Phase diagram for non-axisymmetric plasma balls and black holes

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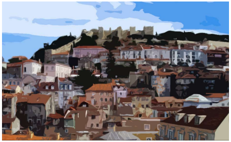


based on arxiv:0910.0020, JHEP (in press)
in collaboration with Vitor Cardoso and Óscar J. C. Dias

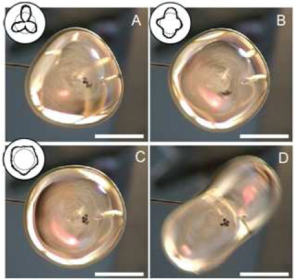
Context And Goal

- The suggestive 'membrane paradigm' [Thorne, Macdonald, Price '86; Cardoso, Dias, Gualtieri '07] hints at a connection between black holes (BH) and hydrodynamics: horizons are associated with membranes with dissipative properties (electrical resistivity, shear viscosity, etc.).
- The fluid-gravity correspondence [Bhattacharyya, Hubeny, Minwalla, Rangamani '07] provides a *precise* duality wherein the dissipative fluid lives on the boundary of AdS in which the BH is placed.
- In this talk we will be interested in fluid/plasma configurations in 2+1 dims, with the aim of determining the phase structure of BHs in 4+1 dims (in some specific background).

Context And Goal

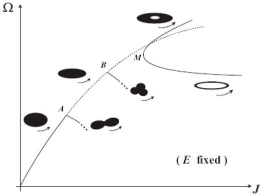
- Gauge theories displaying a confinement/deconfinement phase transition can, at temperatures near the critical temperature $\mathcal{T}_c$, present regions of (deconfined) plasma immersed in a 'sea' of confined vacuum. [Aharony, Minwalla, Wiseman '05]
- Known stationary plasma configurations include plasma balls and plasma rings (both thin and fat). [Lahiri, Minwalla '07]
- What happens if we spin up these plasma lumps?

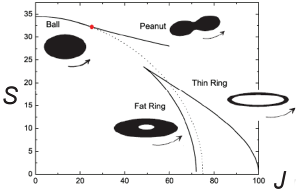
In the non-relativistic regime, droplets have been observed to acquire lobes when spinning above a critical rotation. [Hill, Eaves '08]



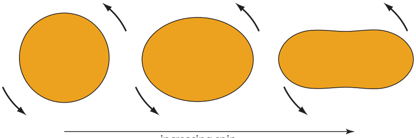
Context And Goal

- The phase diagram (at fixed energy) for axisymmetric rotating lumps of plasma in 3d is known. [Lahiri, Minwalla '07]





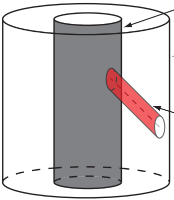
- Bifurcation of plasma balls to m-lobed plasmas has been studied recently by looking for an instability. [Cardoso, Dias '09]
  Above a critical rotation plasma balls become unstable to non-axisymmetric perturbations.



Fluid Dynamics From Gravity [Bhattacharyya, Hubeny, Minwalla, Rangamani '07]

- Consider a boosted black brane in AdS_{d+1} (in Eddington-Finkelstein coords), parametrized by temperature $b \propto \mathcal{T}$ and boosts β_i.
- Next, allow the parameters to vary along the boundary coordinates,

$$b = b(x^\mu) \quad , \quad \beta_i = \beta_i(x^\mu)$$



- Metric with non-constant $\{b, \beta_i\}$ is generically *not* a solution of Einstein's eqs.
- But if variation is slow (with lengthscale $\sim L$) then it is a good approximation to a solution and we may consider a perturbative expansion:

$$g = g^{(0)}(b, \beta_i) + \varepsilon g^{(1)}(b, \beta_i) + \varepsilon^2 g^{(2)}(b, \beta_i) + \mathcal{O}(\varepsilon^3)$$

Each power of ε corresponds to a suppression by $\sim 1/(\mathcal{T} L)$.

