

\[
 a_r \equiv \frac{R_{\text{out}} - R_{\text{in}} - b_r}{\sinh (s(1 - y_b)) + \sinh (s y_b) - s}
\]

The meaning of the parameters is more readily gleaned when looking at \( \frac{\partial r}{\partial y} \):

\[
 \frac{\partial r}{\partial y} = b_r + s a_r \left[ \cosh (s(y - y_b)) - 1 \right]
\]
Binary Warped Gridding scheme

![Graph showing Fisheye Grids for Accreting BBHs]
Bondi Accretion

- Steady-state solution for spherically-symmetric, adiabatic accretion onto a black hole.
Disk with single black hole
Disk with black hole binary
Outline

1. Introduction

2. Setup

3. Warped coordinate transformation
   - Warped Cartesian coordinates
   - Warped Spherical coordinates

4. Final Remarks
Final Remarks

- We have tools to model single black hole accretion disks
- We have tools to make observational predictions from these simulations;
- We are in the process of applying these tools to the binary case:
  - Implemented dynamic warped gridding scheme in the Harm3d code
  - This construction is very general and in no way relies in MHD, or BH evolutions.
  - Successfully passes “Field Loop” and “Bondi” tests
- Goal: evolve circumbinary accretion disk around binary black hole