Post-Newtonian expansion of gravitational waves from a test particle in circular orbits around a Kerr black hole

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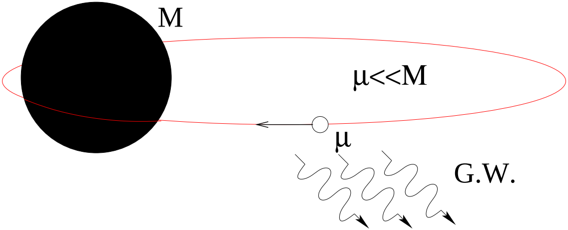
Extreme mass ratio inspirals (EMRIs)

- Gravitational waves from a compact star orbiting around a supermassive black hole in the center of a galaxy



- ⋆ One of the main targets of LISA-type space detector
  ∼10-1000 events/yr (Gair et al.,2004)
- ⋆ Masses of binary, spin, distance, position, test of general relativity and etc...

⇒ We want to compute waveforms very accurately to extract information by matched filtering

Contents

- Introduction
  - ⋆ Extreme mass ratio inspirals (EMRIs)
- Black hole perturbation
  - ⋆ Teukolsky equation
  - ⋆ Mano-Suzuki-Takasugi method
- Results
  - ⋆ 10th post-Newtonian expansion of GWs from a Kerr BH
  - ⋆ Comparison with high precision numerical results
- Summary

Gravitational waves (GWs)

- One of the predictions from general relativity
  - ⋆ New observational window of astrophysics

    binary coalescence, test of general relativity, early universe...
- Indirect detection
  - ⋆ Decay of orbital period of binary pulsar PSR 1913+16

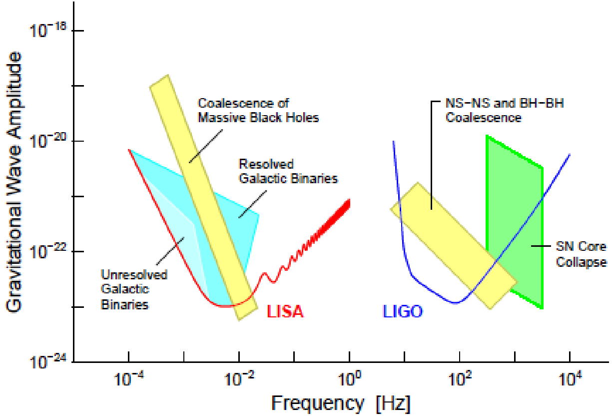
    [Hulse and Taylor (1974)]
- Toward direct detection by laser interferometer detectors
  - ⋆ Ground based
    - 1999∼ TAMA, LIGO, GEO600, VIRGO
    - 2015∼ Adv. LIGO, Adv. VIRGO, KAGRA
  - ⋆ Space based
    - 2028(?)∼ LISA-type mission, NGO (eLISA)

Frequency of GW

Frequency of GW $\sim$ (Typical time scale of the system)$^{-1}$

$$f_{\rm GW} \sim \sqrt{G\rho} \sim 10^4 \left(\frac{M_\odot}{M}\right) [\rm Hz]$$

$$\to 10^{-2}[\rm Hz] \text{ for } M = 10^6 M_\odot \text{ (ex. EMRIs)}$$



Accuracy necessary for observations of EMRIs

- Total cycle of GW $= N_{\rm GW} \sim f_{\rm GW} T_{\rm obs} \sim 10^5 - 10^6$

  ($f_{\rm GW} \sim \rm mHz$, eLISA mission time $= T_{\rm obs} \sim 2-3$ yrs?)

- For data analysis by matched filtering, error of total cycle $\Delta N_{\rm GW} \leq 1$

- Accuracy necessary for observations of EMRIs

  $$\frac{\Delta N_{\rm GW}}{N_{\rm GW}} \sim \frac{\Delta(dE/dt)}{dE/dt} \leq 10^{-5} \text{ (for circular orbits)}$$

  $\Rightarrow$ Very accurate numerical computations for circular orbits:
  $\sim 10^{-14}$ [RF and Tagoshi (2004,2005)]

- Number of templates is very large [Gair et al. (2004)]

  $\Rightarrow$ We need efficient templates : Numerical calculation is costly
  $\rightarrow$Post-Newtonian (PN) expansion

  $\Rightarrow$ One can estimate the region of validity in PN expansion by using the high precision numerical computations

Post-Newtonian works in the test particle limit

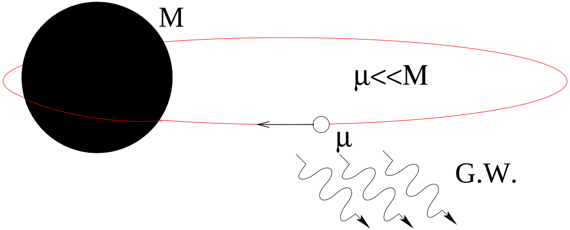
n-PN means $(v^2/c^2)^n$ correction to leading order
2.5PN means (v^5/c^5) correction to leading order