- Einstein's eqs can now be solved perturbatively to determine the bulk solution. At each order n of perturbation, a subset of Einstein's equations are constraint equations — these turn out to be the boundary fluid dynamics equations:

$$\nabla_\mu T^{\mu\nu}_{(n)} = 0$$

The Boundary Stress Tensor

- Once the bulk solution is known one can compute the boundary stress tensor in the vein of AdS/CFT, following [Balasubramanian, Kraus '99].
- The result, at a given order of perturbation, determines the transport coefficients at that order.

Example

For the $\mathcal{N} = 4$, $SU(N)$ SYM (conformal) fluid in d dimensions,

$$T^{\mu\nu} = \underbrace{\alpha \mathcal{T}^d (g^{\mu\nu} + d u^\mu u^\nu)}_{\text{ideal fluid } (0^{th} \text{ order})} - \underbrace{2\,\eta\,\sigma^{\mu\nu}}_{\text{dissipative part } (1^{st} \text{ order})} + \ldots$$

where $\alpha = \frac{\pi^d}{16\pi G_N^{(d+1)}}$ and the shear viscosity is $\eta = \frac{\pi}{8} N^2 \mathcal{T}^3$. [Policastro, Son, Starinets '01]

$u^\mu \longrightarrow$ velocity field of the fluid; $\sigma^{\mu\nu} \longrightarrow$ shear tensor ($\sim \nabla u$).

- The perturbative expansion in the bulk is in correspondence with the derivative expansion of the boundary stress tensor.

Approaching a Gravity/QCD Duality

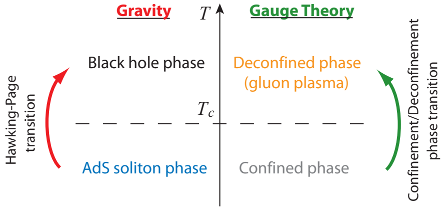
- The simplest instance of gauge/gravity duality relates

  IIB string theory on $AdS_5 \times S^5 \quad \longleftrightarrow \quad 4d\ \mathcal{N} = 4$ SU(N) SYM

  The latter (a) is conformal, (b) is SUSY, (c) does not exhibit confinement. Very different from QCD...

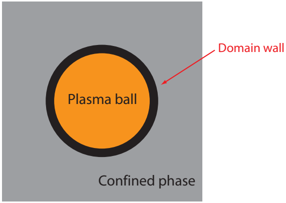
- However, a Scherk-Schwarz (SS) compactification on a S^1 of a 4d CFT is non-conformal, non-SUSY and exhibits a confinement/deconfinement phase transition at some temperature $\mathcal{T}_c$.
  Low energy dynamics is effectively 3d.

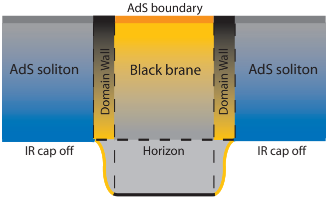
- There are 2 candidate bulk geometries dual to the boundary field theory.
  The two geometries compete to minimize the free energy.



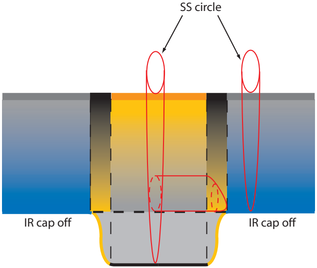
Plasma Lumps

- Near $\mathcal{T}_c$ both phases can coexist in different regions of spacetime, separated by domain walls. On the boundary, this corresponds to a bubble of deconfined plasma immersed in a confined phase.



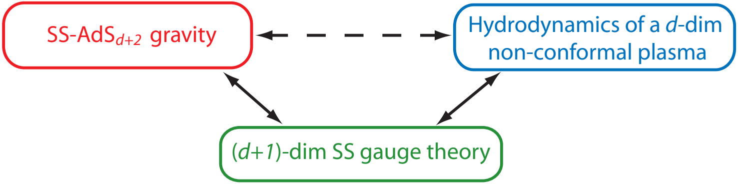


The surface tension of the domain wall is $\sigma \sim \pi^2 N^2 \mathcal{T}_c^2$. [Aharony, Minwalla, Wiseman '05]



Dual Theories And The Equation Of State

- The complete duality chart is as follows:



- The SS-AdS$_5$ system is dual to 3d fluid dynamics with an equation of state describing the SS plasma:

$$P = \frac{\rho - 4\rho_0}{3}\,, \qquad s = 4\alpha^{\frac{1}{4}}\left(\frac{\rho-\rho_0}{3}\right)^{\frac{3}{4}}\,, \qquad \mathcal{T} = \left(\frac{\rho-\rho_0}{3\,\alpha}\right)^{\frac{1}{4}}$$

with $\alpha = \frac{\pi^2 N^2}{8\mathcal{T}_c}$ and $\rho_0 = \alpha\mathcal{T}_c^4$ being constants. [Lahiri, Minwalla '07]

Stationarity => Rigid Rotation

- Consider a fluid in a $3d$ Minkowski background with coordinates $\{t, r, \phi\}$.

$$\text{stationary configuration} \quad \Rightarrow \quad \text{no dissipation}$$

It can be shown that the velocity field must be a linear combination of the background Killing vector fields $\xi = \partial_t$ and $\chi = \partial_\phi$: [Caldarelli, Dias, Emparan, Klemm '08]

$$u = \gamma\,(\xi + \Omega\chi) = \frac{1}{\sqrt{1 - r^2\Omega^2}}\,(\partial_t + \Omega\partial_\phi)$$

i.e., the plasma must be rigidly rotating, with angular velocity Ω.

- The Young-Laplace equation (surface term of $\nabla_\mu T^{\mu\nu} = 0$) reduces to

$$P_< - P_> = \sigma K$$

where K is the trace of the extrinsic curvature of the plasma's surface.

- For the SS plasma in equilibrium the pressure is determined by the equation of state:

$$P = \frac{\rho_*}{3}\,\gamma^4 - \rho_0$$

where $\rho_* \equiv 3\alpha T^4$ is a constant.

Plasma Balls And Plasma Rings

- Plasma balls have a single axisymmetric outer surface at $r = R_o$ and $P_> = 0$. Plasma rings have also an axisymmetric inner surface at $r = R_i$ (where $P_< = 0$).
- It is convenient to use dimensionless variables:

$$\widetilde{\Omega} = \frac{\sigma\Omega}{\rho_0}, \qquad \tilde{r} = \frac{\rho_0 r}{\sigma}, \qquad v = \Omega r = \widetilde{\Omega}\tilde{r}, \qquad k = \frac{\rho_*}{3\rho_0}$$

- For plasma balls, the Young-Laplace equation reduces to the condition:

$$\left(1 + \frac{\widetilde{\Omega}}{v_o}\right)\left(1 - v_o^2\right)^2 = k, \quad \text{with} \quad 0 \leq v_o \leq 1$$

- Similarly, plasma rings must satisfy

$$\left(1 + \frac{\widetilde{\Omega}}{v_o}\right)\left(1 - v_o^2\right)^2 = \left(1 - \frac{\widetilde{\Omega}}{v_i}\right)\left(1 - v_i^2\right)^2$$

  Thus, plasma rings exist only for large enough rotations, $v_o \geq v_o^*$.
- There are two families of plasma rings: – the fat ring ($\widetilde{\Omega} \leq v_i \leq v_i^*$)
  – the thin ring ($v_i^* \leq v_i \leq 1$)

  At $v_i = v_i^*$ the two families meet at a regular solution.

Conserved Charges

- Introduce dimensionless thermodynamic quantities:

$$\widetilde{E} = \frac{\rho_0 E}{\pi\sigma^2}\,, \quad \widetilde{J} = \frac{\rho_0^2 J}{\pi\sigma^3}\,, \quad \widetilde{S} = \frac{\rho_0^{5/4} S}{\pi\alpha^{1/4}\sigma^2}\,, \quad \widetilde{T} = T\left(\frac{\alpha}{\rho_0}\right)^{1/4}$$

- The energy, angular momentum and entropy of plasma rings are given by

$$\begin{aligned} \widetilde{E} &= \frac{4(v_o^2 - v_i^2) - (v_o^4 - v_i^4) + 5\widetilde{\Omega}(v_o + v_i) - \widetilde{\Omega}(v_o^3 + v_i^3)}{\widetilde{\Omega}^2} \\ \widetilde{J} &= \frac{2(v_o^4 - v_i^4) + 2\widetilde{\Omega}(v_o^3 + v_i^3)}{\widetilde{\Omega}^3} \\ \widetilde{S} &= \frac{4}{\widetilde{\Omega}^2}\left[v_o^2\sqrt{1 - v_o^2}\left(1 + \frac{\widetilde{\Omega}}{v_o}\right)^{3/4} - v_i^2\sqrt{1 - v_i^2}\left(1 - \frac{\widetilde{\Omega}}{v_i}\right)^{3/4}\right] \end{aligned}$$

- For plasma balls just set $v_i = 0$.

