Symmetries & Spin-statistics relation in Quantum Space-time

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Introduction

- Two basic questions about nature are:
  - Can space-time coordinates be measured with arbitrary precision ?
  - Is there a fundamental and elementary length scale in nature ?
- These issues are related to the quantum structure of space-time relevant at the Planck scale.
- Noncommutative Geometry is one of the candidates for describing physics at that regime.

Space-time UR

Heisenberg's Principle
+ $\Longrightarrow$ Space-time uncertainty relations
Einstein's Theory

- Measuring a space-time coordinate with an accuracy δ causes and uncertainty in the momentum $\sim \frac{1}{\delta}$.
- Neglecting rest mass, an energy of the order $\frac{1}{\delta}$ is transmitted to the system and concentrated for some time in the localization region. The associated energy-momentum tensor generates a gravitational field.
- The smaller the uncertainties in the measurement of coordinates, the stronger will be the gravitational field generated by the measurement.

Space-time UR

- To probe physics at Planck Scale l_p, the Compton wavelength $\frac{1}{M}$ of the probe must be less than l_p, hence $M > \frac{1}{l_p}$, i.e. Planck mass.
- When this field becomes so strong as to prevent light or other signals from leaving the region in question, an operational meaning can no longer be attached to the localization.
- Similarly, observations of very short time scales also require very high energies. Such observations can also form black holes and limit spatial resolutions leading to a relation of the form

$$\Delta t \Delta x \geq L^2, \quad L = \text{fundamental length}$$

Space-time UR

- Based on these arguments, Doplicher, Fredenhagen and Roberts (1994) arrived at uncertainty relations between the coordinates, which they showed could be deduced from a commutation relation of the type

$$[q_\mu, q_\nu] = iQ_{\mu\nu}$$

where q_μ are self-adjoint coordinate operators, μ, ν run over space-time coordinates and $Q_{\mu\nu}$ is an antisymmetric tensor, with the simplest possibility that it commutes with the coordinate operators.

Noncommutative geometry

An example of noncommutative geometry is provided by the d-dimensional Groenewold-Moyal spacetime or GM plane, which is an algebra $\mathcal{A}_\theta(\mathbb{R}^N)$ generated by elements $\hat{x}_\mu$ ($\mu \in [0, 1, 2, \cdots, N-1]$) with the commutation relation

$$[\hat{x}_\mu\,, \hat{x}_\nu] = i\theta_{\mu\nu}\mathbf{1}\,,$$

$\theta_{\mu\nu}$ being real, constant and antisymmetric in its indices. This algebra can be represented by functions of commuting variables with a twisted product

$$\mu_\theta(f \otimes g) = f * g = f e^{i/2 \overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu} g = \mu_0(F_\theta f \otimes g).$$

The $*$ product defines the associative but noncommutative algebra $\mathcal{A}_\theta(\mathbb{R}^N)$. The twist element is denoted by F_θ.

NCG

- In the commutative case, we know that symmetry and spin-statistics theorem are the two basic building blocks for any quantum theory.
- How do we implement symmetries and how should we define particle statistics in a noncommutative framework?
- For that, we first discuss how a symmetry group in the commutative framework acts on a multiparticle Hilbert space and then generalize that to the noncommutative algebra.

Symmetry on algebra

- Let a symmetry group $\mathcal{G}$ with elements α act on single particle Hilbert space H by a unitary representation $\alpha \to D(\alpha)$. Then, in the usual case, $\mathcal{G}$ acts on $H \otimes H$ by the representation

$$\alpha \to [D \otimes D](\alpha \otimes \alpha)\,.$$

  The homomorphism

$$\Delta : \mathcal{G} \to \mathcal{G} \otimes \mathcal{G} \qquad \alpha \to \Delta(\alpha) = \alpha \otimes \alpha$$

  is called the coproduct on $\mathcal{G}$.

- The action of $\mathcal{G}$ on multiparticle states involves more than just group multiplication as it requires the coproduct.

