Quasiblack holes with pressure

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1. Introduction

- Growing interest in gravitational systems with charge (Yang-Mills—Higgs, Maxwell)
- Yield QBHs: systems that are BHs, almost
  up to now without matter pressure Lemos Weinberg PRD 2004; Lemos Zanchin PRD 2005, JMP 2006, PRD 2008, 2009; Lemos Zaslavskii PRD 2007-2009
- Present here QBHs with pressure. More stable against kinetic arguments
- Einstein-Maxwell eqs.
  - Weyl (Ann. Ph. 1917)

$$ds^2 = -W^2 dt^2 + h_{ij}\, dx^i dx^j$$
$$A = \phi\, dt + A_i\, dx^i$$

try $W = W(\phi)$ in vacuo $\Rightarrow$ $W^2 = (-\varepsilon\phi + b)^2 + c$

$\varepsilon = \pm 1$ b, c constants

Perfect square $\Rightarrow$ MP 1947

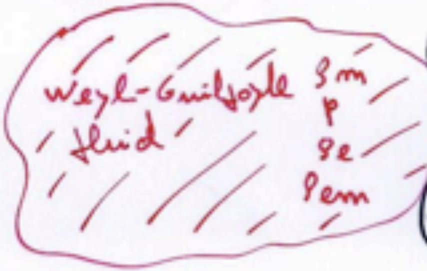
can study the relation in matter, for fluids

- Guilfoyle (GRG 1999)

generalized $W^2 = a\,(-\varepsilon\phi + b)^2 + c$

in matter Weyl-Guilfoyle fluids have pressure

2. Weyl-Guilfoyle fluids and their properties (Lemos, Zanchin PRD 2009)



$$\left\{\begin{array}{l} U_\mu = -W\delta_\mu^0 \\ A_\mu = -\phi\,\delta_\mu^0 \\ g_{00} = W^2 = a(-\varepsilon\phi + b)^2 + c \\ g_{ij} = h_{ij} \end{array}\right. \quad + \quad G_{\mu\nu} = 8\pi T_{\mu\nu} \Longrightarrow \left\{\begin{array}{l} \nabla^2 W = 4\pi W\left(\rho_m + \frac{1}{3}p + \rho_{em}\right) \\ \left(\nabla_i \frac{1}{W}\nabla^i\phi\right) = -4\pi\rho_e \\ \text{with } \rho_{em} \equiv -\frac{1}{4\pi}\frac{(\nabla_i\phi)^2}{W^2} \end{array}\right.$$

combine: $$\nabla_i\left(\frac{1}{W}\nabla^i\left(W^2 - a(-\varepsilon\phi+b)^2 + c\right)\right) = 8\pi\left(\left(\rho_m + \frac{1}{3}p + (1-a)\rho_{em}\right)W + \right.$$
$$\left. - \varepsilon a \rho_e(-\varepsilon\phi + b)\right)$$

Theorem (Lemos - Zanchin 2009)

(i) In any Einstein-Maxwell charged pressure fluid, if the metric and electric potential are such that $W^2 - a(-\varepsilon\phi+b)^2 + c$ vanishes everywhere, then the fluid quantities satisfy the constraint

$$ab\,\rho_e = \varepsilon\left[\left(\rho_m + \frac{1}{3}p\right)W + \phi\rho_e + (a-1)(\phi\rho_e - W\rho_{em})\right]$$

(ii) conversely, in any Einstein-Maxwell charged pressure fluid, if the fluid quantities are such that the equality above holds, and there is a closed surface, with no singularities, holes, or alien matter inside it, where $W^2 - a(-\varepsilon\phi+b)^2 + c$ vanishes, then it vanishes everywhere inside it.

Proof. see paper

3. Spherical solutions and QBHs with pressure (Lemos, Zanchin 2010)

(a) The solution

$$ds^2 = -W^2 dt^2 + h_{ij}\, dx^i dx^j \xrightarrow[\text{Schw. coord}]{\text{sph. symm.}} ds^2 = -B(r)\,dt^2 + A(r)\,dr^2 + r^2 d\Omega^2$$

$$W^2 = a(-\epsilon\phi + b)^2 + c \qquad a > 0$$

RN spacetime

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)dt^2 + \frac{dr^2}{1 - \frac{2m}{r} + \frac{q^2}{r^2}} + r^2 d\Omega^2$$

$$\phi = \frac{q}{r}$$

$q = m \Rightarrow$ extremal

Guilfoyle's solutions:

assumptions
$$\begin{cases} \rho_m(r) + \dfrac{Q^2(r)}{8\pi r^4} = \text{const} \\ A(r) = \dfrac{1}{1 - r^2/R^2} \quad R \text{ const} \\ c = 0 \end{cases}$$