The Profile Equation

- Ansatz for the surface of the plasma adapted to rigid rotation:

$$f(t, r, \phi) \equiv r - R(t - \phi/\Omega) = 0$$

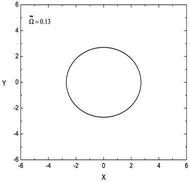
- Introduce dimensionless quantities:

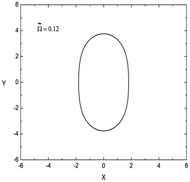
$$\psi = \phi - \Omega t\,, \qquad v_\text{o}(\psi) = \Omega R(x)\,, \qquad v_\text{o}'(\psi) = R'(x)$$

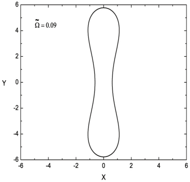
Young-Laplace determines the equation for the profile of an *m*-lobed plasma:

$$\frac{v_\text{o} v_\text{o}''(1 - v_\text{o}^2) - v_\text{o}'^2(2 - v_\text{o}^2) - v_\text{o}^2}{\left[v_\text{o}^2 + v_\text{o}'^2(1 - v_\text{o}^2)\right]^{3/2}} + \frac{1}{\widetilde{\Omega}}\left[k(1 - v_\text{o}^2)^{-2} - 1\right] = 0$$

- The profile equation can be solved numerically:





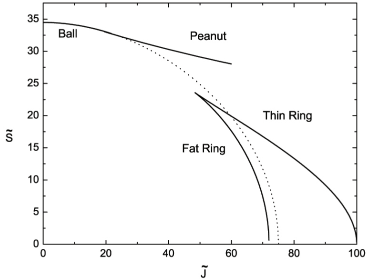


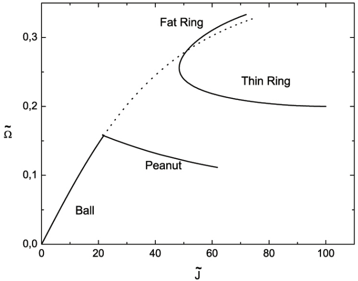
The Plasma Peanut Branch In The Phase Diagram

- Given a profile $v_o(\psi)$, the configuration's charges can be found by certain integrals over ψ, e.g.

$$\widetilde{E} = \frac{1}{2\pi\widetilde{\Omega}^2}\int_0^{2\pi} d\psi \left[v_o^2 + k\,\frac{v_o^2(3-v_o^2)}{(1-v_o^2)^2} + 2\widetilde{\Omega}\,\frac{v_o^2+v_o'^2}{\sqrt{v_o^2+v_o'^2(1-v_o^2)}} \right]$$

- Focus on $m = 2$ lobes, i.e., configurations with periodicity $\psi \sim \psi + \pi$. These are plasma peanuts.
- Fixing the energy ($\widetilde{E} = 40$) one determines the phase diagrams $\widetilde{S}(\widetilde{J})$, $\widetilde{\Omega}(\widetilde{J})$:





Lobed Plasmas As Perturbations Of Plasma Balls

- Consider a small non-axisymmetric perturbation of the plasma ball:

$$v_o(\psi) = \widehat{v}_o \left[1 + \varepsilon\, \nu(\psi) + \mathcal{O}(\varepsilon^2)\right], \qquad \widetilde{\Omega} = \widehat{\Omega} + \varepsilon\omega + \mathcal{O}(\varepsilon^2), \qquad k = \widehat{k} + \varepsilon\kappa + \mathcal{O}(\varepsilon^2)$$

