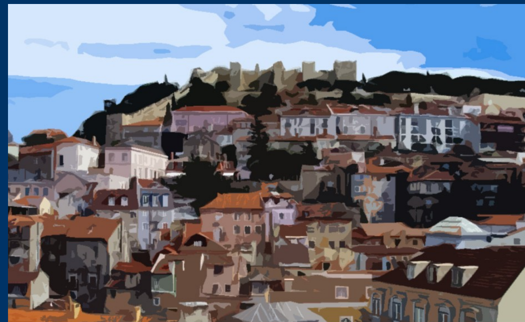

II Workshop on black holes

Lisbon, 21-22 Dec 2009

Charged dilatonic black holes

Phase transitions and holographic duals

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A 15' presentation on:

- Dilatonic Einstein-Maxwell theory in AdS
- Instability of RN-AdS and phase transitions
- Charged dilatonic black holes
- AdS/CFT: The dual field theory
- Electrical conductivity in a strongly coupled field theory

Our model: Dilatonic Einstein-Maxwell in AdS

$$\mathcal{L} = R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{f(\phi)}{4}F^2$$

Nonminimal coupling between the dilaton and the Maxwell field

$$V(\phi) = -\frac{6}{L^2} + \frac{m^2}{2}\phi^2 + \mathcal{O}(\phi^3)\,, \qquad f(\phi) = 1 + \frac{\alpha}{2}\phi^2 + \mathcal{O}(\phi^3)$$

- **RN-AdS is solution**
- **Quite well motivated model**

Is RN-AdS the only static BH solution?

Instability of RN-AdS.

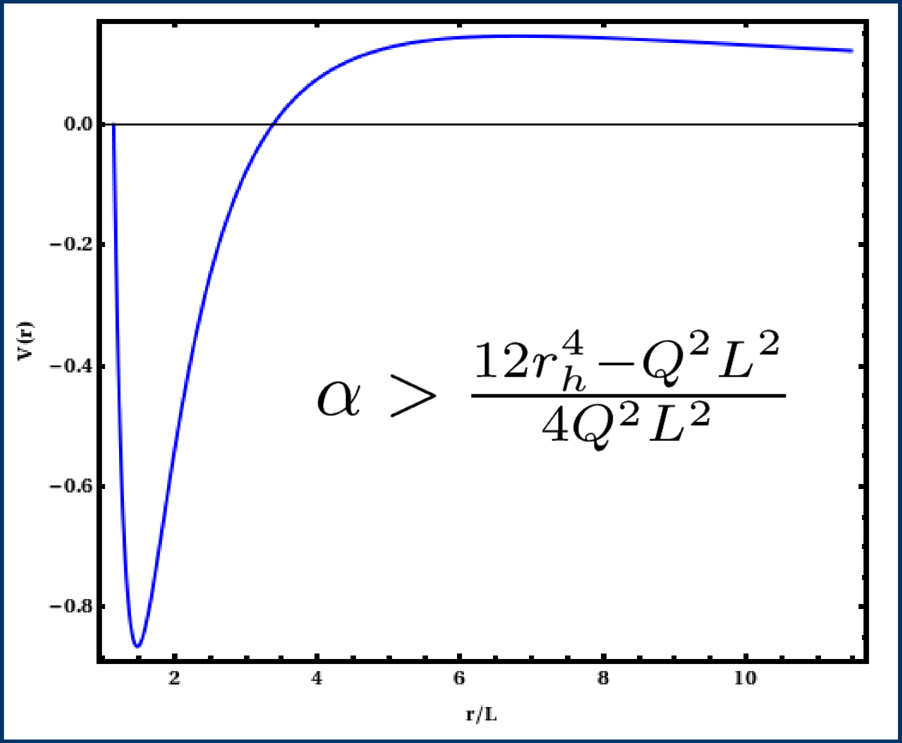
- **Scalar linear perturbations:** $\phi = \sum_{l,m} R(r) Y_{lm}(\theta,\varphi) e^{-i\omega t}$
- **Can be recast into a Schroedinger-like equation:**

$$\frac{d^2\Psi(z)}{dz^2} + \left[\omega^2 - V_l(r) - m_{\rm eff}^2(r)\right]\Psi(z) = 0$$