- Schwarzschild black hole (circular orbits)
  - $\ln(v)$ terms appear from 3PN (Numerical fitting) [Tagoshi and Nakamura(1994)]
  - 4PN waveforms and energy flux [Tagoshi and Sasaki(1994)]
  - 5.5PN energy flux [Tanaka et al.,(1996)]
  - 22PN waveforms and energy flux [RF(2012)] $\sim \frac{\Delta(dE/dt)}{dE/dt} \leq 10^{-5}$
- Kerr black hole (circular and equatorial orbits)
- 1.5PN waveforms [Poisson(1993)]
- 4PN energy flux [Tagoshi et al.,(1996)]
- 10PN energy flux [This work]

Black hole perturbation theory

- Teukolsky equation
- Mano-Suzuki-Takasugi method

Black hole perturbation theory



- Black hole perturbation at the first order of the mass ratio
  - $g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + h_{\alpha\beta}, \quad h_{\alpha\beta} \sim \frac{\mu}{M}$
  - $\mu/M << 1$ but v/c can be $O(1)$
- Scales in BH perturbation theory (with $c = G = 1$)
  - $M_\odot \leq \mu \leq 10^2\, M_\odot, \quad 10^5\, M_\odot \leq M \leq 10^7\, M_\odot$
  - $E \sim \mu, \quad \frac{dE}{dt} \sim h^2_{\alpha\beta} \sim \left(\frac{\mu}{M}\right)^2$
  - $T_{\rm orbit} \sim M \sim 10^2 \frac{M}{10^6 M_\odot} [{\rm s}], \quad T_{\rm radiation} = \frac{E}{dE/dt} \sim \frac{M^2}{\mu}$

$T_{\rm orbit} \ll T_{\rm radiation} \Rightarrow$ Adiabatic gravitational radiation reaction to geodesic orbits

Black hole perturbation

- Master equation for the metric perturbation
  - ⋆ Schwarzschild BH : Regge-Wheeler-Zerilli eq.
  - ⋆ Kerr BH : NOT known
- Master equation for the curvature perturbation
  - ⋆ Kerr BH : Teukolsky eq.

Teukolsky equation I

- Perturbation equation for Weyl scalar $\Psi \sim C_{\alpha\beta\gamma\delta} n^\alpha \bar{m}^\beta n^\gamma \bar{m}^\delta$

$$\begin{aligned}\Psi &= \frac{1}{2}(\ddot{h}_+ - i\ddot{h}_\times) \quad \text{for} \quad r \to \infty, \\ &= \sum_{\ell m} \int d\omega e^{-i\omega t + im\varphi} \; {}_{-2}S_{\ell m}(\theta) Z^\infty_{\ell m\omega}(r),\end{aligned}$$

- Teukolsky equation in the frequency domain [Teukolsky (1973)]

$$\Delta^2 \frac{d}{dr}\left(\frac{1}{\Delta}\frac{dZ_{\ell m\omega}}{dr}\right) - V(r) Z_{\ell m\omega} = T_{\ell m\omega},$$

$$\begin{cases} V(r) = -\frac{K^2 + 4i(r-M)K}{\Delta} + 8i\omega r + \lambda : \text{Long range potential} \\ \Delta = r^2 - 2Mr + q^2, \; K = (r^2+q^2)\omega - m\,q, \\ r = r_\pm : \text{Regular singular points}, \\ r = \infty : \text{Irregular singular point}. \end{cases}$$

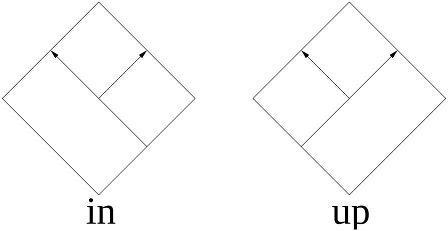
We solve the Teukolsky equation by Green function method

Teukolsky equation II

- Boundary conditions for two kinds of homogeneous solutions

$$R^{\rm in}_{lm\omega} \to \begin{cases} B^{\rm trans}_{lm\omega}\Delta^2 e^{-ikr*} & \text{for } r \to r_+, \\ r^3 B^{\rm ref}_{lm\omega} e^{i\omega r*} + r^{-1} B^{\rm inc}_{lm\omega} e^{-i\omega r*} & \text{for } r \to +\infty, \end{cases}$$

$$R^{\rm up}_{lm\omega} \to \begin{cases} C^{\rm up}_{lm\omega} e^{ikr*} + \Delta^2 C^{\rm ref}_{lm\omega} e^{-ikr*} & \text{for } r \to r_+, \\ r^3 C^{\rm trans}_{lm\omega} e^{i\omega r*} & \text{for } r \to +\infty. \end{cases}$$



Teukolsky equation II

- Boundary conditions for two kinds of homogeneous solutions

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$$R^{\rm up}_{lm\omega} \to \begin{cases} C^{\rm up}_{lm\omega} e^{ikr*} + \Delta^2 C^{\rm ref}_{lm\omega} e^{-ikr*} & \text{for } r \to r_+, \\ r^3 C^{\rm trans}_{lm\omega} e^{i\omega r*} & \text{for } r \to +\infty. \end{cases}$$

- How to compute the homogeneous solutions:
  - Transformation from Teukolsky eq. to Sasaki-Nakamura eq.
  - Mano-Suzuki-Takasugi (MST) method to solve Teukolsky eq.