Symmetry on algebra

- Let $\mathcal{A}$ be an algebra. $\mathcal{A}$ comes with a rule for multiplying its elements. For $f, g \in \mathcal{A}$ there exists the multiplication map μ such that

$$\mu : \mathcal{A} \otimes \mathcal{A} \to \mathcal{A} \,,$$
$$f \otimes g \to \mu(f \otimes g) \,.$$

- Now let $\mathcal{G}$ be the group of symmetries acting on $\mathcal{A}$ by a given representation $D : \alpha \to D(\alpha)$ for $\alpha \in \mathcal{G}$. We can denote this action by

$$f \longrightarrow D(\alpha) f \,.$$

Symmetry on algebra

The action of $\mathcal{G}$ on $\mathcal{A} \otimes \mathcal{A}$ is formally implemented by the coproduct Δ

$$\Delta : \mathcal{G} \longrightarrow \mathcal{G} \otimes \mathcal{G}$$

The action is compatible with μ only if a certain compatibility condition between $\Delta(\alpha)$ and μ is satisfied. This action is

$$f \otimes g \longrightarrow (D \otimes D)\Delta(\alpha) f \otimes g \,,$$

and the compatibility condition requires that

$$\mu \left((D \otimes D)\Delta(\alpha) f \otimes g \right) = D(\alpha)\, \mu(f \otimes g) \,.$$

Symmetry on algebra

The compatibility condition can be expressed in terms of the following commutative diagram :

$$\begin{tikzcd} f \otimes g \arrow[rr, "\Delta"] \arrow[d, "\mu"'] & & (D \otimes D)\Delta(\alpha) f \otimes g \arrow[d, "\mu"] \\ \mu(f \otimes g) \arrow[rr] & & D(\alpha)\mu(f \otimes g) \end{tikzcd}$$

If a Δ satisfying the above compatibility condition exists, then $\mathcal{G}$ is an automorphism of $\mathcal{A}$. If such a Δ cannot be found, then $\mathcal{G}$ does not act on $\mathcal{A}$.

Commutative Diffeos

- Diffeos are generated by vector fields defined by

$$\xi = \xi^{\mu} \frac{\partial}{\partial x^{\mu}}$$

- Denote the space of vector fields by V. Commutator of two vector fields $\xi, \eta \in V$ is another vector field in V given by by

$$[\xi, \eta] = (\eta^{\mu}(\partial_{\mu}\xi^{\rho}) - \xi^{\mu}(\partial_{\mu}\eta^{\rho})) \frac{\partial}{\partial x^{\rho}}$$

Leibniz Rule

- The Leibniz rule for the diffeos is given by

$$(\xi(f.g)) = (\xi f).g + f.(\xi g)$$

where $f, g \in \mathcal{A}_0(\mathbb{R}^N)$ and are multiplied by the usual commutative pointwise multiplication rule.

- Leibniz rule is equivalent to the coproduct for the diffeos

$$\Delta_0 : V \longrightarrow V \otimes V, \qquad \Delta_0(\xi) = \xi \otimes \mathbf{1} + \mathbf{1} \otimes \xi$$

- This coproduct or the Leibniz rule is compatible with the multiplication map on the algebra of vector fields

$$[\Delta_0(\xi), \Delta_0(\eta)] = \Delta_0([\xi, \eta])$$

NC Diffeos

In the noncommutative case, we have the algebra $\mathcal{A}_\theta(\mathbb{R}^N)$ with the multiplication map μ_θ. Various works, based mainly on ideas of Drinfeld have shown that

- The coproduct Δ_0 is not compatible with the multiplication map μ_θ.
- One can define a new twisted coproduct

$$\Delta_\theta = F_\theta^{-1} \Delta_0 F_\theta$$

  which is compatible with μ_θ.
- This implies that the Leibniz rule is modified when $\theta \neq 0$.

