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The extremal black hole bomb

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Introduction

- Superradiant instability: Press & Teukolsky (1972-73)
  - scattering of waves in Kerr background (ergoregion)
  - for modes with $\omega < m\Omega$, scattered wave is amplified
- Black hole bomb: Press & Teukolsky (1972), Cardoso et al. (2004), etc
  - multiple scatterings induced by mirror around BH
  - can extract large amounts of energy and spin from the BH
- Natural mirrors:
  - inner boundary of accretion disks Putten (1999), Aguirre (2000)
  - massive field bound states

Introduction

- Estimates of superradiant instability growth rate (scalar field):
  - small mass limit: $\omega_I M \sim (\mu M)^9$ Detweiller (1980), Furuhashi & Nambu (2004)
  - large mass limit: $\omega_I M \sim e^{-1.84\mu M}$ Zouros & Eardley (1979)
  - numerical: $(\omega_I M)_{max} \simeq 1.5 \times 10^{-7}$ Dolan (2007)
  - recent analytical: $(\omega_I M)_{max} \simeq 1.7 \times 10^{-3}$ Hod & Hod (2009)

  } 4 orders of magnitude discrepancy!

- Phenomenology: $\mu M \sim 1 \quad \Rightarrow \quad \mu \lesssim 10^{-10}\ \mathrm{eV}$
  - pions: mechanism effective if instability develops before decay, but only for small primordial black holes $\tau_\pi = 8.4 \times 10^{-17}\ \mathrm{sec}$, $M \sim 10^{12}\ \mathrm{kg}$
  - "string axiverse": Arvanitaki et al. (2009)
    many pseudomoduli acquire small masses from non-perturbative string instantons and are much lighter than QCD axion! $\mu \gtrsim H_0 \sim 10^{-33}\ \mathrm{eV}$

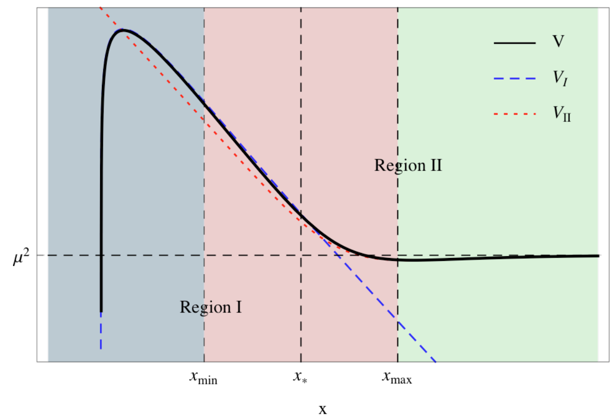
- Main goals:
  - compute superradiant spectrum of massive scalar states
  - focus on extremal BH and P-wave modes (l=m=1) → maximum growth rate
  - check discrepancy!

The massive Klein-Gordon equation

- Massive Klein-Gordon equation in curved spacetime: $(\nabla^\mu\nabla_\mu - \mu^2)\Phi = 0$
  - field decomposition: $\Phi_{lm}(t,r,\theta,\phi) = e^{-i\omega t}R_{lm}(r)S_{lm}(\theta)e^{im\phi}$
  - angular part given in terms of spheroidal harmonics
- Schrodinger-like radial eq. for extremal BH: $\frac{d^2}{dx^2}\Psi_{lm} + \left[\omega^2 - V(\omega,x)\right]\Psi_{lm} = 0$
  - change of variables: $x \equiv \frac{r-r_+}{r_+} = \frac{r}{M} - 1\ ,$
  - re-scaled function: $\Psi_{lm}(x) = xR_{lm}(x)$
  - radial potential:

$$V(\omega,x) = \underbrace{-\frac{4\varpi^2}{x^4} - \frac{8\omega\varpi}{x^3}}_{\text{ergoregion}} + \underbrace{\frac{\beta(\beta-1)}{x^2}}_{\text{a.m. barrier}} \underbrace{- \frac{2q\nu}{x} + \mu^2}_{\text{potential well}}$$

$$q \equiv \sqrt{\mu^2-\omega^2}\ , \qquad \nu \equiv \frac{2\omega^2-\mu^2}{q}\ , \qquad \varpi \equiv \omega - m\Omega\ , \qquad \beta \simeq l+1\ ,$$



