Goldstone gravity from spontaneous Lorentz violation

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Outline

Masslessness from symmetry or broken symmetry

Gauge Symmetries

Generator of unbroken gauge symmetry $\Rightarrow$ massless vector boson

General Relativity

Diffeomorphism invariance $\Rightarrow$ massless gravitons

Spontaneously Broken Global Symmetry

Spontaneously broken global symmetry $\Rightarrow$ massless Nambu-Goldstone boson

The Nambu-Goldstone theorem

Nambu-Jona-Lasinio model (1961)

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi + \frac{\lambda}{4}\left((\bar{\psi}\psi)(\bar{\psi}\psi) - (\bar{\psi}\gamma^5\psi)(\bar{\psi}\gamma^5\psi)\right)$$

invariant under ordinary and chiral phase rotations:

$$\psi \to e^{i\alpha}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{-i\alpha}$$

$$\psi \to e^{i\alpha\gamma_5}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}.$$

Mass term breaks chiral symmetry.
But: possibility of chiral condensate $\langle\bar{\psi}\psi\rangle$:

- Spontaneously breaks chiral symmetry
- Yields effective mass term
- Broken symmetry leads to massless Goldstone boson

The Nambu-Goldstone theorem

Bjorken (1963): "emergent photons"

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \frac{G}{2}(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)$$

- Nonvanishing fermion condensate carrying vacuum current possible
- Dynamics can be interpreted in terms of photon in temporal gauge
- Lorentz-violating effects can be suppressed by taking G very large

The Nambu-Goldstone theorem

Nambu (1968): QED in nonlinear gauge

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ - m)\psi - e A_\mu \bar{\psi}\gamma^\mu \psi$$

with A_μ subject to the constraint

$$A_\mu^2 = M^2$$

- $M \neq 0$ implies Lorentz-violating expectation value for A_μ
- No Lorentz-violating physical effects assumed: constraint merely implies Lorentz-violating choice of gauge

The Nambu-Goldstone theorem

Can Lorentz invariance be spontaneously broken in gauge theories?

1. In usual models of particle physics with gauge symmetry non-derivative potential terms are not gauge invariant
2. However, in string field theory terms like $\phi A_\mu A^\mu$ appear. If ϕ acquires a vev of the right sign, this can trigger a vev for the vector field

   (Kostelecky, R.P. '92)
3. Other models with possibility of spontaneous LIV: loop quantum gravity, spacetime foam, ...
4. Recently-performed lattice simulations suggest that Lorentz-breaking fermionic condensates can form in large N strongly-coupled lattice gauge theories. (Tomboulis '10, '11)

(Effective) field theory without gauge invariance

Assume nonderivative potential for vector field: (Kostelecky,Samuel '89; Krauss, Tomboulis '02; Nielsen et.al. '07)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(A_\mu A^\mu) + \mathcal{L}_{matter}(\psi, A_\mu)$$

Here V is a potential that has a local minimum for either timelike or spacelike A_μ, at which we have the constraint

$$A_\mu^2 = \pm M^2$$

Example 1: Mexican hat potential $V(A_\mu A^\mu) = -\mu^2 A_\mu A^\mu + \kappa (A_\mu A^\mu)^2$

Example 2: Lagrange multiplier potential $V(A_\mu A^\mu) = \lambda (A_\mu A^\mu \pm M^2)$

Consequence: A_μ acquires vacuum expectation value $\bar{A}_\mu$

photons as Nambu-Goldstone modes

Fluctuations of A_μ around vacuum expectation value: $A_\mu = \bar{A}_\mu + a_\mu$
Lagrangian for fluctuations:

$$\mathcal{L} = -\frac{1}{4} f_{\mu\nu} - V(\bar{A}_\mu, a_\mu)$$

Goldstone bosons identified by inifitesimal Lorentz transformations on vector vev's:

$$a_\mu = -\Theta_\mu{}^\nu(x)\bar{A}_\nu$$

with

$$\Theta_\mu{}^\nu = \begin{pmatrix} 0 & \beta_1 & \beta_2 & \beta_3 \\ -\beta_1 & 0 & \theta_3 & -\theta_2 \\ -\beta_2 & -\theta_3 & 0 & \theta_1 \\ -\beta_3 & \theta_2 & -\theta_1 & 0 \end{pmatrix} \qquad \beta_i = \bar{\beta}_i e^{ik\cdot x}, \quad \theta_i = \bar{\theta}_i e^{ik\cdot x}$$

photons as Nambu-Goldstone modes

Example: purely timelike vector with only $A_0 \neq 0$
Three Goldstone bosons:

$$a_\mu = -\Theta_{\mu 0} = \begin{pmatrix} 0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