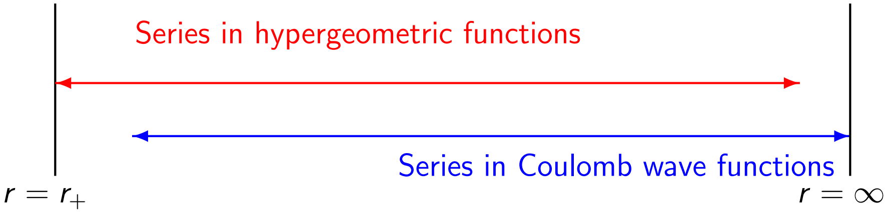
We use MST method since it is easy to perform high PN calculation

Mano-Suzuki-Takasugi method

- Mano-Suzuki-Takasugi (MST) method (1996)

$$R_{\ell m\omega}(r) \sim \sum_n a_n F_n(r), \; F_n(r) = \begin{cases} \text{Hypergeometric fn.}(r \sim r_+) \\ \text{Coulomb wave fn.}(r \sim \infty) \end{cases}$$

- Region of convergence for series expansions



Mano-Suzuki-Takasugi method

- Mano-Suzuki-Takasugi (MST) method (1996)

$$R_{\ell m\omega}(r) \sim \sum_n a_n F_n(r), \; F_n(r) = \begin{cases} \text{Hypergeometric fn.}(r \sim r_+) \\ \text{Coulomb wave fn.}(r \sim \infty) \end{cases}$$

$\Rightarrow$ Teukolsky equation is reduced to recurrence relation for a_n

- Post-Newtonian expansion of a_n up to 3PN

$$\begin{aligned} a_0 &= 1, \\ a_1 &= i\, v^3 \frac{(\ell+3)^2}{(\ell+1)\,(2\,\ell+1)} + v^6 \frac{2\,(\ell+3)^2}{(\ell+1)^2\,(2\,\ell+1)}, \\ a_2 &= -v^6 \frac{(\ell+3)^2\,(\ell+4)^2}{(\ell+1)\,(2\,\ell+1)\,(2\,\ell+3)^2}, \\ a_{-1} &= a_1(\ell \leftrightarrow -\ell-1), \; a_{-2} = a_2(\ell \leftrightarrow -\ell-1), \cdots \\ a_{|n|} &\sim O(v^{3\,|n|}). \end{aligned}$$

$\Rightarrow$ MST is powerful method for PN approx. (& numerical calc.)

Summary to compute GWs in BH perturbation

- $\mu/M << 1$ but v/c can be $O(1)$
- Circular orbit $\Rightarrow E,\ L_z$ and $\omega = m\,\Omega_\phi$
- The expansion coefficient: $a_{|n|} \sim O(v^{3\,|n|})$
- Homogeneous solutions:

$$R_{\ell m\omega}(r) \sim \sum_n a_n\, F_n(r),$$

- Weyl scalar Ψ and the energy flux:

$$\Psi = -\frac{2}{r}\sum_{\ell m} e^{-i\omega t + im\varphi}\ {}_{-2}S_{\ell m}(\theta) Z^{\infty}_{\ell m\omega}(r) \to \frac{1}{2}(\ddot{h}_+ - i\ddot{h}_\times),$$

$$\frac{dE}{dt} = \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} \frac{|Z^{\infty}_{\ell m\omega}|^2}{4\pi\omega^2} + O(\mathrm{post-Teukolsky\ \ corrections}).$$

Results

- 10th post-Newtonian expansion of GWs from a Kerr BH
- Comparison with high precision numerical results
  - ⋆ Energy flux with/without resummation
  - ⋆ Dephase during two-year inspiral