- **Effective mass:**

$$m_{\rm eff}(r)^2 = m^2 - \alpha\frac{Q^2}{2r^4}$$

Possible source of instability



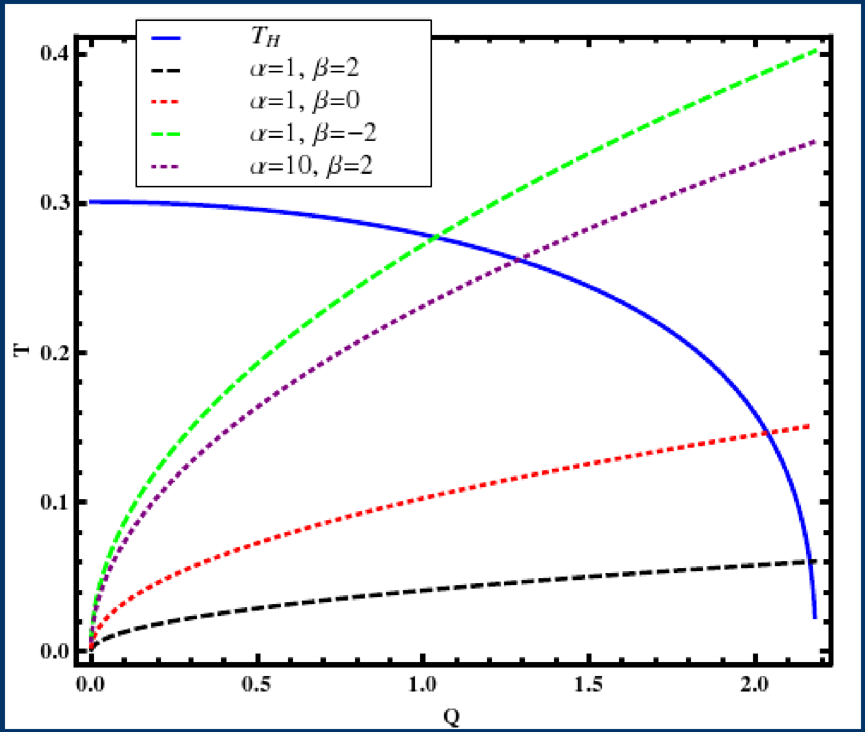
$$\alpha > \frac{12r_h^4 - Q^2L^2}{4Q^2L^2}$$

Instability of RN-AdS. Naïve argument

- In AdS a scalar perturbation produces an instability if

$$m^2_{\text{eff}} \ll -\frac{9}{4L^2}$$

Breitenlohner-Freedman bound



Instability below a critical temperature

$$T \ll T_c(\alpha, m^2)$$

also confirmed by

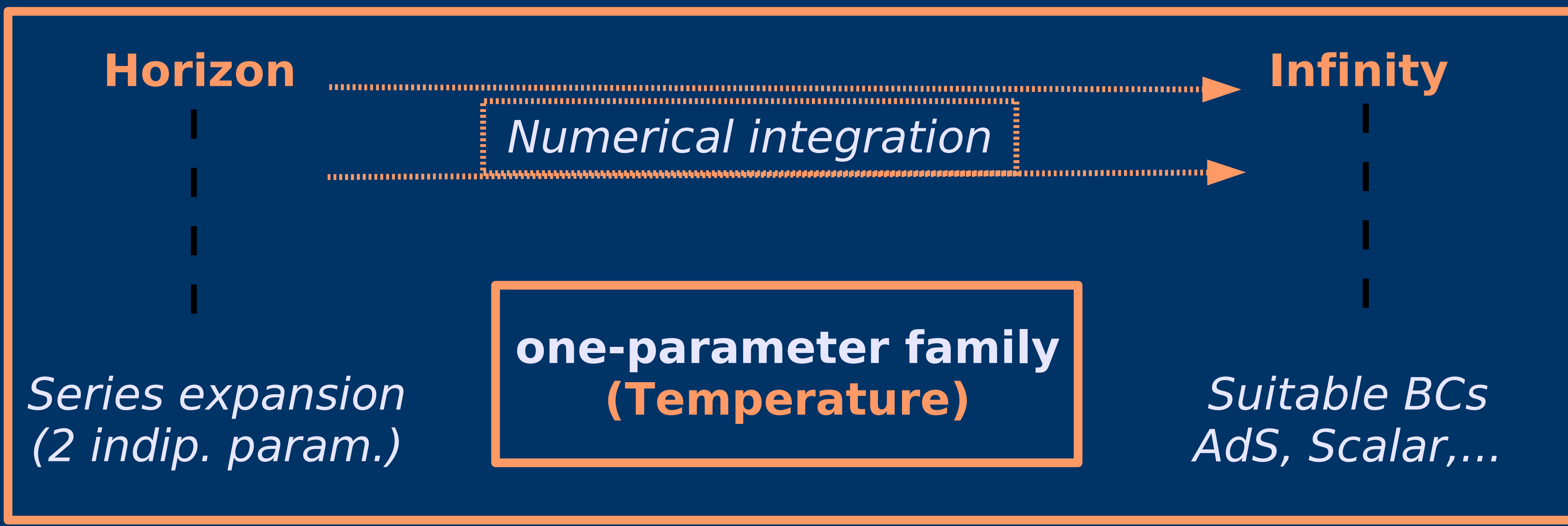
- analytical arguments
- numerical integration

What is the endpoint of the instability??