Every choice of vev corresponds to a different gauge: temporal, axial, ...
Three Goldstone bosons can be decomposed in:

- 2 transverse modes: $k^\mu \epsilon_\mu^{trans} = 0$
- 1 longitudinal mode: $\epsilon_\mu^{long} = k_\mu - \frac{\bar{A}^\alpha a_\alpha}{\bar{A}^\alpha \bar{A}_\alpha} \bar{A}_\mu$

Longitudinal mode does not propagate due to gauge invariant kinetic term

photons as Nambu-Goldstone modes

Bumblebee Lagrangian:

- has no gauge invariance
- Lorentz invariance is spontaneously broken
- Masslessness of vector field is direct consequence of Lorentz breaking (Goldstone boson)
- Maxwell theory is “emergent"phenomenon

Further properties

Quantum corrections:

- In general, it is expected that integrating out massive modes can yield gauge non-invariant derivative terms in effective action:
- Possible terms with two derivatives: $f_1(A^2)\partial_\mu A_\nu \partial^\mu A^\nu$, $f_2(A^2)\partial_\mu A_\nu \partial^\nu A^\mu$, $f_3(A^2)A^\mu A^\alpha \partial_\mu A_\nu \partial_\alpha A^\nu$, $f_4(A^2)A^\nu A^\alpha \partial_\mu A_\nu \partial_\alpha A^\mu$, $f_5(A^2)A^\mu A^\nu A^\alpha \partial_\mu A_\nu \partial^\mu A_\alpha$, $f_6(A^2)A^\nu A^\alpha \partial_\mu \partial_\nu A_\alpha$, $f_7(A^2)A^\mu A^\nu A^\alpha A^\beta \partial_\mu A_\nu \partial_\alpha A_\beta$
- This will give rise to (small) Lorentz-violating physical effects, such as:
  - Anisotropic propagation
  - Birefringence
  - longitudinal mode can become propagating

Renormalization

Fixed points of Renormalization Group

- Interesting to consider behaviour of theory under (Wilson) renormalization group
- Gaussian fixed point exists that is UV stable in certain directions of linearized RG flow (Altschul, Kostelecky '05)
- These relevant directions of RG flow correspond to asymptotically free theory with nonpolynomial interactions, similar to behaviour for scalar fields (Halpern, Huang '95)
- These potentials exhibit stable nontrivial minima for $A_\mu A^\mu$, implying "spontaneous bumblebee potential"!

Linearized "Cardinal" dynamics (V.A. Kostelecky and R.P., GRG 37 (2005) 1675)

gravitons as Nambu-Goldstone modes

$$L = \frac{1}{2} C^{\mu\nu} K_{\mu\nu\alpha\beta} C^{\alpha\beta} + V(C^{\mu\nu}, \eta_{\mu\nu})$$

$$K_{\mu\nu\alpha\beta} = -\partial^2(\eta_{\mu\alpha}\eta_{\nu\beta} - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}) + \partial_\mu \eta_{\nu\alpha}\partial_\beta + \partial_\nu \eta_{\mu\alpha}\partial_\beta$$

- $K_{\mu\nu\alpha\beta}$: ghost-free quadratic kinetic operator for spin 2; $C^{\mu\nu}$: tensor density; $\eta_{\mu\nu}$: background metric
- V: scalar potential built out of the 4 independent scalars $X_1 = C^{\mu\nu}\eta_{\nu\mu}$, $X_2 = (C \cdot \eta \cdot C \cdot \eta)^{\mu}_{\mu}, \ldots$
- kinetic term invariant under $C^{\mu\nu} \to C^{\mu\nu} - \partial^\mu \Lambda^\nu - \partial^\nu \Lambda^\mu$; invariance broken by V!
- V acquires minimum for $C^{\mu\nu} = c^{\mu\nu} \equiv \langle C^{\mu\nu} \rangle \neq 0$: spontaneous breaking of Lorentz symmetry
- Generally all six Lorentz generators are broken; Special situation may arise with three or five broken generators (Carroll et.al. '09)