solutions
$$B(r) = \left[\frac{2-a}{a^{1+1/a}}\left(k_0 R^2 \sqrt{1 - \frac{r^2}{R^2}} - k_1\right)\right]^{\frac{2a}{a-2}}$$

$$Q(r) = \frac{\epsilon\sqrt{a}}{2-a}\, \frac{k_0 r^3}{k_0 R^2 \sqrt{1 - \frac{r^2}{R^2}} - k_1}$$

$$8\pi p(r) = -\frac{1}{R^2} + \frac{a}{(2-a)^2}\frac{k_0^2 r^2}{\left(k_0 R^2 \sqrt{1 - \frac{r^2}{R^2}} - k_1\right)^2} + \frac{2k_0 a}{2-a}\frac{\sqrt{1 - \frac{r^2}{R^2}}}{k_0 R^2 \sqrt{1 - \frac{r^2}{R^2}} - k_1}$$

k_0, k_1 constants of integration found through matching to Reissner-Nordström solution $\qquad p(r_0) = 0$

$k_0 = k_0(m, q, r_0)$

$k_1 = k_1(m, q, r_0)$

Want $c_s^2 \leq 1 \Rightarrow 1 \leq a \leq 4/3$ physical solutions

(recall Schw. interior $c_s = \infty$!)

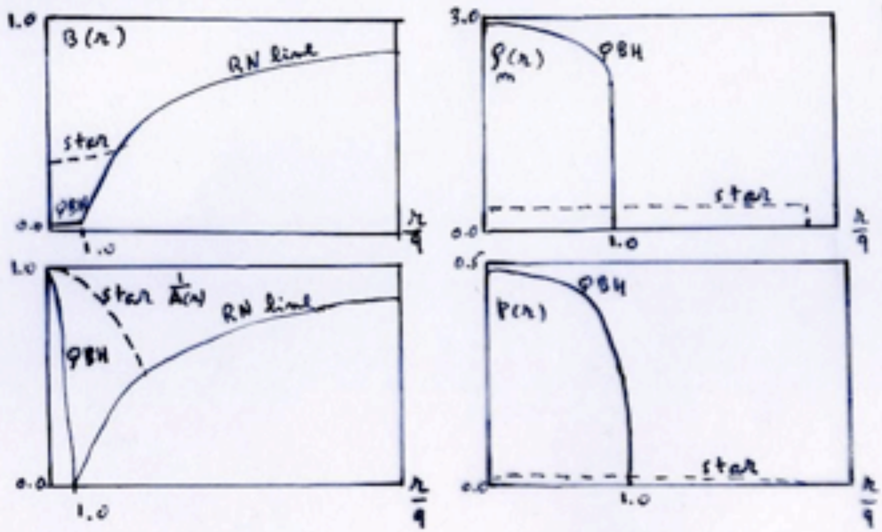
(b) QBHs object on the verge of becoming an extremal BH but distinct, it has matter inside its quasi-horizon

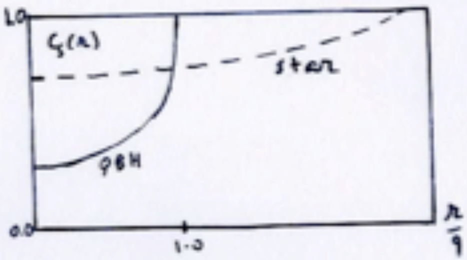
Analytical results: in the solution fix q and decrease r_0 (towards horizon)
$\Rightarrow$ m decreases (to have a solution), like main sequence stars
unlike white dwarfs.

Near quasihorizon solution is extremal (Lemos, Zaslavskii PRD 2007)

So do $r_0(1-\delta) \to m \to q$, $\delta \ll 1$
$$\Rightarrow \begin{cases} q/r_0 \simeq 1-\delta \\ m/r_0 \simeq 1-\delta \\ m/q \simeq 1 + \frac{a-1}{2a}\delta^2 \end{cases} \Rightarrow \begin{cases} B(r) \sim \delta^2 & 0 \le r \le r_0 \\ \frac{1}{A}(r_0) \sim \delta^2 & r = r_0 \end{cases}$$

Numerical results:





pressure behavior:
unlike main sequence stars
like white dwarfs

4. Conclusions

- QBHs with pressure improve stability.
  Kinematic instability argument against QBH does not hold anymore.
- $c_s < 1$ a great improvement. Plus all energy conditions hold.

- Lemos, Zaslavskii (2010) general result

  $$p^{in}(r_+) = -\frac{1}{8\pi r_+^2}$$

  If no electrom. field $\Rightarrow$ tension (can show that pressure is continuous)

  If electrom. field (objects considered above)

  $p_{matter}(r_+) = 0 \quad p_{em} = -\frac{1}{8\pi r_+^2}$ as it should