Numerical resolution. Planar symmetry

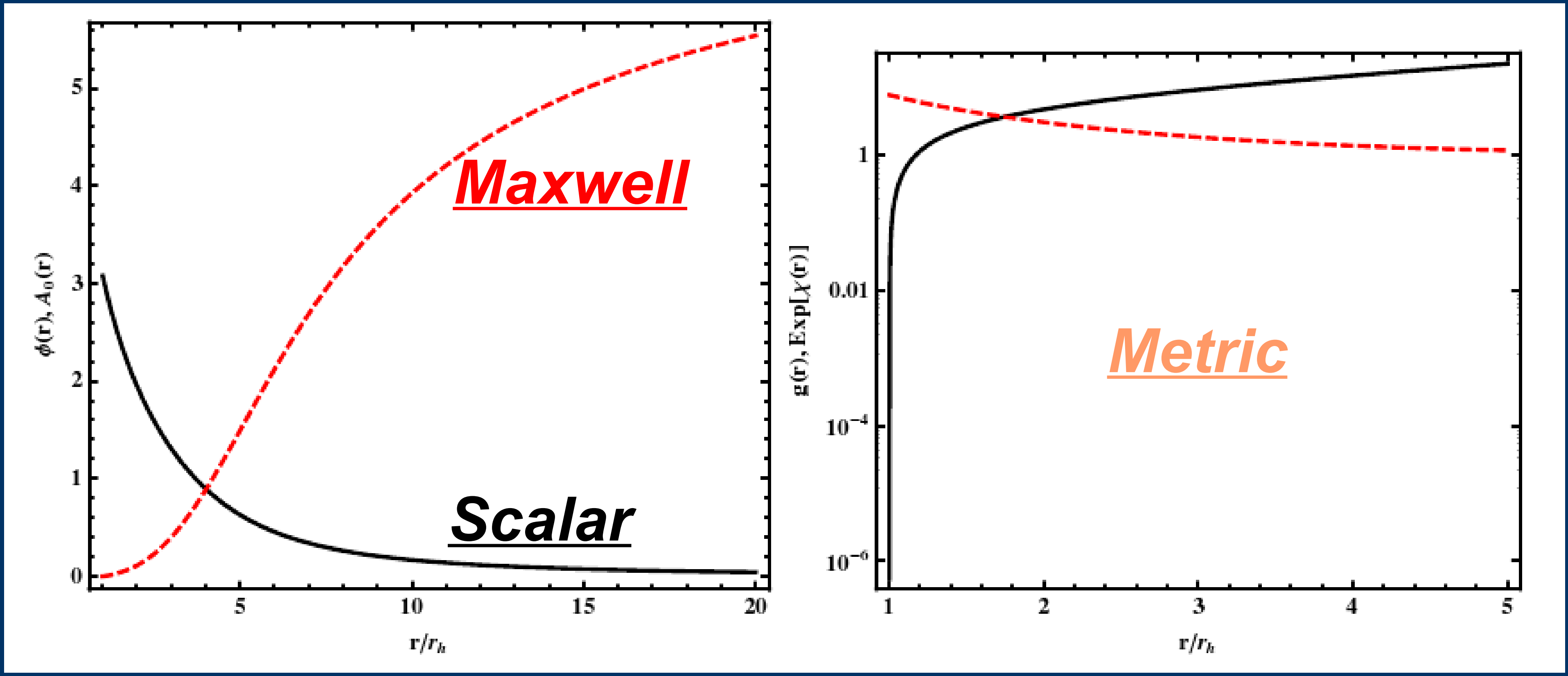
$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2)$$

- **4 fields** (**2** metric functions, **1** Maxwell field, **1** dilaton)
- **4 independent eqs.** (**2** Einstein's, **1** Maxwell's, **1** scalar eq.)



New solutions: Charged dilatonic BHs in AdS

Numerical resolution imposing **suitable boundary conditions**



RN-AdS will decay into this when $T<T_c$

Fine, that was nice but...

Universe is ~ flat and at most Λ>0, so...

<u>Why are you studying Λ<0 gravity??</u>

Because Maldacena's conjecture!

AdS/Condense Matter dictionary:

Metric tensor ⟷ Conserved energy-stress tensor

Black hole temperature ⟷ Temperature of the theory

Scalar field:

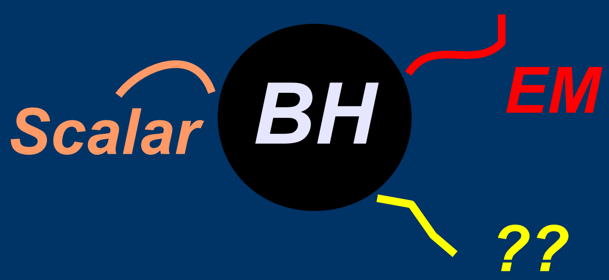
$$\phi \to \frac{\mathcal{O}_-}{r^{\lambda_-}} + \frac{\mathcal{O}_+}{r^{\lambda_+}}$$