Linearized "Cardinal" dynamics

At low energy, assume V can be approximated by sum of delta-functions that fix the 4 independent scalars: $V = \sum_{n=1}^{4} \frac{\lambda_n}{n} X_n$

Fluctuations around vev: $C^{\mu\nu} = c^{\mu\nu} + \tilde{C}^{\mu\nu}$

equations of motion:

$$K_{\mu\nu\alpha\beta}\tilde{C}^{\alpha\beta} - \lambda_1\eta_{\mu\nu} - \lambda_2(\eta c\eta)_{\mu\nu} - \lambda_3(\eta c\eta c\eta)_{\mu\nu} - \lambda_4(\eta c\eta c\eta c\eta)_{\mu\nu} = 0$$

constraints:

$$\tilde{C}^{\mu}_{\mu} = 0 \qquad c^{\mu\nu}\tilde{C}_{\mu\nu} = 0 \qquad (c\eta c)^{\mu\nu}\tilde{C}_{\mu\nu} = 0 \qquad (c\eta c\eta c)^{\mu\nu}\tilde{C}_{\mu\nu} = 0$$

Low-energy dynamics of $\tilde{C}_{\mu\nu}$-fluctuations around vev equal to linearized general relativity (in axial-type gauge)!

Counting degrees of freedom

Propagating massless degrees of freedom

- Can be considered Nambu-Goldstone modes of spontanously broken Lorentz generators ${\mathcal{E}_\mu}^\alpha$:

$$\tilde{C}_{\mu\nu} = {\mathcal{E}_\mu}^\alpha c_{\alpha\nu} + {\mathcal{E}_\nu}^\alpha c_{\mu\alpha}$$

- Equations of motion imply masslessness $\partial^2 \tilde{C}_{\mu\nu} = 0$ and Lorenz conditions $\partial^\mu \tilde{C}_{\mu\nu} = 0$
- Number of propagating massless degrees of freedom: $6 - 4 = 2$

Comparison between bumblebee and cardinal model

	Photon	Graviton
# Goldstone modes	3	6
Equivalent gauge condition	Temporal / axial	Cardinal
# massive modes	1	4
# transverse modes	2	2
# longitudinal modes	1	1
Kinetic term	Maxwell	Einstein-Hilbert

Bootstrap in General Relativity

Write metric as Minkowski + fluctuations (gravitons):

$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$$

Quadratic action for (free) gravitons: $\mathcal{L}^L_{GR} = \frac{1}{2} h^{\mu\nu} K_{\mu\nu\alpha\beta} h^{\alpha\beta}$

Linear coupling to matter EM

$$\mathcal{L} \supset h^{\mu\nu} \tau_{\mu\nu}$$

- $\tau_{\mu\nu}$: trace-inversed energy-momentum tensor
- linear coupling to EM-tensor gives rise to linearized Einstein equation

$$K_{\mu\nu\alpha\beta} h^{\alpha\beta} \equiv R^L_{\mu\nu} = \tau_{\mu\nu}$$

consistent coupling

consistent coupling to *total* EM tensor

- require coupling to total EM tensor, including contribution of gravitational fluctuations (Gupta '52; Kraichnan '55; Feynman '57)

$$K_{\mu\nu\alpha\beta}h^{\alpha\beta} = {\tau_h^{(1)}}_{\mu\nu}$$

- ${\tau_h^{(1)}}_{\mu\nu}$ is quadratic in $h^{\mu\nu}$, necessitating inclusion of cubic term in lagrangian
- cubic term yields new contribution to EM-tensor ${\tau_h^{(2)}}_{\mu\nu} \Rightarrow$ quartic term in Lagrangian
- etc., etc.

After resumming all terms one obtains Einstein-Hilbert action!