Commutative Statistics

- In the commutative case, the physical wavefunctions describing two identical particles are either symmetric or antisymmetric under the particle exchange, corresponding to bosons and fermions, and given by

$$f \otimes_{S,A} g = \frac{1}{2}(f \otimes g \pm g \otimes f)$$

- If $\mathcal{G}$ is a symmetry of the theory, then the particle statistics must not change under its action.

Commutative Statistics

- The commutative flip operator τ_0 given by

$$\tau_0(f \otimes g) = g \otimes f \,.$$

The physical Hilbert space is constructed from the elements

$$\left(\frac{1 \pm \tau_0}{2}\right)(f \otimes g)\,.$$

- The coproduct Δ_0 usually commutes with the action of τ_0,

$$\tau_0 \Delta_0 = \Delta_0 \tau_0 \,,$$

and the statistics is superselected.

Twisted Statistics

- In the noncommutative case, in general we have that

$$\tau_0 \Delta_\theta \neq \Delta_\theta \tau_0 \, .$$

Hence the usual flip operator τ_0 is not compatible with the action of the standard symmetries. So τ_0 cannot be used to construct the physical Hilbert space.

- We can define a twisted flip operator τ_θ by

$$\tau_\theta = F_\theta^{-1} \tau_0 F_\theta \, ,$$

which commutes with the twisted coproduct Δ_θ,

$$\tau_\theta \Delta_\theta = \Delta_\theta \tau_\theta \, .$$

Twisted Statistics

- The two-particle physical Hilbert space is now constructed out of the states

$$f \otimes_S g = \left(\frac{1+\tau_\theta}{2}\right)(f \otimes g)\,,$$

$$f \otimes_A g = \left(\frac{1-\tau_\theta}{2}\right)(f \otimes g)\,,$$

- This encodes a twisted spin-statistics relation which has profound consequences for observables and quantum field theories.

Twisted Statistics

- Since the Pauli exclusion principle is now modified, some of the processes that are Pauli forbidden in the commutattive case may now become possible. Using this idea, and using the experimental bounds on the branching ratios of such forbidden transitions, there have been attempts to put bounds on the noncommutativity parameter.
- In quantum field theory, the commutation relation between the creation and annihilation operators are now modified, which affects almost every calculation in QFT.
- We demonstrate some of these for a NC black hole.

BTZ

- The metric for the BTZ black hole in terms of Schwarzschild-like coordinates (r, t, ϕ) is given by

$$ds^2 = \left(M - \frac{r^2}{\ell^2} - \frac{J^2}{4r^2}\right) dt^2 + \left(-M + \frac{r^2}{\ell^2} + \frac{J^2}{4r^2}\right)^{-1} dr^2 + r^2 \left(d\phi - \frac{J}{2r^2} dt\right)^2 ,$$

$$0 \leq r < \infty \; , \; -\infty < t < \infty \; , \; 0 \leq \phi < 2\pi \; ,$$

  where M and J are the mass and spin, respectively, and $\Lambda = -1/\ell^2$ is the cosmological constant.

- For $0 < |J| < M\ell$, there are two horizons, the outer and inner horizons, corresponding respectively to $r = r_+$ and $r = r_-$, where

$$r_\pm^2 = \frac{M\ell^2}{2} \left\{ 1 \pm \left[1 - \left(\frac{J}{M\ell} \right)^2 \right]^{\frac{1}{2}} \right\}$$

- The metric is diagonal in the coordinates (χ_+, χ_-, r), where

$$\chi_\pm = \frac{r_\pm}{\ell} t - r_\mp \phi \; ,$$

BTZ

- The manifold of the BTZ black hole solution is the quotient space of the universal covering space of AdS^3 by some elements of the group of isometries of AdS^3.
- Let AdS^3 be spanned by coordinates (t_1, t_2, x_1, x_2) satisfying

$$-t_1^2 - t_2^2 + x_1^2 + x_2^2 = -\ell^2$$

- Alternatively, one can introduce 2×2 real matrices

$$g = \frac{1}{\ell}\begin{pmatrix} t_1 + x_1 & t_2 + x_2 \\ -t_2 + x_2 & t_1 - x_1 \end{pmatrix} \qquad \det g = 1\ ,$$

belonging to the defining representation of $SL(2, R)$.

