Dynamic Fisheye Grids for Accreting Black Hole Binaries Simulations

Miguel Zilhão   Scott Noble

Center for Computational Relativity and Gravitation
Rochester Institute of Technology

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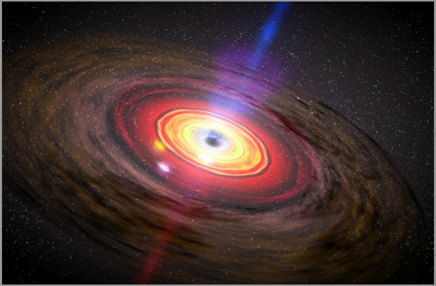
Outline

Context

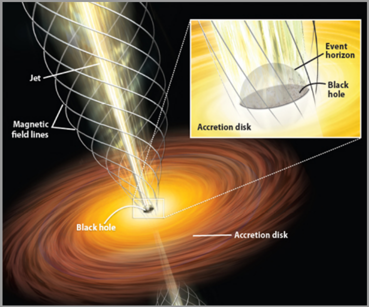
General Relativity + Magnetohydrodynamics (GRMHD):

- quasars
- gamma-ray bursts
- active galactic nuclei
- accretion disks



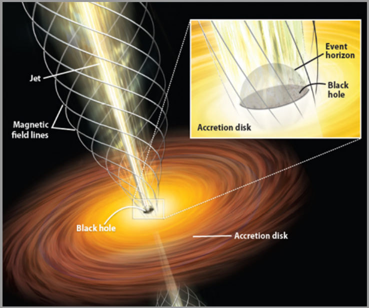
Binary Black Hole Accretion Disks

- Circumbinary accretion disks should form around massive binary black holes systems
- Understanding circumbinary accretion flows is essential for identification of binary black holes



Binary Black Hole Accretion Disks

- Gravitational Waves (GW) and light (EM) originate in different mechanisms, independently constraining models
- Either GW or EM observations of close supermassive BH binaries would be the first of its kind!
- Follow up observations can often be made via coordinated alert systems



Binary Black Hole Accretion Disks

- time-dependent gravity moves matter around
- gas is heated and becomes luminous
- light emitted reaches observer
- we can make predictions for this emitted light

Goals

- simulate EM waves coming from these objects
- focusing on inspiral, merger, and ringdown phase
- GW observations of these events in tandem with EM observations
- explore dynamics of high-energy plasma and strong field regime of gravity
- make predictions so that people now what to look for in the data

Outline

GRMHD

General Relativity + Magnetohydrodynamics (GRMHD):

$$
\begin{aligned}
R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} &= 8\pi \left(T^{\mathrm{H}}_{\mu\nu} + T^{\mathrm{EM}}_{\mu\nu} \right) \\
T^{\mathrm{H}}_{\mu\nu} &= \rho h u_\mu u_\nu + P g_{\mu\nu} \\
T^{\mathrm{EM}}_{\mu\nu} &= F_{\mu\lambda} F_\nu{}^\lambda - \frac{g_{\mu\nu}}{4} F^2
\end{aligned}
$$

where ρ, ϵ, P, u_μ and $h \equiv 1 + \epsilon + P/\rho$ are the fluid rest mass density, specific internal energy, gas pressure, 4-velocity, and specific enthalpy

Interlude

Challenges

- Equations of the type

$$q_{,t} + q q_{,x} = 0 \qquad \text{truly non-linear (hydrodynamics)} \qquad (1)$$

$$q_{,t} + v q_{,x} = 0 \qquad \text{linearly degenerate (pure GR)} \qquad (2)$$

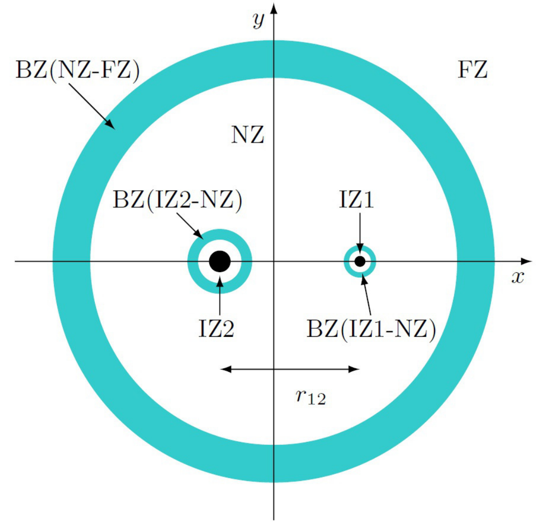
  characteristic speed of the former depends on the solution itself while this is not the case for the latter;
- Courant condition greatly limits timestep of coupled system (the timestep is determined by c, even when MHD speeds are significantly smaller)

