Instabilities and new phases of higher dimensional black holes

Ricardo Monteiro
DAMTP, University of Cambridge

arXiv:0907.2248, with Óscar Dias (DAMTP), Pau Figueras (Durham), Jorge Santos (DAMTP) and Roberto Emparan (Barcelona)

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Main points

- Find onset of instability of ultraspinning Myers-Perry BHs.
- Interpret onset of instability as bifurcation of BH phases.
- Relate new bifurcated BH phases to expected solutions: black rings, black saturns, etc.
- Propose a simple necessary condition for this instability.

Single spinning Myers-Perry BH

Single spinning case: 'Kerr-part' (t, r, θ, ϕ) + S^{D-4}

$$
\begin{aligned}
ds^2 &= -dt^2 + \frac{r_m^{D-3}}{r^{D-5}\Sigma}\left(dt + a\sin^2\theta\, d\phi\right)^2 + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) \\
&\quad + (r^2 + a^2)\sin^2\theta\, d\phi^2 + r^2\cos^2\theta\, d\Omega^2_{(D-4)}\,,
\end{aligned}
$$

$$
\Sigma = r^2 + a^2\cos^2\theta\,, \quad \Delta = r^2 + a^2 - \frac{r_m^{D-3}}{r^{D-5}}\,, \quad \text{horizon} : \Delta(r_+) = 0
$$

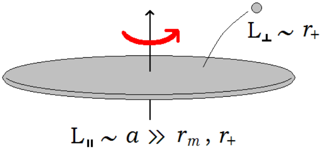
Parameters: mass-radius r_m and rotation parameter a

$$
r_m^{D-3} = \frac{16\pi G M}{(D-2)\Omega_{D-2}}\,, \qquad a = \frac{D-2}{2}\frac{J}{M}
$$

- $D = 4$ Kerr: $r_m = 2GM$, $a = J/M$, extremality $|a| \leq r_m/2$
- $D = 5$: $|a| < r_m$, naked singularity for $|a| = r_m$
- $D \geq 6$: $\underline{a \text{ unbounded}}$!

Ultraspinning instability of MP BHs (Emparan, Myers '03)

- $D \geq 6$: rotation parameter unbounded. Spin it up!



Horizon scales:
$L_{||}$ is (θ, ϕ) pancake scale
$L_\perp$ is transverse S^{D-4} scale
$L_{||} \gg L_\perp$

- Zoom near axis: locally Schwarz$_{D-2} \times \mathbb{R}^2$
- Black brane! But black branes are unstable (Gregory-Laflamme)

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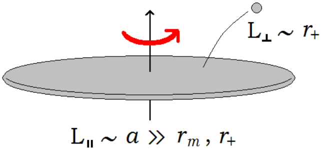

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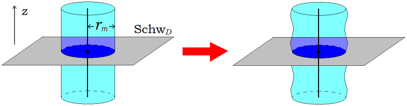
$\Rightarrow$ Ultraspinning MP black holes should be unstable!

Recall Gregory-Laflamme instability '93

Schwarzschild brane, $(D+n)$-dim: $ds^2 = ds^2(\mathrm{Schw}_D) + d\vec{z}\cdot d\vec{z}$



Perturb along $\vec{z}$: $H_{AB} = e^{i\vec{k}\cdot\vec{z}} \begin{pmatrix} h_{ab} & 0 \\ 0 & 0 \end{pmatrix}$, $h_{ab} \sim e^{\Omega t}$.

E.o.m. in TT gauge: $\boxed{(\Delta_{\rm L} h)_{ab} \equiv -\nabla^c\nabla_c h_{ab} - 2R_a{}^c{}_b{}^d h_{cd} = -k^2 h_{ab}}$

Find $0 < |\vec{k}| < k_c \sim r_m^{-1}$ giving $\Omega > 0$ $\Rightarrow$ Instability!

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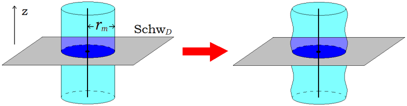

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Threshold $|\vec{k}| = k_c$ signals family of non-uniform branes (Gubser, Wiseman and others) connecting to sequence of black holes $\rightarrow$ $\rightarrow$ O O

MP ultraspin. instability: pinched black holes (r,θ)-dependent connecting to rings: $\rightarrow \infty \rightarrow$ O⁀O , and to saturns, etc. for more pinches.