BTZ

- The isometries correspond to the left and right actions on g,

$$g \to h_L g h_R \ , \qquad h_L, h_R \in SL(2,R)$$

- Since (h_L, h_R) and $(-h_L, -h_R)$ give the same action, the connected component of the isometry group for AdS^3 is

$$SL(2,R) \times SL(2,R)/Z_2 \approx SO(2,2)$$

.

BTZ

The BTZ black-hole is obtained by discrete identification of points on the universal covering space of AdS_3. This ensures periodicity in ϕ, $\phi \sim \phi + 2\pi$. The condition is

$$g \sim \tilde{h}_L g \tilde{h}_R \ , \qquad \tilde{h}_L, \tilde{h}_R \in SO(2,2)$$

where

$$\tilde{h}_L = \begin{pmatrix} e^{\pi(r_+ - r_-)/\ell} & 0 \\ 0 & e^{-\pi(r_+ - r_-)/\ell} \end{pmatrix} , \qquad \tilde{h}_R = \begin{pmatrix} e^{\pi(r_+ + r_-)/\ell} & 0 \\ 0 & e^{-\pi(r_+ + r_-)/\ell} \end{pmatrix}$$

Thus,

$$\mathrm{BTZ} = \frac{AdS^3}{< (\tilde{h}_L, \tilde{h}_R) >}$$

where $< (\tilde{h}_L, \tilde{h}_R) >$ denotes the group generated by $(\tilde{h}_L, \tilde{h}_R)$.

BTZ

- The identification breaks the $SO(2,2)$ group of isometries to a two-dimensional subgroup $\mathcal{G}_{BTZ}$, consisting of only the diagonal matrices in $\{h_L\}$ and $\{h_R\}$.
- $\mathcal{G}_{BTZ}$ is the isometry group of the BTZ black hole.

We shall now discuss the deformation of this solution.

NC BTZ

For generic spin, $0 < |J| < M\ell$ (and $M > 0$), we shall search for Poisson brackets for the matrix elements of g which are polynomial of lowest order. They should be consistent with the quotienting, as well as the unimodularity condition and the Jacobi identity.
Writing the $SL(2, R)$ matrix as

$$g = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \qquad \alpha\delta - \beta\gamma = 1 \ ,$$

Under the quotienting, we get

$$\begin{aligned} \alpha &\sim e^{\ 2\pi r_+/\ell}\,\alpha \\ \beta &\sim e^{-2\pi r_-/\ell}\,\beta \\ \gamma &\sim e^{\ 2\pi r_-/\ell}\,\gamma \\ \delta &\sim e^{-2\pi r_+/\ell}\,\delta \end{aligned}$$

All quadratic combinations of matrix elements scale differently, except for $\alpha\delta$ and $\beta\gamma$, which are invariant under the quotienting.

NC BTZ

Lowest order polynomial expressions for the Poisson brackets of α, β, γ and δ which are preserved under the quotienting are quadratic and have the form

$$\{\alpha, \beta\} = c_1 \alpha\beta \;\; \{\alpha, \gamma\} = c_2 \alpha\gamma \;\; \{\alpha, \delta\} = f_1(\alpha\delta, \beta\gamma)$$

$$\{\beta, \delta\} = c_3 \beta\delta \;\; \{\gamma, \delta\} = c_4 \gamma\delta \;\; \{\beta, \gamma\} = f_2(\alpha\delta, \beta\gamma)$$

where c_{1-4} are constants and $f_{1,2}$ are functions.
They are constrained by

$$\begin{aligned} c_1 + c_2 &= c_3 + c_4 \\ f_1(\alpha\delta, \beta\gamma) &= (c_1 + c_2)\beta\gamma \\ f_2(\alpha\delta, \beta\gamma) &= (c_2 - c_4)\alpha\delta \;, \end{aligned}$$

after demanding that det$g$ is a Casimir of the algebra. There are three independent constants c_{1-4}.