Code

The Harm3d code

- Ideal-MHD on curved spacetimes (does not evolve Einstein's Equations)
- Spacetime described through a vacuum post-Newtonian (PN) approximation
- 8 coupled nonlinear 1st-order hyperbolic PDEs ; 1 constraint eq.
- Finite Volume conservative scheme; Method of Lines with 2nd-order Runge-Kutta
- Mesh refinement via coordinate transformation: Eqs. solved on uniform "numerical" coordinates related to "physical" coordinates via nonlinear algebraic expressions
- Parallelization via uniform domain decomposition; 1 subdomain per process

Metric



- Inner Zones (IZ): close to BHs;
- Near Zone (NZ): “intermediate” region;
- Far Zone (FZ): gravitational wave region;
- Buffer Zones (BZ): transition regions.

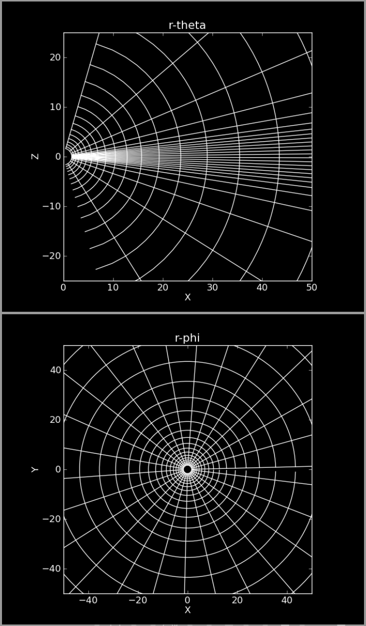
“Diagonal” gridding scheme

MHD equations are solved in a uniformly discretized space of spatial coordinates $\left\{x^{(i)}\right\}$ isomorphic to spherical coordinates $\{r, \theta, \phi\}$:

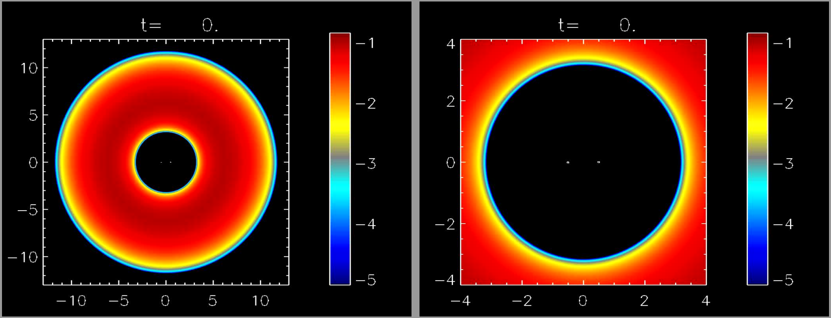
$$
\begin{aligned}
r(x^{(1)}) &= M e^{x^{(1)}} \\
\theta(x^{(2)}) &= \frac{\pi}{2}\left[1 + (1-\xi)\left(2x^{(2)} - 1\right)\right. \\
&\qquad \left. + \left(\xi - \frac{2\theta_c}{\pi}\right)\left(2x^{(2)} - 1\right)^n\right] \\
\phi(x^{(3)}) &= x^{(3)}
\end{aligned}
$$

“Diagonal” gridding scheme

- Radial cell extents are smaller at smaller radii in order to resolve smaller scale features of the accretion flow there.
- More cells are concentrated near the plane of the disk and the binary’s orbit.



Example: circumbinary disk



$\log_{10}$ of density integrated in θ (surface density)

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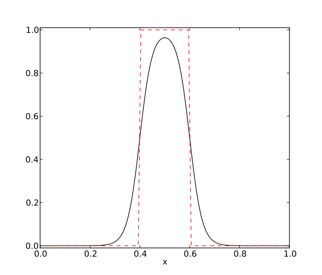
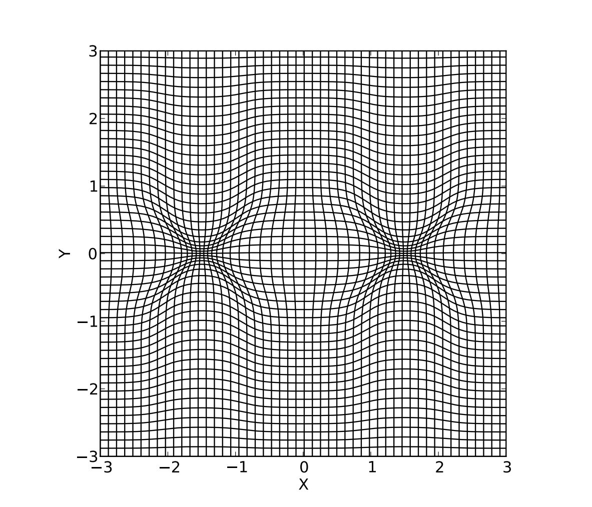
Warped Cartesian coordinates