Cardinal bootstrap (V.A. Kostelecky and R.P., Phys. Rev. D (2009))

Deser's procedure

- bootstrap can be done in one step using procedure developed by Deser for GR (Deser '70)
- use trace-reverted field: $\mathfrak{C}^{\mu\nu} = -C^{\mu\nu} + \frac{1}{2}\eta^{\mu\nu}C^{\alpha}_{\alpha}$
- can rewrite original linearized cardinal dynamics using Palatini formalism with auxiliary field $\Gamma^{\alpha}_{\mu\nu}$:

$$\mathcal{L}^L = \mathfrak{C}^{\mu\nu}(\Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma_{\mu,\nu}) + \eta^{\mu\nu}(\Gamma^{\alpha}_{\mu\nu}\Gamma_{\alpha} - \Gamma^{\alpha}_{\beta\mu}\Gamma^{\beta}_{\alpha\nu})$$

1 step coupling

- Have to include coupling to energy-momentum tensor of $\mathcal{L}^L$ in self-consistent manner
- Use Rosenfeld method, promoting $\eta^{\mu\nu}$ to variable metric density and partial derivatives to $\eta^{\mu\nu}$-covariant ones.

Cardinal bootstrap

It follows: $-\frac{1}{2}\tau_{\mu\nu} = \frac{\delta\mathcal{L}^L}{\delta\eta^{\mu\nu}} = \Gamma^\alpha_{\mu\nu}\Gamma_\alpha - \Gamma^\alpha_{\beta\mu}\Gamma^\beta_{\alpha\nu}$ + total derivative

Full nonlinear action obtained by coupling nonderivative part of $\tau_{\mu\nu}$ as source for $\mathfrak{C}^{\mu\nu}$:

$$\mathcal{L} = \mathcal{L}^L + \mathfrak{C}^{\mu\nu}(\Gamma^\alpha_{\mu\nu}\Gamma_\alpha - \Gamma^\alpha_{\beta\mu}\Gamma^\beta_{\alpha\nu})$$

final result for kinetic term

recursive process yields nonlinear "bootstrapped" action

$$\int d^4x \left(\mathfrak{C}^{\mu\nu}(\Gamma^\alpha_{\mu\nu,\alpha} - \Gamma_{\mu,\nu}) + (\eta + \mathfrak{C})^{\mu\nu}(\Gamma^\alpha_{\mu\nu}\Gamma_\alpha - \Gamma^\alpha_{\beta\mu}\Gamma^\beta_{\alpha\nu})\right)$$

$$\equiv \int d^4x(\eta + \mathfrak{C})^{\mu\nu} R_{\mu\nu}(\Gamma)$$

Thus $(\eta + \mathfrak{C})^{\mu\nu}$ is naturally interpreted as curved-space metric density!

Bootstrap of matter tensor and scalar potential

Bootstrap can also be applied to scalar potential and matter EM tensor

- flat-space matter EM-tensor yields curved-space matter lagrangian with metric density $(\eta + \mathfrak{C})^{\mu\nu}$: $\mathcal{L}_{M,\mathfrak{C}} = \sqrt{|\eta + \mathfrak{C}|}\mathcal{L}^{L}_{M,\mathfrak{C}}|_{\eta\to\eta+\mathfrak{C}}$
- procedure compatible with scalar potential depending only on $C^{\mu\nu}$ and background Minkowski metric $\eta_{\mu\nu}$

integrability conditions

- Bootstrap requires V satisfy integrability conditions
- Conditions satisfied only by particular solutions, e.g.:

$$1, \qquad \mathfrak{X}_1, \qquad \mathfrak{X}_2 - \frac{\mathfrak{X}_1^2}{2}, \qquad \mathfrak{X}_3 - \frac{3\mathfrak{X}_1\mathfrak{X}_2}{4} + \frac{\mathfrak{X}_1^3}{8}, \qquad \ldots$$

$$(\mathfrak{X}_1 = \mathfrak{C}^{\mu\nu}\eta_{\nu\mu}, \mathfrak{X}_2 = (\mathfrak{C}\cdot\eta\cdot\mathfrak{C}\cdot\eta)^{\mu}_{\mu}, \ldots \text{ etc.})$$

- Particular linear combination yields cosmological constant $\sqrt{|\eta + \mathfrak{C}|}$