Potential for l=m=1 and μM=0.4

- Boundary conditions:

$$\Psi_{lm}(x) \to \begin{cases} xe^{\frac{2i\varpi}{x}}, & x \to 0 \\ e^{-qx}, & x \to +\infty \end{cases}\ .$$

"ingoing wave" ($x \to 0$)

bound state ($x \to +\infty$)

Functional matching

- Solutions in terms of confluent hypergeometric functions:

$$\begin{aligned}\Psi_I(x) &= A_I e^{\frac{2i\varpi}{x}} x^\beta U(1-\beta-2i\omega,\ 2-2\beta,\ -4i\varpi/x)\ , \qquad & x \ll (2q)^{-1}\\ \Psi_{II}(x) &= A_{II} e^{-qx} x^\beta U(\beta-\nu,\ 2\beta,\ 2qx)\ , \qquad & x \gg -4\varpi\end{aligned}$$

- Approximate forms in overlap region: $-4\varpi \ll x \ll (2q)^{-1}$ Abramowitz & Stegun (1964)

$$\begin{aligned}\Psi_I(x) &= -A_I \frac{\pi}{\sin(2\pi\beta)}\left[\frac{x^\beta}{\Gamma[\beta-2i\omega]\Gamma[2-2\beta]} - \frac{(-4i\varpi)^{2\beta-1}x^{1-\beta}}{\Gamma[1-\beta-2i\omega]\Gamma[2\beta]}\right]\ ,\\ \Psi_{II}(x) &= A_{II} \frac{\pi}{\sin(2\pi\beta)}\left[\frac{x^\beta}{\Gamma[1-\beta-\nu]\Gamma[2\beta]} - \frac{(2q)^{1-2\beta}x^{1-\beta}}{\Gamma[\beta-\nu]\Gamma[2-2\beta]}\right]\end{aligned}$$

Matching possible for:
$-8\varpi q < 1$

- Matching condition:

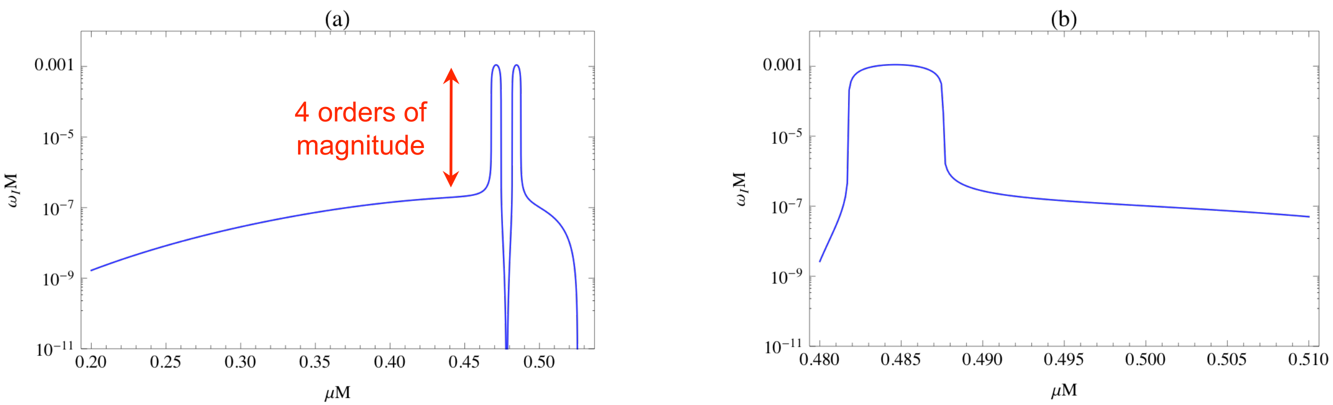
$$\frac{\Gamma[1-\beta-\nu]}{\Gamma[\beta-\nu]} = (-8i\varpi q)^{2\beta-1}\frac{\Gamma[\beta-2i\omega]}{\Gamma[1-\beta-2i\omega]}\left(\frac{\Gamma[2-2\beta]}{\Gamma[2\beta]}\right)^2$$