NC BTZ

Further restrictions on the constants come from the Jacobi identity, which leads to the following two possibilities:

$$\text{A.} \quad c_2 = c_4 \qquad \text{and} \qquad \text{B.} \quad c_2 = -c_1$$

Both cases define two-parameter families of Poisson brackets. Say we call c_2 and c_3 the two independent parameters. The two cases are connected by an $SO(2,2)$ transformation.

NC BTZ

We can write the Poisson brackets for the various cases in terms of the Schwarzschild-like coordinates (r, t, ϕ). For the two-parameter families A and B we get

A.

$$
\begin{aligned}
\{\phi, t\} &= \frac{\ell^3}{2} \frac{c_3 - c_2}{r_+^2 - r_-^2} \\
\{r, \phi\} &= -\frac{\ell r_+ (c_3 + c_2)}{2r} \frac{r^2 - r_+^2}{r_+^2 - r_-^2} \\
\{r, t\} &= -\frac{\ell^2 r_- (c_3 + c_2)}{2r} \frac{r^2 - r_+^2}{r_+^2 - r_-^2}
\end{aligned}
$$

B.

$$
\begin{aligned}
\{\phi, t\} &= \frac{\ell^3}{2} \frac{c_3 - c_2}{r_+^2 - r_-^2} \\
\{r, \phi\} &= -\frac{\ell r_- (c_2 + c_3)}{2r} \frac{r^2 - r_-^2}{r_+^2 - r_-^2} \\
\{r, t\} &= -\frac{\ell^2 r_+ (c_2 + c_3)}{2r} \frac{r^2 - r_-^2}{r_+^2 - r_-^2}
\end{aligned}
$$

NC BTZ

These Poisson brackets are invariant under the action of the isometry group $\mathcal{G}_{BTZ}$ of the BTZ black hole. A central element of the Poisson algebra can be constructed out of the Schwarzschild coordinates for both cases. It is given by

$$\rho_\pm = (r^2 - r_\pm^2)\exp\left\{-\frac{2\kappa\chi_\pm}{\ell}\right\}, \qquad c_2 \neq c_3 \ ,$$

where the upper and lower sign correspond to case A and B, respectively,

$$\kappa = \frac{c_3 + c_2}{c_3 - c_2} \ ,$$

The $\rho_\pm =$constant surfaces define symplectic leaves, which are topologically $\mathbb{R}^2$ for generic values of the parameters (more specifically, $c_2 \neq \pm c_3$). We can coordinatize them by χ_+ and χ_-. One then has a trivial Poisson algebra in the coordinates $(\chi_+, \chi_-, \rho_\pm)$:

$$\{\chi_+, \chi_-\} = \frac{\ell^2}{2}(c_3 - c_2) \qquad \{\rho_\pm, \chi_+\} = \{\rho_\pm, \chi_-\} = 0$$

The action of the $\mathcal{G}_{BTZ}$ transforms one symplectic leaf to another, except for the case $c_2 = -c_3$, on which we focus from now on.

NC BTZ

- For $c_2 = -c_3$, the radial coordinate is in the center of the algebra.
- $r =$constant define $\mathbb{R} \times S^1$ symplectic leaves, and they are invariant under the action of $\mathcal{G}_{BTZ}$.
- The coordinates ϕ and t parametrizing any such surface are canonically conjugate:

$$\{\phi, t\} = \frac{c_3 \ell^3}{r_+^2 - r_-^2} \qquad \{\phi, r\} = \{t, r\} = 0$$

NC BTZ

- Upon passing to the “quantum” theory, in terms of the operators $\hat{\phi}$, $\hat{t}$ and $\hat{r}$, we have

$$[\hat{\phi}, \hat{t}] = i\theta \qquad [\hat{\phi}, \hat{r}] = [\hat{t}, \hat{r}] = 0$$

  where the constant θ is linearly related to $\ell^3/(r_+^2 - r_-^2)$.