$$\frac{1}{X_{\max}-X_{\min}}\frac{\partial X}{\partial x} = 1 \quad -a_{x1}\,\tilde{\tau}\left(y,y_1,\delta_{y3}\right)\left[\tilde{\tau}\left(x,x_1,\delta_{x1}\right)-2\delta_{x1}\right]$$
$$-a_{x2}\,\tilde{\tau}\left(y,y_2,\delta_{y4}\right)\left[\tilde{\tau}\left(x,x_2,\delta_{x2}\right)-2\delta_{x2}\right]$$

$$\frac{1}{Y_{\max}-Y_{\min}}\frac{\partial Y}{\partial y} = 1 \quad -a_{y1}\,\tilde{\tau}(x,x_1,\delta_{x3})\left[\tilde{\tau}\left(y,y_1,\delta_{y1}\right)-2\delta_{y1}\right]$$
$$-a_{y2}\,\tilde{\tau}(x,x_1,\delta_{x4})\left[\tilde{\tau}\left(y,y_2,\delta_{y2}\right)-2\delta_{y2}\right]$$



Example

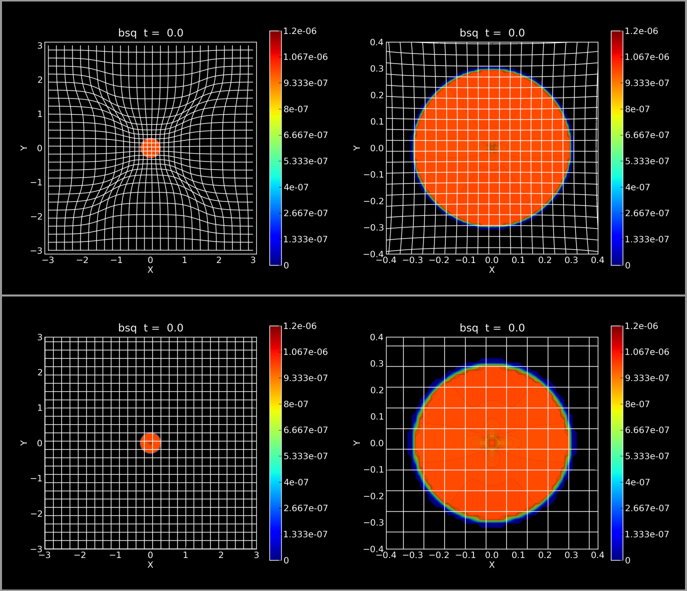


Field loop evolution

Advected magnetic field loop

$$B_x,\ B_y = \begin{cases} -A_{loop} y/r_c,\ A_{loop} x/r_c; & r < R_{loop} \\ 0; & r \geq R_{loop} \end{cases},$$

Field loop evolution



Outline

Warped Spherical Coordinates

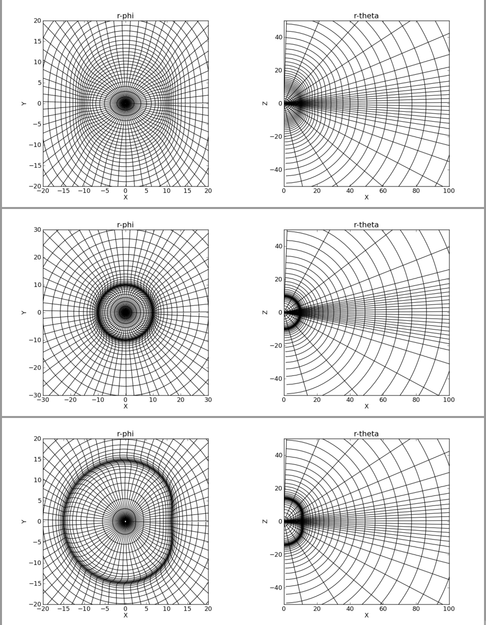
$$r(y) = R_{\text{in}} + (b_r - s a_r)\, y + a_r \left[\sinh\left(s(y - y_b)\right) + \sinh\left(s y_b\right)\right]$$

$$a_r \equiv \frac{R_{\text{out}} - R_{\text{in}} - b_r}{\sinh\left(s(1 - y_b)\right) + \sinh\left(s y_b\right) - s}$$