Gravitational wave energy flux at 10PN

10PN formula for circular orbits around a Kerr black hole

$$
\begin{aligned}
\frac{dE}{dt} = \left(\frac{dE}{dt}\right)_N \Bigg[& 1 + (q\text{-independent terms}) - \frac{11}{4} q v^3 + \frac{33}{16} q^2 v^4 - \frac{59}{16} q v^5 + \left(-\frac{65}{6}\pi q + \frac{611}{504} q^2\right) v^6 \\
& + \left(\frac{162035}{3888} q + \frac{65}{8}\pi q^2 - \frac{71}{24} q^3\right) v^7 + \left(-\frac{359}{14}\pi q + \frac{22667}{4536} q^2 + \frac{17}{16} q^4\right) v^8 \\
& + \left\{ \left(-\frac{9828207709}{52390800} + \frac{40939}{315} \ln 2 - \frac{43}{3}\pi^2 + \frac{6841}{105}\gamma + \frac{6841}{105}\ln v\right) q \right. \\
& \left. + \frac{8447}{672}\pi q^2 - \frac{112025}{4536} q^3 \right\} v^9 \\
& + \left\{ \frac{23605}{144}\pi q + \left(\frac{93301799461}{628689600} - \frac{27499}{420}\ln 2 + \frac{43}{4}\pi^2 - \frac{4601}{140}\gamma - \frac{4601}{140}\ln v\right) q^2 \right. \\
& \left. - \frac{45}{4}\pi q^3 + \frac{731}{126} q^4 \right\} v^{10} + \cdots + O(v^{20}) \Bigg],
\end{aligned}
$$

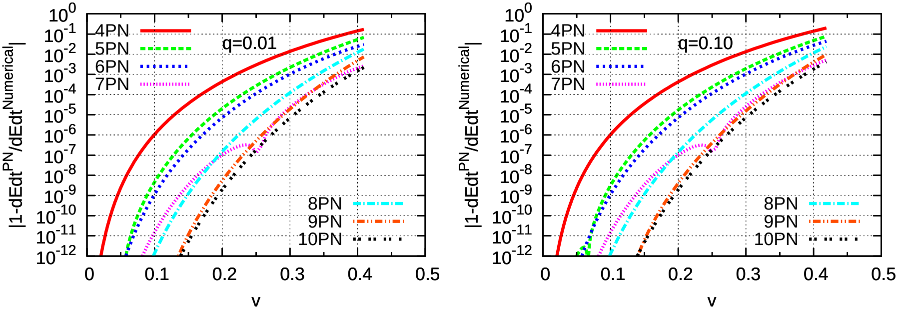
where $\left(\frac{dE}{dt}\right)_N = \frac{32}{5}\left(\frac{\mu}{M}\right)^2 v^{10}$ and spin dependent terms at $O(v^9) - O(v^{20})$ are new terms.

⋆ cf. 1.5PN expression for comparable mass binaries

$$
\frac{dE}{dt} = \left(\frac{dE}{dt}\right)_N \left[1 + \left(-\frac{1247}{336} - \frac{35}{12}\frac{\mu}{M}\right) v^2 + 4\pi v^3 + O(v^4)\right].
$$

10PN energy flux and numerical energy flux

- 4–10PN: Circular orbits around a Kerr BH ($q = 0.01,\ 0.1$)



[Numerical results for Teukolsky eq.: RF and H. Tagoshi (2004,2005)]

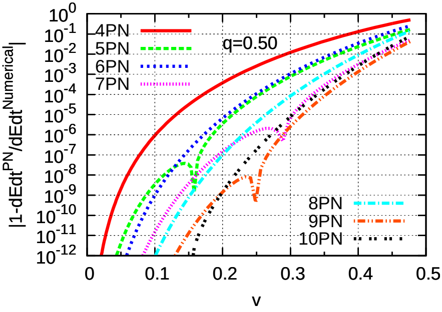
⋆ Convergence region becomes larger.

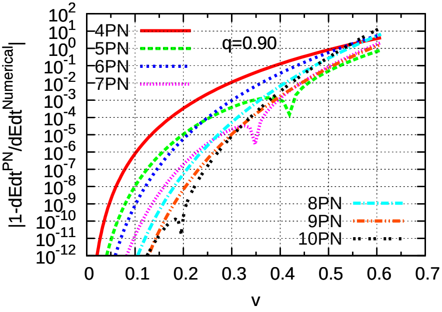
$$\frac{\Delta(dE/dt)}{dE/dt} \leq 10^{-5} \text{ for } v \leq 0.3 \ (v \leq 0.13) \text{ at 10PN (4PN)}$$

⋆ Accuracy at ISCO becomes better for higher PN order.

10PN energy flux and numerical energy flux

- 4–10PN: Circular orbits around a Kerr BH ($q = 0.5$, 0.9)





[Numerical results for Teukolsky eq.: RF and H. Tagoshi (2004,2005)]

⋆ Convergence region becomes larger.

$$\frac{\Delta(dE/dt)}{dE/dt} \leq 10^{-5} \text{ for } v \leq 0.3 \ (v \leq 0.13) \text{ at 10PN (4PN)}$$

⋆ Accuracy at ISCO does not always become better for higher PN order.