Bootstrap of scalar potential

integrable scalar potentials

Particularly interesting: Scalar potentials of the form

$$V(\{\mathfrak{X}_i\}) = \frac{1}{2}\sum_{i,j} m_{ij}(\mathfrak{X}_i - \mathfrak{x}_i)(\mathfrak{X}_j - \mathfrak{x}_j) + \mathcal{O}(\mathfrak{X}_i - \mathfrak{x}_i)^3$$

with local minimum at $\mathfrak{X}_i = \mathfrak{x}_i$ $(i = 1 \ldots 4)$

- Represent possibly stable vacuum
- Integrability and stability highly nontrivial conditions
- Limit $m_{ij} \to \infty$ corresponds to bootstrap of linearized limit

$$V^L = \lambda_1(\mathfrak{X}_1 - \mathfrak{x}_1) + \lambda_2(\mathfrak{X}_2 - \frac{\mathfrak{X}_1^2}{2} - \mathfrak{x}_2 + \frac{\mathfrak{x}_1^2}{2}) + \ldots$$

Vacuum energy-momentum tensor

Bootstrapped Lagrangian

$$(\eta + \mathfrak{C})^{\mu\nu} R_{\mu\nu}(\Gamma) - \sqrt{-|\eta + \mathfrak{C}|}\, V(\mathfrak{X}_1, \mathfrak{X}_2, \mathfrak{X}_3, \mathfrak{X}_4) + L_{matter}(\mathfrak{C}, \eta, \phi_i, \partial_\mu \phi_i)$$

Linearized equations of motion

$$K_{\mu\nu\alpha\beta} h^{\alpha\beta} = (\eta_{\mu\nu}\partial_1 + 2\eta_{\mu\alpha} c^{\alpha\beta}\eta_{\beta\nu}\partial_2 + \ldots)V + \tau^{(m)}_{\mu\nu}(\eta, \phi_i, \partial_\mu \phi_i)$$

$$\partial_n \equiv \frac{\partial}{\partial X_n} \qquad X_n = (C \cdot \eta)^n \qquad (n = 1 \ldots 4)$$

"vacuum energy-momentum tensor"

$$T^{(vac)}_{\mu\nu} = \tau^{(vac)}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\left(\tau^{(vac)}\right)^{\alpha}_{\alpha}.$$

where

$$\tau^{(vac)}_{\mu\nu} = (\eta_{\mu\nu}\partial_1 + 2\eta_{\mu\alpha} C^{\alpha\beta}\eta_{\beta\nu}\partial_2 + \ldots)V$$

Vacuum energy-momentum tensor (cont.)

Explicit solutions

- Explicit solutions of linearized equations of motion can be obtained for $h^{\mu\nu}$ with nonzero vacuum energy-momentum tensor
- Initial/boundary values can be defined on suitable initial timelike/spacelike spacetime slices (4 independent functions)

Conservation and initial conditions

If matter EM tensor conserved independently, same is true for vacuum EM tensor

then

Choosing $T^{(vac)}_{\mu\nu}$ to be zero at suitable initial timelike/spacelike spacetime slices ensures it is zero at all spacetime

Quantum effective action

Quantum effective action

- Kinetic term not protected by gauge invariance
- Upon integrating out massive modes, one can expect quantum corrections to the kinetic term for gravitons; also, the 4 auxiliary modes may become propagating (Carroll et.al. '09)
- This yields Lorentz-violating corrections to Lagrangian; possible effects:
  - Anisotropic graviton propagation
  - Graviton bi-refringence
  - Possibility of cosmic rays emitting gravi-Cherenkov radiation

Conclusions

- Construction of alternative theory of gravity possible
- Natural explanation of masslessness of gravitons as Nambu-Goldstone modes of spontaneously broken Lorentz symmetry; no need to invoke gauge invariance
- Nonlinear lagrangian from requirement of consistent coupling to total energy-momentum tensor
- Low-energy Lagrangian corresponds to Einstein-Hilbert action
- Full Lagrangian includes 4 massive graviton modes
- Integrability conditions for potential very restrictive
- "Vacuum energy-momentum tensor" may arise
- Quantum effective action can turn auxiliary modes propagating at low energy

Future work

- Classification of all integrable and bootstrapped potentials
- What is the behaviour of the theory at high energies/temperatures?
- Cosmological implications of vacuum energy-momentum tensor?
- Investigate stability in presence of quantum corrections
- Investigate behaviour of theory under (Wilson) renormalization group: does possibility exist of renormalization group flow with nonpolynomial potential in IR?
- Extension of cardinal model: extra massless modes (ex. combination with bumblebee)?