⟷ Expectation values of scalar operators

Gauge field:

$$A_0 \to \mu - \frac{\rho}{r}$$

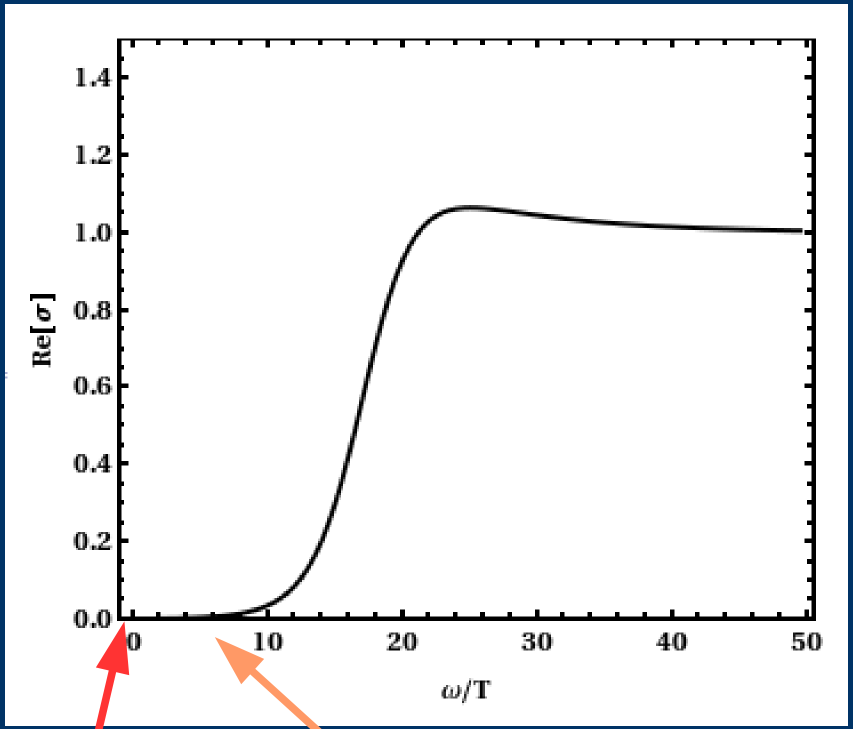
⟷ Chemical potential, charge density



⟷ **Real condense matter systems???**
(STRONG COUPLING!)

An example: *High-Tc Superconductors*

Electrical Conductivity



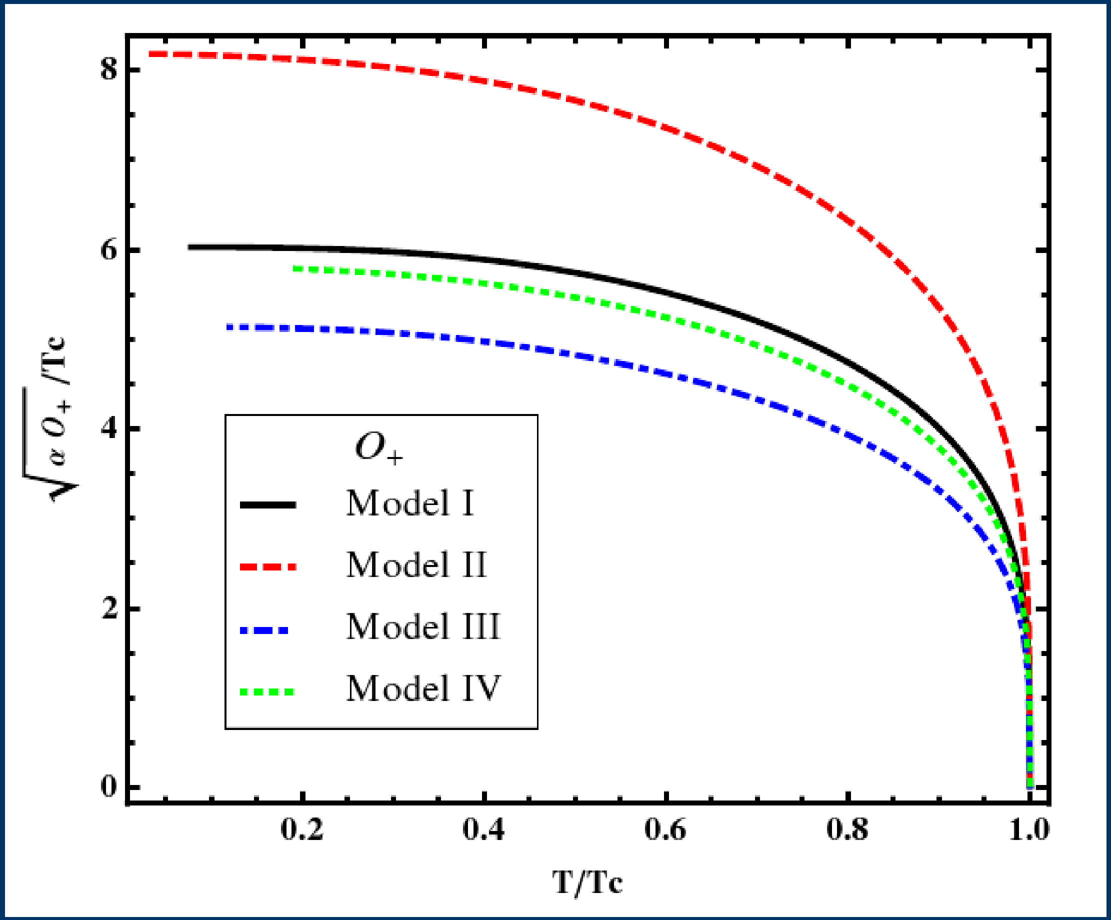
- Frequency gap
- Infinite DC conductivity
- Charged condensate