Factorized resummed waveforms

- Multipolar waveforms for $\ell = m = 2$ mode

$$h_{2,2} = -\sqrt{\frac{16\pi}{5}}\frac{2\mu\, v^2}{r} e^{-2\,i\phi}\left[1 - \frac{107}{42}v^2 + \left(2\pi + 12\,i\,\ln(\frac{v}{v_0})\right)v^3 + O(v^4)\right],$$

- Factorized resummed waveforms [Damour, Iyer and Nagar (2009)]

$$\begin{aligned} h_{\ell m} &= h_{\ell m}^{(N,\epsilon_p)}\,[1 + h_{\ell m}^{(2)} v^2 + O(v^3)] \\ &= h_{\ell m}^{(N,\epsilon_p)}\,\hat{S}_{\rm eff}^{(\epsilon_p)}\, T_{\ell m}\, e^{i\delta_{\ell m}}(\rho_{\ell m})^{\ell} \end{aligned}$$

$$\begin{cases} h_{\ell m}^{(N,\epsilon_p)} & : \text{Newtonian amplitude} \\ \hat{S}_{\rm eff}^{(\epsilon_p)} & : \text{Source factor (E or vL}_z) \\ T_{\ell m} & : \text{Leading tail factor and } \frac{\Gamma(\ell+1-2imv^3)}{\Gamma(\ell+1)} e^{m\pi v^3} \\ e^{i\delta_{\ell m}} & : \text{Supplementary phase factor} \\ h_{\ell m}^{(2)} & : -\ell + 1/2 + f(\ell, m) \\ \rho_{\ell m} & : \ell\text{−th root of residual amplitude} \end{cases}$$

Factorization itself improves the convergence of $h_{\ell m}$
$\rho_{\ell m}$ deals with ℓ-linear term of $h_{\ell m}$ at 1PN

Factorized resummed waveforms

1PN term of $\rho_{\ell m}$ is smaller than that of $h_{\ell m}$:

$$h^{(2)}_{\ell m} = -\ell + 1/2 + f(\ell, m), \quad \rho^{(2)}_{\ell m} = -1 + 1/(2\,\ell) + f(\ell, m)/\ell$$

⇒Does the convergence improve?

Factorized resummed waveforms : ρ_{22}

1PN term of $\rho_{\ell m}$ is smaller than that of $h_{\ell m}$:

$h^{(2)}_{\ell m} = -\ell + 1/2 + f(\ell, m)$, $\rho^{(2)}_{\ell m} = -1 + 1/(2\,\ell) + f(\ell, m)/\ell$

⇒Does the convergence improve?

$$\rho_{\ell m} = \left(\frac{\mid h_{\ell m} \mid}{h^{(N,\epsilon_p)}_{\ell m}\, \hat{S}^{(\epsilon_p)}_{\rm eff} \mid T_{\ell m} \mid} \right)^{1/\ell},$$

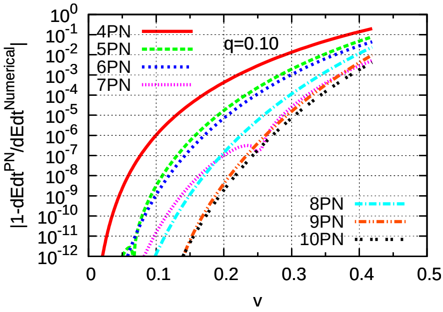
$$\rho_{2,2} = 1 - \frac{43}{42}\, v^2 - \frac{20555}{10584}\, v^4 + O(v^6),$$

$$[\text{cf.}\ \ h_{2,2} \propto 1 - \frac{107}{42}\, v^2 + \left(2\pi + 12\, i\, \ln(\frac{v}{v_0}) \right) v^3 + O(v^4)],$$

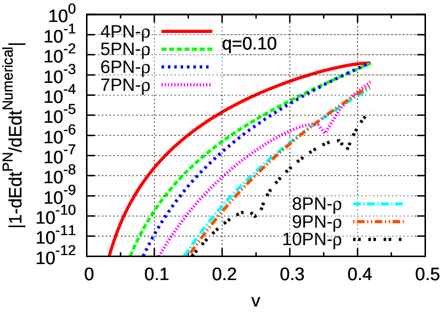
PN energy flux w/o factorized resummation

- $q = 0.1$

⋆ Taylor expansion



⋆ Factorized resummation



[Numerical results for Teukolsky eq.: RF and H. Tagoshi (2004,2005)]

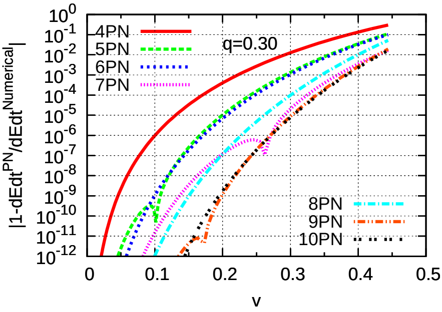
⋆ $\frac{\Delta(dE/dt)}{dE/dt} \leq 10^{-5}$ at 10PN: $v \leq 0.3$(Taylor)$\rightarrow$ $v \leq v_{\mathrm{ISCO}}$(Factorized)

⋆ Relative error at ISCO for 10PN: 10^{-3} (Taylor) $\rightarrow 10^{-5}$ (Factorized)