- Hydrogen-like spectrum:

$$\beta-\nu=-n \ \Rightarrow \qquad \omega \approx \mu\left(1-\frac{(\mu M)^2}{2(l+1+n)^2}\right)$$

Functional matching

- Numerical solution of matching condition:
  - smooth increase for $\mu M \ll 1$ and sharp decrease close to endpoint at $\omega \sim m\Omega$
  - peak-like structures close to pole in $\Gamma[2-2\beta]$



- Solution in vicinity of poles: Abramowitz & Stegun (1964)

$$\psi_I(x) = A_I\Big[-\frac{(4i\varpi)^{2\beta-1}}{\Gamma[2\beta]\Gamma[1-\beta-2i\omega]}\big[\log(-4i\varpi)-\log x+\psi(\beta-2i\omega)+\gamma-\psi(2\beta)\big]x^{1-\beta}+\frac{\Gamma[2\beta-1]}{\Gamma[\beta-2i\omega]}x^{\beta}\Big]$$

$$\psi_{II}(x) = A_{II}\Big[\frac{(-1)^{2\beta}}{\Gamma[2\beta]\Gamma[1-\beta-\nu]}\big[\log(2q)+\log x+\psi(\beta-\nu)+\gamma-\psi(2\beta)\big]x^{\beta}+\frac{\Gamma[2\beta-1]}{\Gamma[\beta-2i\omega]}(2q)^{1-2\beta}x^{1-\beta}\Big]$$

No matching possible in vicinity of poles!

Point matching

- Alternative: match functions and derivatives at a single point:
  - eliminate need for approximations
  - no exact prescription for choosing the match point

- Match point choice:
  - expect small variation within overlap region
  - geometric mean of boundaries gives small and equal arguments for both functions:

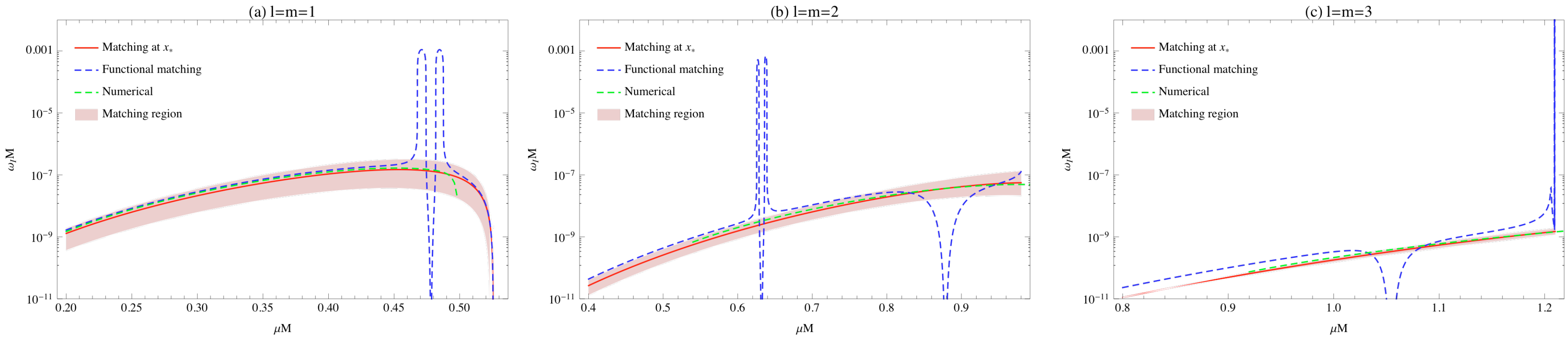
$$x_* = \sqrt{\frac{-2\varpi}{q}}$$

- Matching condition:

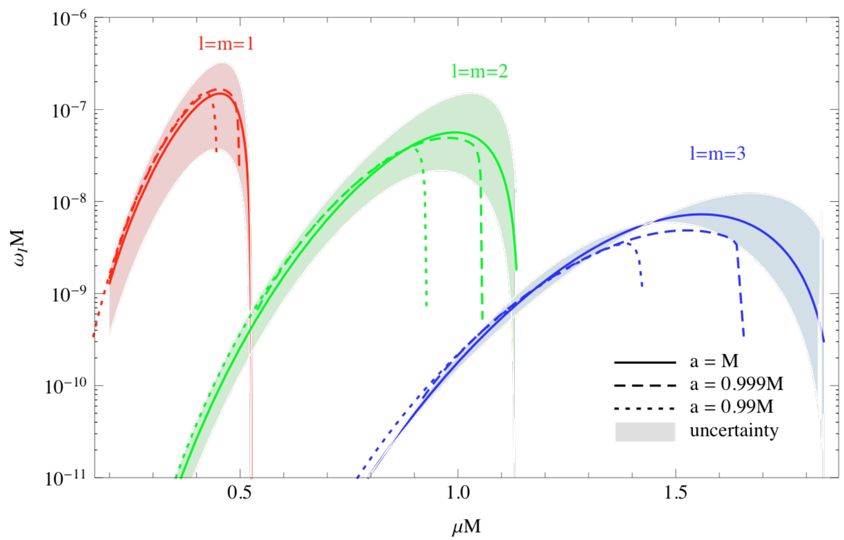
$$1 + 2(1-\beta-2i\omega)\frac{U(2-\beta-2i\omega, 3-2\beta, iz_*)}{U(1-\beta-2i\omega, 2-2\beta, iz_*)} = i\left(1 + 2(\beta-\nu)\frac{U(\beta-\nu+1, 2\beta+1, z_*)}{U(\beta-\nu, 2\beta, z_*)}\right) , \qquad z_* = \sqrt{-8\varpi q} < 1$$

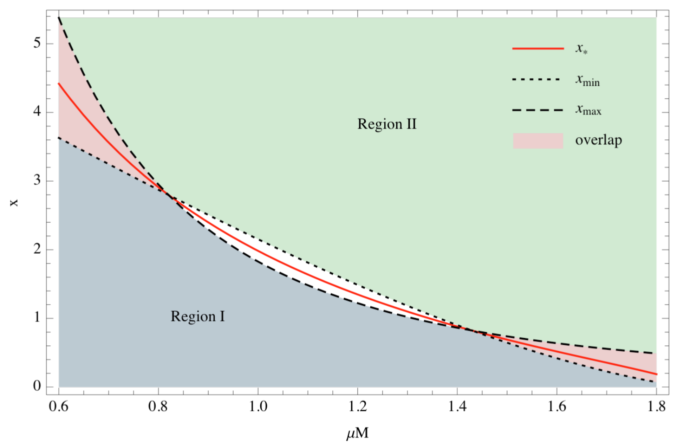
Vary the match point within the overlap region to obtain effective uncertainty

Point matching results



Agreement with numerical analysis of Dolan (2007)!





No overlap region for some masses in l=m=3 case!

Summary

- Functional matching:
  - exhibits unphysical poles in matching condition
  - cannot be used in vicinity of poles
  - unreliable for higher multipoles

- Point matching:
  - growth rate is smooth function of scalar mass (no poles)
  - good agreement with numerical results except close to endpoint
  - uncertainty due to choice of match point
  - good approximation even in absence of matching region

Maximum growth rate (l=m=1):

$$\omega_I M \simeq 1.49 \times 10^{-7}$$

$$\mu M \simeq 0.454$$

- Future work:
  - extension to non-extremal case (better comparison with numerical results and more realistic)
  - Phenomenology (string axions): gravity waves, axion-photon conversion, etc.