It is just gravity!!!

(No electrons, No Cooper's pairs...)

[Hartnoll, Horowitz, Hertog, Gubser, Roberts, Polchinski, Kachru...] (2008-now)

Results: *second order phase transition*



$$\phi \rightarrow \frac{\mathcal{O}_-(T)}{r} + \frac{\mathcal{O}_+(T)}{r^2}$$

Condensation of a neutral operator

How can the condensation of a neutral operator change the transport proprieties of a strongly coupled theory?

Electrical conductivity

Transport phenomena *in the dual theory can be studied by* ***linear perturbations in the bulk***

$$A_\mu = [A_0(r),0,0,0] + [0,0,A_x(r),0]e^{-i\omega t}$$

$$A_x'' + \left(\frac{g'}{g} - \frac{\chi'}{2} + \frac{f'(\phi)}{f(\phi)}\phi'\right)A_x' + \left[\left(\frac{\omega^2}{g^2} - \frac{{A_0'}^2 f(\phi)}{g}\right)e^\chi\right]A_x = 0$$

In the AdS/CFT dictionary the electrical conductivity can be computed by the asymptotical behavior of the perturbations

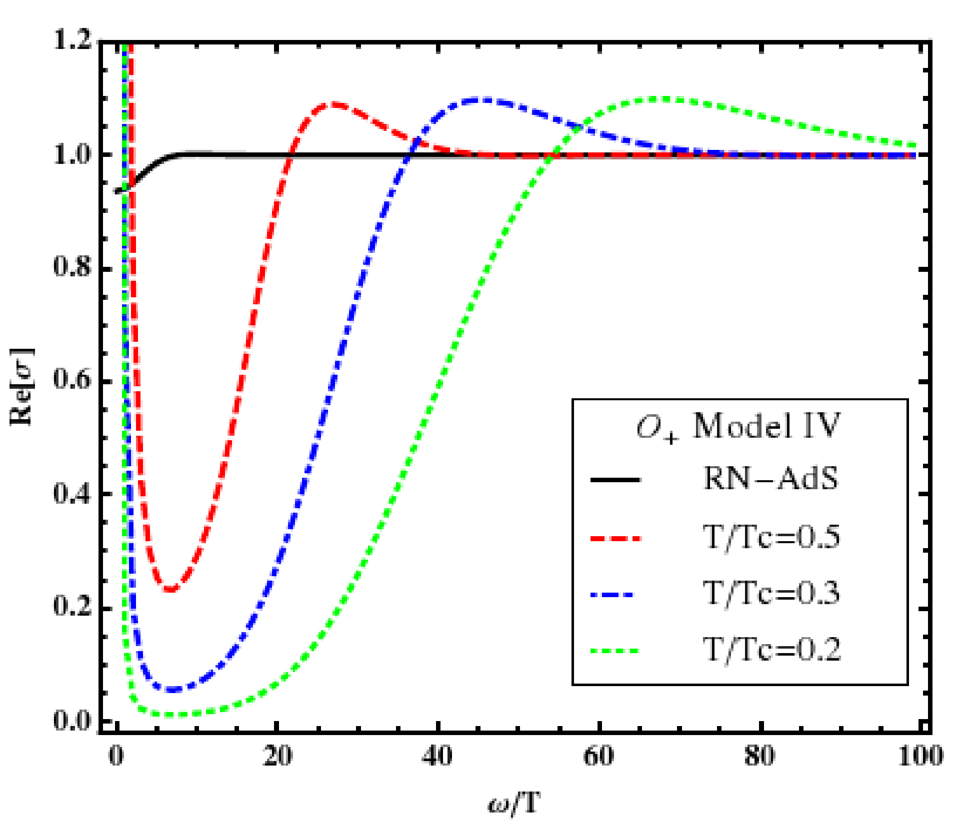
$$A_x \sim A_x^{(0)} + \frac{A_x^{(1)}}{r}$$

$$\sigma = -i\frac{A_x^{(1)}}{\omega A_x^{(0)}} = \frac{1-\mathcal{R}}{1+\mathcal{R}}$$