- Deformation of BTZ provides an example of the general space-time noncommutativity given by

$$[\hat{x}_0, \hat{x}_1] = i\theta$$

  .

NC BTZ

- Since the coordinate ϕ is periodic, it is better to consider the operators $\hat{t}, e^{i\hat{\phi}}$ and $\hat{r}$, which satisfy :

$$[e^{i\hat{\phi}}, \hat{t}] = \theta e^{i\hat{\phi}} \qquad [\hat{r}, \hat{t}] = [\hat{r}, e^{i\hat{\phi}}] = 0 \ ,$$

This is similar to κ-Minkowski space-time.

- There are now two central elements in the algebra:

$$i) \ \hat{r} \qquad \text{and} \qquad ii) \ e^{-2\pi i\hat{t}/\theta}.$$

NC BTZ

- In an irreducible representation, the central element is proportional to the identity

$$e^{-2\pi i\hat{t}/\theta} = e^{i\chi}\mathbf{1}$$

- The spectrum of the time operator $\hat{t}$ is then discrete

$$n\theta - \frac{\chi\theta}{2\pi}\ , \quad n \in \mathbb{Z}$$

- If there is a Hamiltonian description for this analysis, then the corresponding energy is conserved modulo $\frac{2\pi}{\theta}$.

Connection with other works

- Exact Black Hole Solutions in Noncommutative Gravity
  P. Schupp and S. N. Solodukhin [arXiv:0906.2724]
- Cosmological and Black Hole Spacetimes in Twisted Noncommutative Gravity
  T. Ohl and A. Schenkel, JHEP 10, 052 (2009)
- Both these find NC cylinder as special cases.
- We know that the near-horizon geometry of a large class of commutative black holes contain a BTZ factor. Has the recurrance of NC cylinder something to do with that?

QFT in NC BH Background

- In standard QFT, the quantization of a field is done by mode expansion, imposition of suitable commutation relations on the creation and annihilation operators depending on the statistics of the field and finally obtaining a Fock space representation.
- In addition, both continuous and discrete symmetries must act properly on the fields.
- We start with a brief description of the κ-Minkowski algebra.

κ-Minkowski

- The κ-Minkowski space is described by the commutation relations

$$[\hat{x}_i, \hat{x}_j] = 0, \qquad [\hat{x}_0, \hat{x}_i] = ia\hat{x}_i \ ,$$

where $a = \frac{1}{\kappa}$ is the noncommutativity parameter. In terms of the Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1......, 1)$, we can define $x^\mu = \eta^{\mu\alpha} x_\alpha$ and $\partial^\mu = \frac{\partial}{\partial x_\mu} = \eta^{\mu\alpha}\partial_\alpha$ which satisfy the relations

$$[x_\mu, x_\nu] = 0, \qquad [\partial_\mu, \partial_\nu] = 0, \qquad [\partial^\mu, x_\nu] = \eta^\mu_\nu, \qquad [\partial_\mu, x_\nu] = \eta_{\mu\nu}$$

We also define $p_\mu = -i\partial_\mu$ so that $[p_\mu, x_\nu] = -i\eta_{\mu\nu}$.

κ-Minkowski

- We seek realizations of the noncommutative coordinates in terms of the commuting coordinates x_μ and corresponding derivatives ∂_μ as a power series. A class of such realizations is given by

$$\hat{x}_\mu = x^\alpha \Phi_{\alpha\mu}(\partial).$$

It is easy to see that these coordinates obey $[\partial_\mu, \hat{x}_\nu] = \Phi_{\mu\nu}(\partial)$. An example of such realizations is given by

$$\hat{x}_i = x_i \varphi(A) \qquad \hat{x}_0 = x_0 \psi(A) + i a x_i \partial_i \gamma(A),$$

where $A = -ia\partial_0$ and $\varphi(0) = 1, \psi(0) = 1$ and $\gamma(0) = \varphi'(0) + 1$ is finite and all are positive functions.