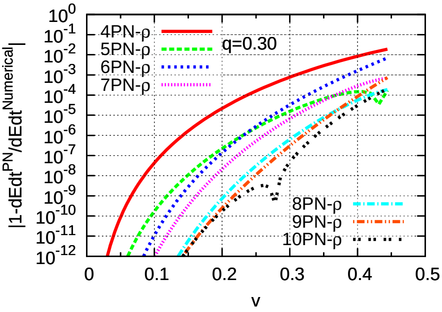
PN energy flux w/o factorized resummation

- $q = 0.3$

⋆ Taylor expansion



⋆ Factorized resummation



[Numerical results for Teukolsky eq.: RF and H. Tagoshi (2004,2005)]

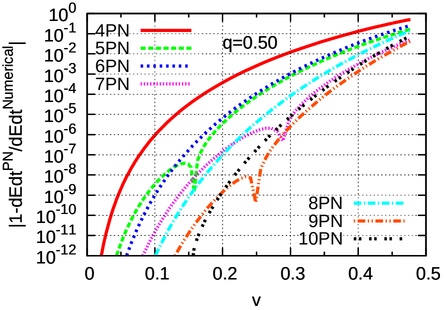
⋆ $\frac{\Delta(dE/dt)}{dE/dt} \leq 10^{-5}$ for 10PN: $v \leq 0.3$ (Taylor) $\rightarrow v \leq 0.37$ (Factorized)

⋆ Relative error at ISCO for 10PN: 10^{-2} (Taylor) $\rightarrow 10^{-4}$ (Factorized)

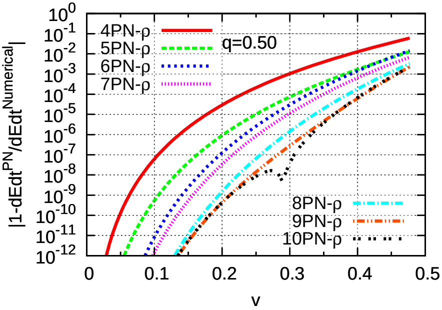
PN energy flux w/o factorized resummation

- $q = 0.5$

⋆ Taylor expansion



⋆ Factorized resummation



[Numerical results for Teukolsky eq.: RF and H. Tagoshi (2004,2005)]

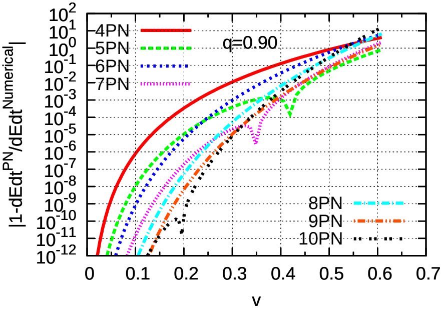
⋆ $\frac{\Delta(dE/dt)}{dE/dt} \leq 10^{-5}$ for 10PN: $v \leq 0.3$ (Taylor) $\to v \leq 0.35$ (Factorized)

⋆ Relative error at ISCO for 10PN: 10^{-1} (Taylor) $\to 10^{-3}$ (Factorized)

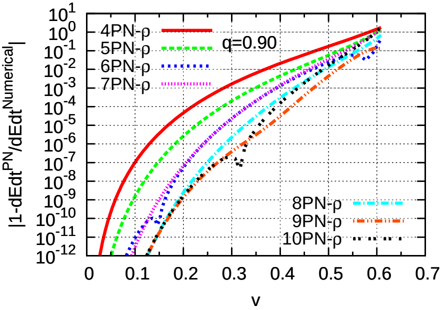
PN energy flux w/o factorized resummation

- $q = 0.9$

⋆ Taylor expansion



⋆ Factorized resummation



[Numerical results for Teukolsky eq.: RF and H. Tagoshi (2004,2005)]

⋆ $\frac{\Delta(dE/dt)}{dE/dt} \leq 10^{-5}$ for 10PN: $v \leq 0.3$ (Taylor) $\rightarrow v \leq 0.35$ (Factorized)

⋆ Relative error at ISCO for 10PN: 10^1 (Taylor) $\rightarrow O(1)$ (Factorized)

Dephase $\delta\psi_{22}$ between 10PN and numerical results

- Dephase $\delta\psi_{22}$ due to two-year inspiral

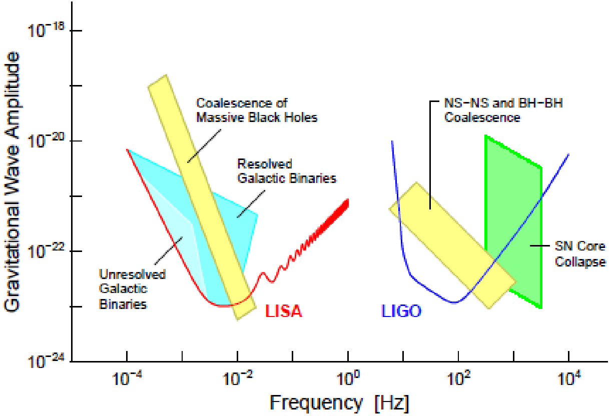
$$(h_{22} = |h_{22}| e^{i\psi_{22}}, \psi_{\ell m} \sim m \int \Omega_\phi(t) dt = m \int v(t)^3 dt)$$