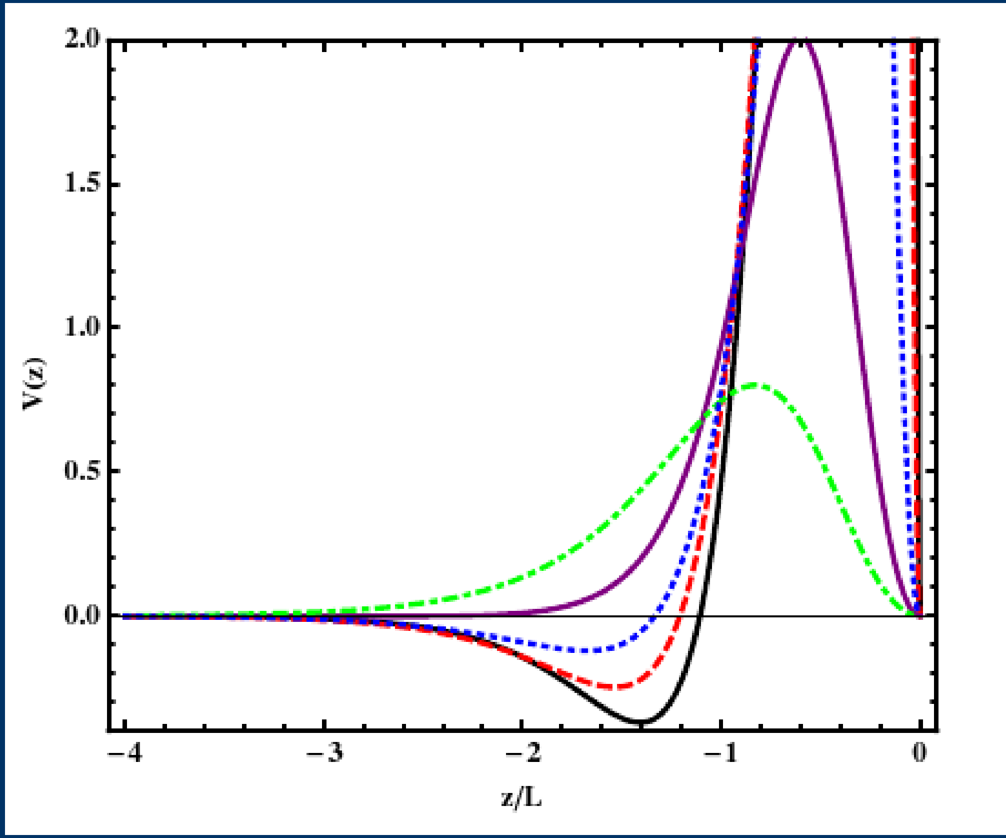
AdS/CFT result

Numerical results

Conductivity:



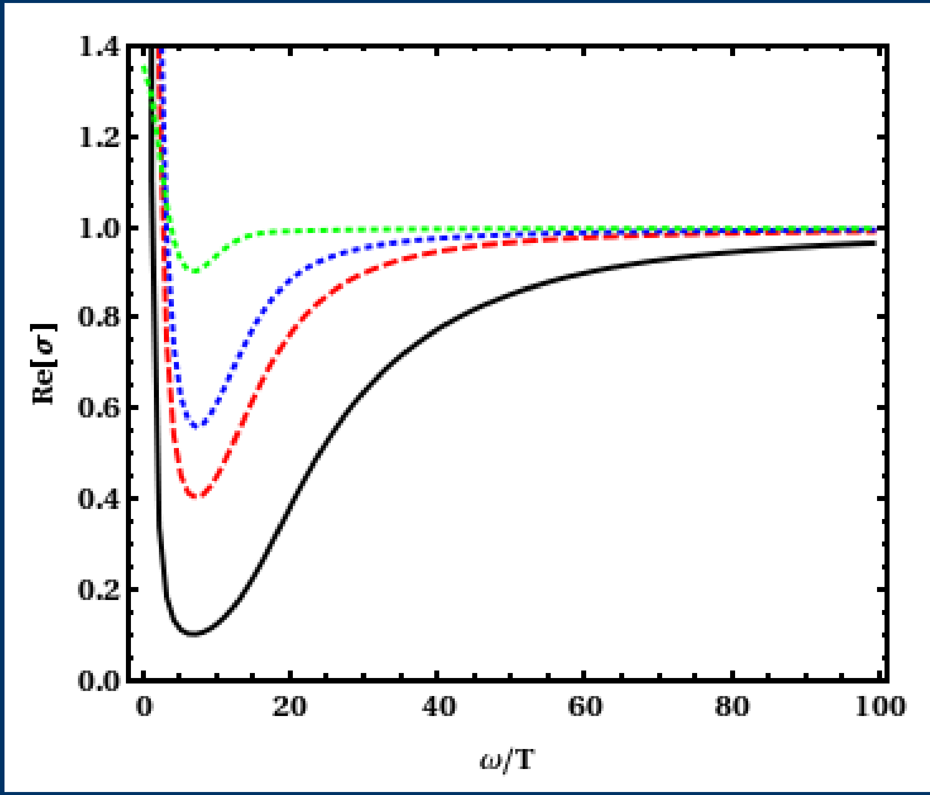
Schroedinger potential:



- **Negative** minimum of V(z) → **slope** of the conductivity at low frequency
- No bound states
- **Reminiscent of <u>"Drude peak"</u> in real materials!**

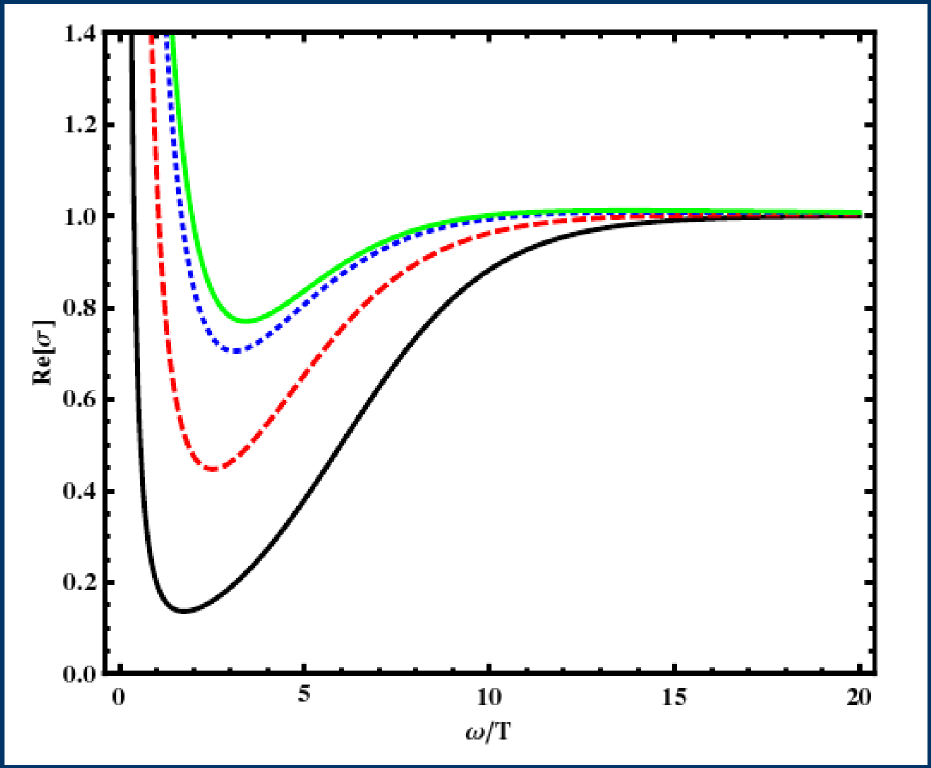
Electrical conductivity

Numerical results:



Conductivity in graphene:

(theory and experiment)



- **Universal behavior** at high frequency
- **Minimum** at low frequency
- **Slope in the DC limit**

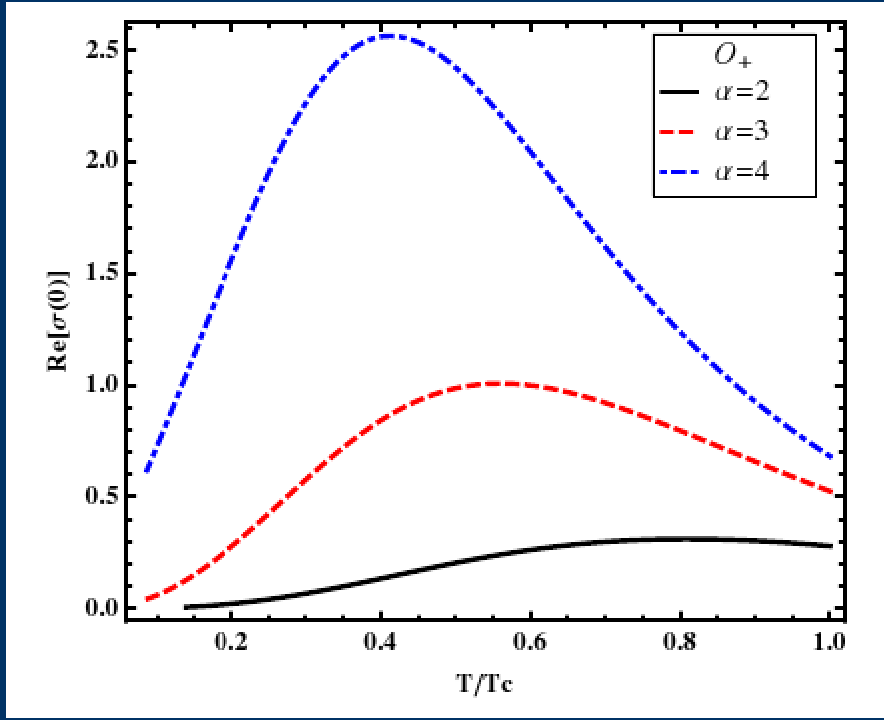
DC conductivity. (Work in progress)