- Next, linearize the Young-Laplace equation. The solution becomes oscillatory,

$$\nu'' + m^2\nu = \Delta \qquad \text{(we are interested in } m = 2\text{)}$$

- The conserved charges may also be computed perturbatively:

$$\widetilde{E} = \widetilde{E}_{ball} + O(\varepsilon), \qquad \widetilde{J} = \widetilde{J}_{ball} + O(\varepsilon), \qquad \widetilde{S} = \widetilde{S}_{ball} + O(\varepsilon)$$

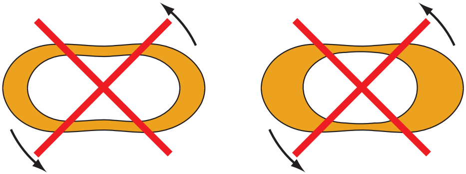
- The slope, at the bifurcation point, of the plasma peanut curve is

$$\left(\frac{\partial \widetilde{S}}{\partial \widetilde{J}}\right)^{\text{peanut}}_{\widetilde{E}} = \lim_{\varepsilon\to 0} \frac{\left(\frac{\partial \widetilde{S}}{\partial \varepsilon}\right)_{\widetilde{E}}}{\left(\frac{\partial \widetilde{J}}{\partial \varepsilon}\right)_{\widetilde{E}}} = -\frac{4\widehat{v}_o^3}{(3-4\widehat{v}_o^2)^{1/4}\left[(1-\widehat{v}_o^2)(3-11\widehat{v}_o^2+4\widehat{v}_o^4)\right]^{3/4}}$$

- At the bifurcation point, $\widehat{v}_o \simeq 0.378$ ($\Longleftrightarrow \widehat{\Omega} \simeq 0.14$), we obtain $\partial\widetilde{S}/\partial\widetilde{J} \simeq -0.142$. Numerically we find $\partial\widetilde{S}/\partial\widetilde{J} \sim -0.149$.

What About Plasma Rings With Lobes?

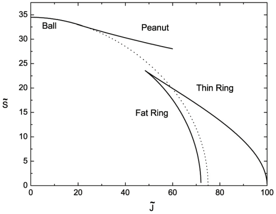
- Is it possible to have *m*-lobed plasma rings?
- A similar analysis excludes this possibility (the plasma ring condition can only be met for $v_{\rm i} \geq v_{\rm o}$).

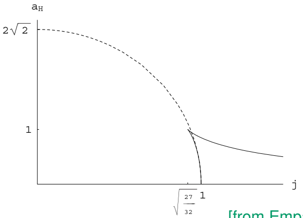


- A lobed plasma ring branch emerging perturbatively from the axisymmetric plasma rings is ruled out.
- Plasma rings can accommodate higher angular momentum by growing larger and thinner. Plasma balls have to break axial symmetry to keep energy fixed.

Discussion

- The phase diagram for stationary plasma configurations in 3d yields the phase structure of the dual SS-AdS_5 BHs. Nevertheless, there are striking similarities with the phase diagram for 5d stationary, *asymptotically flat* BHs.





[from Emparan, Reall '08]

- *m*-lobed plasma balls are dual to rotating *m*-lobed BHs.
  These cannot be stationary – they have a quadrupole moment and so radiate away their lobed deformations. What's going on?
- Presently, the fluid-gravity correspondence does not capture gravitational interactions and radiation.
  Rotating *m*-lobed BHs dual to *m*-lobed plasma balls are necessarily long-lived.

Open Problems

- It would be desirable to improve the fluid-gravity correspondence in order to accommodate gravitational interactions and gravitational radiation.

- Given the similarities between the phase diagram of SS AdS and asymptotically flat BHs it is natural to ask:

  Is the m-lobed instability also present in Myers-Perry(–AdS) black holes?

- Fluids in $d > 3$ dimensions feature pinched plasma balls — these are in correspondence with the ultra-spinning instability of black holes in $D > 5$ dimensions. But for $d > 3$ the m-lobed instability is also present.

  Which one becomes active first?

  One can argue that it is the m-lobed instability that appears at lower rotations. It may be possible to verify this, taking into account the recent results of [Dias, Figueras, Monteiro, Santos, Emparan '09].

Thank you