Gregory-Laflamme and thermodynamics

GL threshold mode ($\Omega = 0$): $(\Delta_{\rm L} h)_{ab} = -k_c^2 \, h_{ab}$

Same as negative mode of Euclidean action (Gross, Perry, Yaffe '82):

$$\boxed{\lambda_{\rm Euclidean} = -k_c^2}$$

Thermodynamic instability $\Rightarrow$ negative mode / GL threshold (Reall '01)
Classical GL $\leftrightarrow$ Local thermod. instability* (Gubser-Mitra conjecture)

* in grand-canonical ensemble: asymp. flat vacuum BHs always unstable

Q: Are all GL instabilities related to thermodynamics?

A: No! New GL instabilities for new black hole phases!

$D > 6$, Myers-Perry single spin (Dias, Figueras, RM, Santos, Emparan '09)

Perturbation h_{ab} preserving all the isometries: $\mathbb{R}_t \times U(1)_\phi \times SO(D-3)$

Solve $(\Delta_{\mathrm{L}} h)_{ab} = -k_c^2 \, h_{ab}$ numerically (spectral method).

Expectations

- no rotation: find negative mode $-k_c^2$, threshold of GL
- rotation: increase k_c of that mode (centrifugal force)
- more physics?

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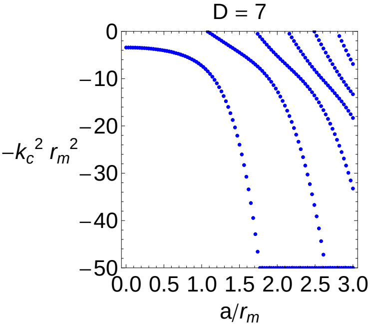
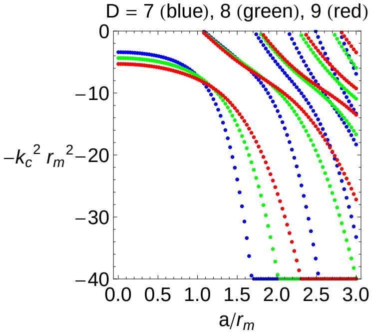




New GL instabilities! These appear when $k_c = 0 \Rightarrow \boxed{(\Delta_L h)_{ab} = 0}$

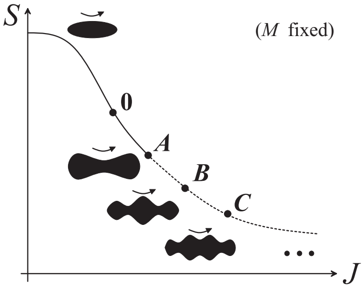
$\Rightarrow$ classical Einstein equation for black hole (not just brane)!

Why so many instabilities? Number curves left to right $l = 0, 1, 2, 3\dots$

BH phase diagram (Dias, Figueras, RM, Santos, Emparan '09)

Look at perturbations $k_c = 0$: new BH phases signaled!

Phase diagram of black holes with a single spin:



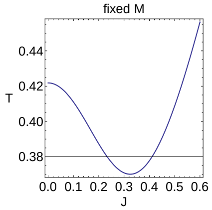
$l = 1$: point 0, inflection!
$l = 2$: point A
$l = 3$: point B
$l = 4$: point C

l is number of nodes
('harmonic' structure)

Suggests connection to black ring, black saturn, concentric rings...

Ultraspinning regime

Emparan-Myers '03 heuristics: when does MP start behaving like brane?



Small rotation: Kerr-like
Large rotation: brane-like

T minimum matches $l = 1$ threshold!
Thermodynamic zero-mode
should give another MP solution.

Modes $l \geq 2$ give true BH bifurcation / onset of classical instability.

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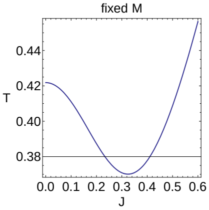

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More spins: new thermod. instabilities appears with negative eigenvalue of

$$H_{ij} = \left(\frac{\partial^2(-S)}{\partial J_i \partial J_j} \right)_M$$

Conjecture:

This instability can appear only after first zero-mode of H_{ij}.

Final remarks

Asymptotically flat higher dimensional black holes can be unstable.

Instabilities may connect different families of black holes (uniqueness $\leftrightarrow$ stability).

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Future work: Jorge's talk!