⋆ Early inspiral

$0.2 \le v \le 0.25$, $\mu/M = 10^{-4}$
$(4 \times 10^{-3}\mathrm{Hz} \le f_{\rm GW} \le 10^{-2}\mathrm{Hz})$

⋆ Late inspiral

$0.3 \le v \le v_{\rm ISCO}$, $\mu/M = 10^{-5}$
$(1.8\times10^{-3}\mathrm{Hz} \le f_{\rm GW} \le 4.4\times10^{-3}\mathrm{Hz})$



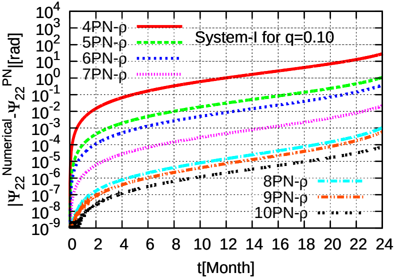
Dephase $\delta\psi_{22}$ between 10PN and numerical results

- Dephase $\delta\psi_{22}$ due to two-year inspiral for $q = 0.1$

$$(h_{22} = | h_{22} | e^{i\psi_{22}}, \psi_{\ell m} \sim m \int \Omega_\phi(t)dt = m \int v(t)^3 dt)$$

★ Early inspiral

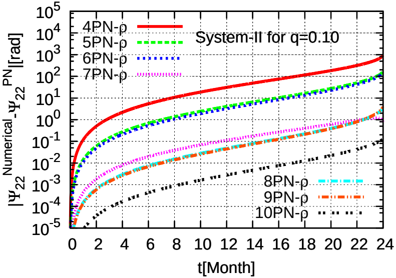
$0.2 \le v \le 0.25$, $\mu/M = 10^{-4}$
($4 \times 10^{-3}\mathrm{Hz} \le f_{\mathrm{GW}} \le 10^{-2}\mathrm{Hz}$)



- 4PN $\sim$ 10rads
- 10PN $\sim 10^{-4}$rads

★ Late inspiral

$0.3 \le v \le v_{\mathrm{ISCO}}$, $\mu/M = 10^{-5}$
($1.8\times10^{-3}\mathrm{Hz} \le f_{\mathrm{GW}} \le 4.4\times10^{-3}\mathrm{Hz}$)



- 4PN $\sim 10^3$rads
- 10PN $\sim 10^{-1}$rads

$\Rightarrow$ Late inspiral can be detected by factorized resummation

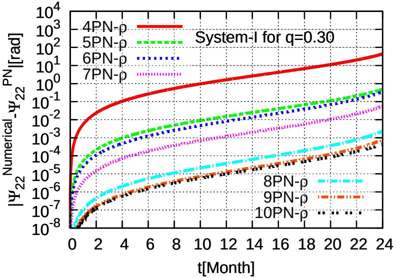
Dephase $\delta\psi_{22}$ between 10PN and numerical results

- Dephase $\delta\psi_{22}$ due to two-year inspiral for $q = 0.3$

$$(h_{22} = \mid h_{22} \mid e^{i\psi_{22}}, \psi_{\ell m} \sim m \int \Omega_\phi(t) dt = m \int v(t)^3 dt)$$

★ Early inspiral

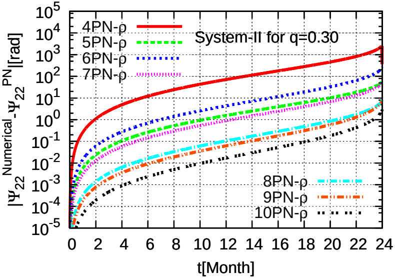
$0.2 \le v \le 0.25,\ \mu/M = 10^{-4}$
$(4 \times 10^{-3}\mathrm{Hz} \le f_{\mathrm{GW}} \le 10^{-2}\mathrm{Hz})$



- 4PN $\sim$ 10rads
- 10PN $\sim 10^{-3}$rads

★ Late inspiral

$0.3 \le v \le v_{\mathrm{ISCO}},\ \mu/M = 10^{-5}$
$(1.8\times10^{-3}\mathrm{Hz} \le f_{\mathrm{GW}} \le 4.4\times10^{-3}\mathrm{Hz})$