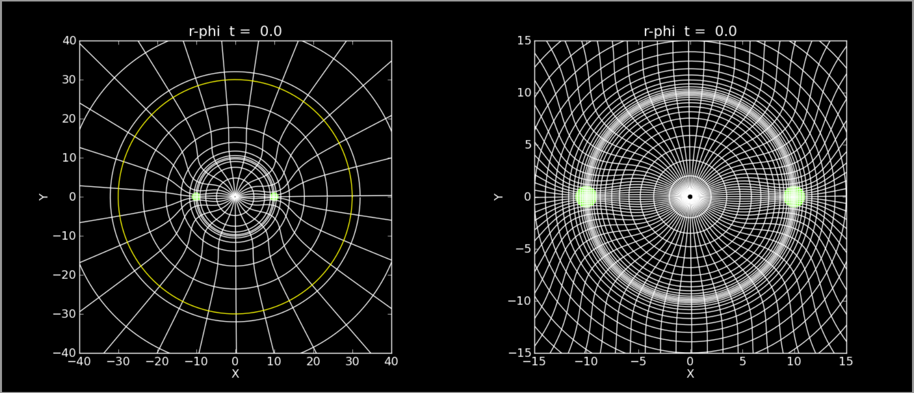
The meaning of the parameters is more readily gleaned when looking at $\partial r / \partial y$:

$$\frac{\partial r}{\partial y} = b_r + s a_r \left[\cosh\left(s(y - y_b)\right) - 1\right]$$

Examples

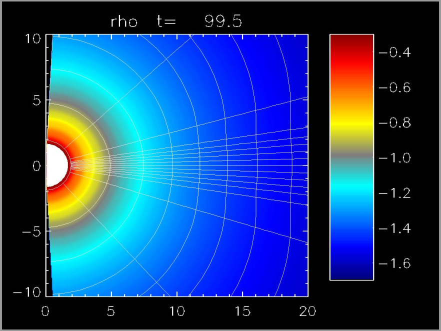


Binary Warped Gridding scheme

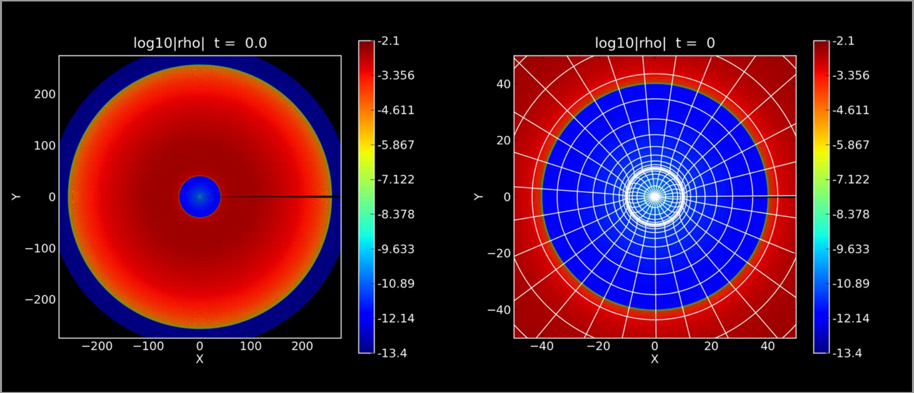


Bondi Accretion

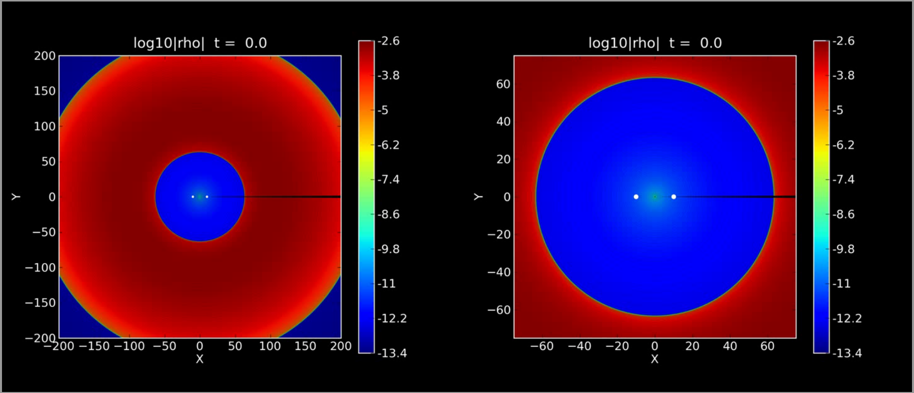
- Steady-state solution for spherically-symmetric, adiabatic accretion onto a black hole.



Disk with single black hole



Disk with black hole binary



Outline

Final Remarks

- We have tools to model single black hole accretion disks
- We have tools to make observational predictions from these simulations;
- We are in the process of applying these tools to the binary case:
  - Implemented dynamic warped gridding scheme in the Harm3d code
  - This construction is very general and in no way relies in MHD, or BH evolutions.
  - Successfully passes “Field Loop” and “Bondi” tests
- Goal: evolve circumbinary accretion disk around binary black hole