Real materials at low temperatures:

$$\sigma_{DC} = \mathrm{Re}[\sigma(0)] \sim \left[a + bT^2 + cT^5 + k\log\left(\frac{\mu}{T}\right)\right]^{-1}$$

Impurities — e-e interaction — e-ph interaction — Kondo's term

Results:



- NO impurity
- Strong coupling at low T
- DC insulators at T = 0

Conclusions

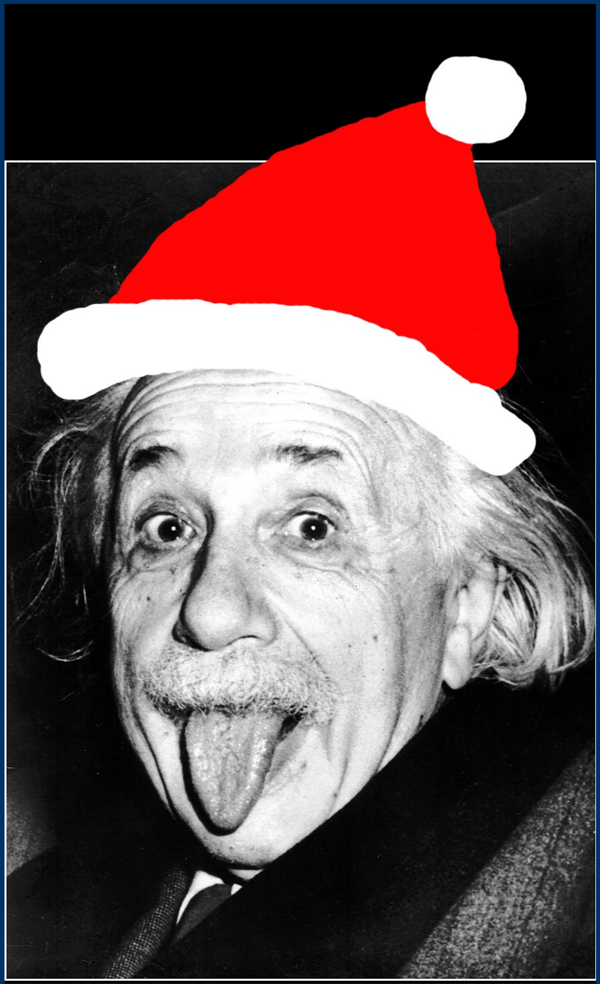
- We study a model in which RN-AdS is unstable
- Second order phase transition toward new charged dilatonic BHs below a critical temperature

- Phase transition: neutral scalar condensation
- Unexpected effects on transport proprieties of a strongly coupled field theory
- Charged dilatonic black holes share some proprieties with real materials

Something to think about:

We still have to learn a lot from gravity/field theories dualities

Thanks!



Backup slides

$$T_c = \gamma^{\frac{1}{4}} \frac{1}{8\pi L} \sqrt{\frac{Q}{L}} \left[12 - \frac{1}{\gamma} + k \frac{L}{Q} \sqrt{\frac{2}{\gamma}} \right]$$

$$\gamma = \frac{\alpha}{\frac{9}{4} + \beta}$$

Num. Sol. of the Field Eqs. Planar symmetry

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2)$$

- **Two nonlinear, coupled differential equations:**

Scalar equation:

$$\phi''(r) + \left(\frac{g'}{g} - \frac{\chi'}{2} + \frac{2}{r}\right)\phi'(r) - \frac{V'(\phi)}{g} + \frac{{A_0'}^2 e^{\chi} f'(\phi)}{2g} = 0$$

Einstein equation:

$$\frac{\phi'^2}{4} + \frac{{A_0'}^2 e^{\chi} f(\phi)}{4g} + \frac{g'}{rg} + \frac{1}{r^2} + \frac{V(\phi)}{2g} = 0$$

The gauge field and one metric function can be directly integrated

Preliminaries:

- Reisser-Nordstrom BH (in flat spacetime)

- Two horizons:

$$r_\pm = M \pm \sqrt{M^2 - Q^2}$$

- The inner horizon is **unstable**
- **Not even** an approximate solution of ST

- Here we deal with AdS black holes

- Particles reach **infinity in a finite amount** of time
- Scalars with **negative mass squared** (tachyons) can exist if

$$m^2 \geq -\frac{9}{4L^2}$$