- 4PN $\sim 10^3$rads
- 10PN $\sim$rads

$\Rightarrow$ **Early inspiral can be detected by factorized resummation**

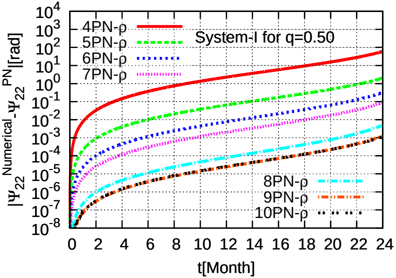
Dephase $\delta\psi_{22}$ between 10PN and numerical results

- Dephase $\delta\psi_{22}$ due to two-year inspiral for $q = 0.5$

$$(h_{22} = \mid h_{22} \mid e^{i\psi_{22}}, \psi_{\ell m} \sim m\int \Omega_\phi(t)dt = m\int v(t)^3 dt)$$

⋆ Early inspiral

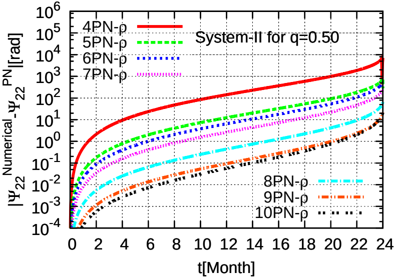
$0.2 \leq v \leq 0.25$, $\mu/M = 10^{-4}$
($4 \times 10^{-3}\mathrm{Hz} \leq f_{\mathrm{GW}} \leq 10^{-2}\mathrm{Hz}$)



- 4PN $\sim 10^2$rads
- 10PN $\sim 10^{-3}$rads

⋆ Late inspiral

$0.3 \leq v \leq v_{\mathrm{ISCO}}$, $\mu/M = 10^{-5}$
($1.8\times10^{-3}\mathrm{Hz} \leq f_{\mathrm{GW}} \leq 4.4\times10^{-3}\mathrm{Hz}$)



- 4PN $\sim 10^4$rads
- 10PN ~ 10rads

$\Rightarrow$ Early inspiral can be detected by factorized resummation

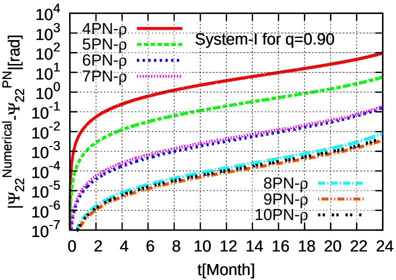
Dephase $\delta\psi_{22}$ between 10PN and numerical results

- Dephase $\delta\psi_{22}$ due to two-year inspiral for $q = 0.9$

$$(h_{22} = \mid h_{22} \mid e^{i\psi_{22}}, \psi_{\ell m} \sim m \int \Omega_\phi(t)dt = m \int v(t)^3 dt)$$

⋆ Early inspiral

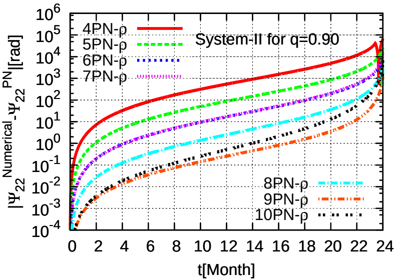
$0.2 \leq v \leq 0.25$, $\mu/M = 10^{-4}$
$(4 \times 10^{-3}\mathrm{Hz} \leq f_{\mathrm{GW}} \leq 10^{-2}\mathrm{Hz})$



- 4PN $\sim 10^2$rads
- 10PN $\sim 10^{-2}$rads

⋆ Late inspiral

$0.3 \leq v \leq v_{\mathrm{ISCO}}$, $\mu/M = 10^{-5}$
$(1.8\times10^{-3}\mathrm{Hz} \leq f_{\mathrm{GW}} \leq 4.4\times10^{-3}\mathrm{Hz})$



- 4PN $\sim 10^5$rads
- 10PN $\sim 10^3$rads

$\Rightarrow$ **Early inspiral can be detected by factorized resummation**

Summary and Future works

- Summary
  - ⋆ GWs from a particle in circular orbits around a Kerr BH
    - 10PN energy flux and waveforms
  - ⋆ Comparison with numerical results
    - Convergence region becomes larger for higher PN order
      $v \leq 0.13$ (4PN) $\rightarrow v \leq 0.3$ (10PN) for $\frac{\Delta(dE/dt)}{dE/dt} \leq 10^{-5}$
      (cf. $v \leq 0.13 \rightarrow r \geq 60M$ and $v \leq 0.3 \rightarrow r \geq 11M$)
    - Resummation can extend the region of convergence
      $v \leq 0.3$ (Taylor) $\rightarrow v \leq 0.35$ (Factorized) for $\frac{\Delta(dE/dt)}{dE/dt} \leq 10^{-5}$
      (cf. $v \leq 0.35 \rightarrow r \geq 8M$)
    - Late (Early) inspiral for $q \leq 0.1$ ($q > 0.1$) can be detected
- Future
  - ⋆ Higher PN order (analytic? numerical fitting?)
  - ⋆ Eccentric and inclined orbits (Sago and RF, in prep.)