κ-Minkowski

- The $*$ product is defined by

$$f \star_\varphi g = m_0(\mathcal{F}_\varphi f \otimes g) = m_\varphi(f \otimes g).$$

- The twist operator is given by

$$\mathcal{F}_\varphi = e^{x_i(\Delta_\varphi - \Delta_0)\partial_i}$$

and Δ_φ is the deformed coproduct.

κ-Minkowski

- The twisted flip operator satisfying the condition

$$[\Delta_\varphi, \tau_\varphi] = 0$$

is given by

$$\tau_\varphi = \mathcal{F}_\varphi^{-1} \tau_0 \mathcal{F}_\varphi.$$

$$\tau_\varphi = e^{i(x_i p_i \otimes A - A \otimes x_i p_i)} \tau_0,$$

where $A = -ia\partial_0$.

- Note that τ_φ is independent of the choice of realizations.

Deformed oscilaltor algebra

- The product of two bosonic fields $\phi(x)$ and $\phi(y)$ under interchange now pick up an additional factor compared to the commutative case.

$$\phi(x) \otimes \phi(y) = e^{-(A\otimes N - N\otimes A)}\phi(y) \otimes \phi(x).$$

- The Fourier transform of the above relation gives

$$\tilde{\phi}(k)\tilde{\phi}(p) = e^{-ia\left[k_0(\partial_{p_i}p_i) - p_0(\partial_{k_i}k_i)\right]}\tilde{\phi}(p)\tilde{\phi}(k).$$

Deformed oscilaltor algebra

Using the mode decomposition

$$\Phi(x) = \int \frac{d^3p}{\sqrt{p_i^2 + m^2}} \left[A(\omega, \vec{p})e^{-ip\cdot x} + A^\dagger(\omega, \vec{p})e^{ip\cdot x}\right]$$

we get

$$A^\dagger(p_0, \vec{p})A(q_0, \vec{q}) - e^{-a(q_0\partial_{p_i}p_i + \partial_{q_i}q_i p_0)}A(q_0, \vec{q})A^\dagger(p_0, \vec{p}) = -\delta^3(p - q),$$
$$A^\dagger(p_0, \vec{p})A^\dagger(q_0, \vec{q}) - e^{-a(-q_0\partial_{p_i}p_i + \partial_{q_i}q_i p_0)}A^\dagger(q_0, \vec{q})A^\dagger(p_0, \vec{p}) = 0,$$
$$A(p_0, \vec{p})A(q_0, \vec{q}) - e^{-a(q_0\partial_{p_i}p_i - \partial_{q_i}q_i p_0)}A(q_0, \vec{q})A(p_0, \vec{p}) = 0.$$

This defines the deformed oscillator algebra.

P,T,CPT

- Consider the relation

$$[x_0, x_1] = i\theta$$

- Under P, $x_1 \to -x_1$, x_0 and $i\theta$ unchanged. Hence P is not an automorphism.
- Under T, $x_0 \to -x_0$, $i\theta \to -i\theta$, x_1 unchanged. Hence T is an automorphism.
- As a result, P, PT and CPT violated.
- Precision measurements can put bounds on the noncommutativity parameter.

Concluding remarks

- In the noncommutative framework, implementation of symmetries in general require a twisted coproduct.
- The twisted coproduct leads to twisted flip operator, twsited statistics and deformed oscillator algebra.
- The QFT in such a space-time behaves very differently compared to the commutative case. In the case of NC black holes this is expected to affect Hawking radiation and other quantum effects..
- Such a model would lead to violation of P, PT and CPT, leading to empirical bounds on the noncommutativity parameter.
- The subject is still in its infancy - lot more work remains to be done.

Concluding remarks

- However, usual description of holography requires the introduction of a sharply defined boundary, which is not possible in the presence of noncommutativity.
- It may be possible to handle this problem in the fuzzy approach or through matrix models. These have not yet been analyzed in detail in the context of gravity.
- Study of noncommutative quantum field theory in curved space would also be interesting.

Collaborators

- E. Harikumar and T. R. Govindarajan (INDIA)
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