Instabilities and rotational symmetries of higher dimensional black holes.

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In collaboration with
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and H. S. Reall - to appear soon

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Motivation

- Super-String theory is one of the most compelling candidates for a quantum theory of gravity, and contains gravity in $d = 10$.
- The AdS/CFT correspondence relates a $(d-1)-$QFT with a $d-$dimensional theory of gravity.
- In large extra dimensions scenarios, $M_{pl}^2 \sim R^n M_{phy}^{2+n}$, and M_{phy} can be of the TeV order. Possible creation of micro black holes at the LHC?
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- Given the plethora of new black holes solutions (Blackfold & Ricardo's talk), ones needs a selection criteria for possible solutions.
- Stability stands as the most natural criteria - unstable solutions should not be observed in nature!

Odd dimensional MP (equal Js) Vs $d-$Schwarzschild - 1/2

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where $ds^2_{\mathbb{CP}^N}$, $\mathbb{J} = d\mathbb{A}/2$ are the metric and Kähler form on $\mathbb{CP}^N$, $\tilde{f}(r) = 1 - r_m^{2N}/r^{2N}$ and $\hat{a}$ runs over the $\mathbb{CP}^N$ coordinates.

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- Odd dimensional MP (equal Js) is a 'deformation' of the former:

$$ds^2 = -\frac{f(r)dt^2}{H(r)} + \frac{dr^2}{f(r)} + r^2 \left\{H(r)[d\psi + \mathbb{A}_{\hat{a}}dx^{\hat{a}} - \Omega(r)dt]^2 + ds^2_{\mathbb{CP}^N}\right\},$$

where $f(r) = H(r) - r_m^{2N}/r^{2N}$, $H(r) = 1 + a^2 r_m^{2N}/r^{2N+2}$ and $\Omega(r) = a r_m^{2N}/[r^{2(N+1)}H(r)]$.

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• $\mathbb{R}_t \times SO[2(N+1)]$ symmetry.	• $\mathbb{R}_t \times U(1)_\psi \times SU(N+1)$ symmetry.
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• S^{2N+1} is an Einstein manifold.	• $\mathbb{CP}^N$ is a Kähler-Einstein manifold.
• Expand perturbations according to how they transform under coordinate transformations in S^{2N+1}.	• Expand perturbations according to how they transform under coordinate transformations in $\mathbb{CP}^N$.

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- Since we are in the scalar sector, the building block: charged Harmonics on $\mathbb{CP}^N$

$$(\mathcal{D}^2 + \lambda)\mathbb{S} = 0$$

where $\mathcal{D}_{\hat{a}} = \hat{\nabla}_{\hat{a}} - im\mathbb{A}_{\hat{a}}$, $\lambda = \ell(\ell + 2N) - m^2$, $\ell = 2\kappa + |m|$ and $\kappa \in \mathbb{N}$.

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where $\gamma_{\hat{a}\hat{b}}$ is the S^{2N+1} metric and

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$$h_{\hat{a}\hat{b}} = H_L \gamma_{\hat{a}\hat{b}}\mathbb{S} + H_T \mathbb{S}_{\hat{a}\hat{b}} + P\mathbb{J}_{\hat{a}}{}^{\hat{c}}\mathbb{J}_{\hat{b}}{}^{\hat{d}}\mathcal{D}_{\hat{c}}\mathcal{D}_{\hat{d}}\mathbb{S} + Q\mathbb{J}_{(\hat{a}}{}^{\hat{c}}\mathcal{D}_{|\hat{c}|}\mathcal{D}_{\hat{b})}\mathbb{S},$$

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Why to expect interesting physics? (Dias, Emparan, Figueras, Monteiro and JES)

- For $N \geq 2$, the following inequality holds:

$$\left(\frac{a}{r_m}\right)_\star = \frac{1}{2^{\frac{N+1}{2N}}} < \left(\frac{a}{r_m}\right)_{\text{ex}} = \frac{\sqrt{N}}{(N+1)^{\frac{N+1}{2N}}}.$$

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- If modes with $\Omega > 0$ are detected, then we undoubtedly have an unstable asym. flat black hole with compact spatial horizon.
- Harmonic expansion on $\mathbb{CP}^N$ is used to study which symmetries are broken by the perturbations. For $N = 3$ $(d = 9)$, the $\ell = 2$ harmonic breaks all the $\mathbb{CP}^3$ symmetries: perturbative black hole saturates generalisation of Hawking's rigidity theorem to higher d (Hollands, Ishibashi and Wald '06, and Isenberg and Moncrief '08).

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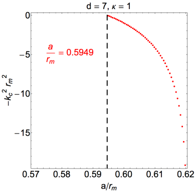

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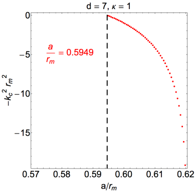

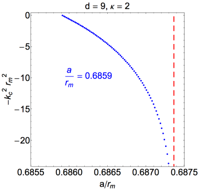

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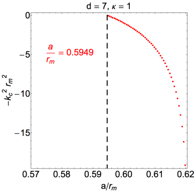

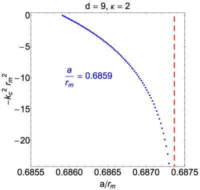

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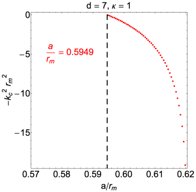

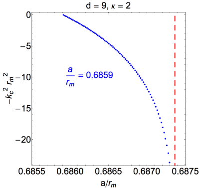

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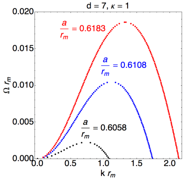

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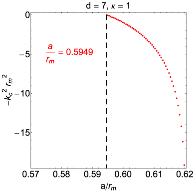
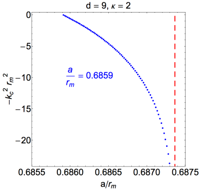
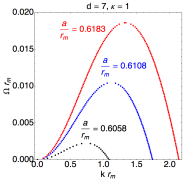
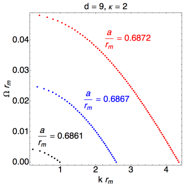




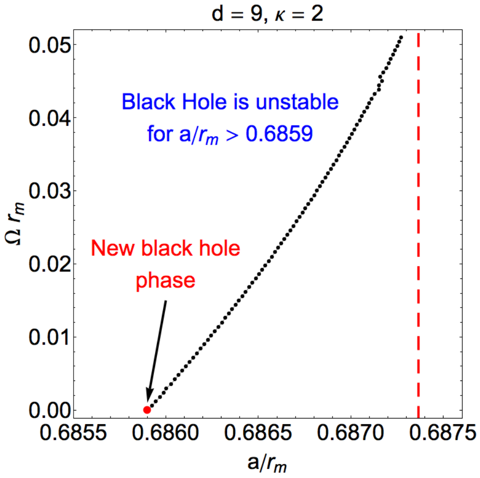
- Dispersion relation $\Omega \neq 0$:
  - $\kappa = 1$ is NOT an instability of the MP, but $\kappa = 2$ IS:





Dispersion relation of the equal angular momenta MP

- For $k = 0$, one finds



Discussion & Conclusions

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- Future directions:
  - Consider the time dependence in the single spinning MP solution (PDEs).
  - Break transverse sphere in the single spinning MP solution (saturate Hawking's rigidity theorem).
  - Consider a background MP with several angular momenta turned on.