BPS Black Holes, the Hesse Potential and the Topological String

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BPS Black Holes

Consider

- supersymmetric (BPS) black holes in four-dimensional $N = 2$ supergravity theories,
- single-center, static, dyonic, with electric/magnetic charges (q_I, p^I),
- supported by complex scalar fields Y^I.

Lagrangian with higher-curvature interactions $\propto$ Weyl^2: (Υ, C^2)

$$L = R + F^{(0)}(Y)\, F^2_{\mu\nu} + \Big(F^{(1)}(Y) + F^{(2)}(Y)\, \Upsilon + \ldots \Big)\, C^2$$

Couplings can be combined into a single function $F(Y, \Upsilon)$:

$$F(Y, \Upsilon) = \sum_{g=0}^{\infty} \Upsilon^g\, F^{(g)}(Y)$$

Couplings $F^{(g)}$ computed by the topological string (subtle)

$\longrightarrow$ BPS microstates counted by the topological string (subtle).

Attractor Mechanism and BPS Free Energy

BPS black holes subject to attractor mechanism: Ferrara, Kallosh, Strominger

$$\text{at horizon} \qquad Y^I \to Y^I(p,q) \quad , \quad \Upsilon \to -64$$

Attractor equations (in the presence of Weyl^2):

$$\begin{aligned} Y^I - \bar{Y}^{\bar{I}} &= i\,p^I \quad , \quad \text{magnetic} \quad , \\ F_I - \bar{F}_{\bar{I}} &= i\,q_I \quad , \quad \text{electric} \quad , \quad F_I = \partial F(Y,\Upsilon)/\partial Y^I \end{aligned}$$

Electro/magnetostatic potentials: $Y^I + \bar{Y}^{\bar{I}} = \phi^I \quad , \quad F_I + \bar{F}_{\bar{I}} = \chi_I$

BPS free energy = Hesse potential H of SG: $H = H(\phi, \chi)$

Question: relation of H to partition function of the topological string?

Duality Transformations

Charges (p^I, q_I) undergo electric/magnetic duality transformations:

$$\begin{aligned} Y^I - \bar{Y}^{\bar{I}} &= i\,p^I \quad , \quad \text{magnetic} \quad , \\ F_I - \bar{F}_{\bar{I}} &= i\,q_I \quad , \quad \text{electric} \quad , \quad F_I = \partial F(Y, \Upsilon)/\partial Y^I \end{aligned}$$

Thus, as $\quad p \to q \quad , \quad q \to -p, \quad$ obtain $Y \to Y' = Y'(Y, \Upsilon)$.

Entanglement with the Weyl background!

Different from topological string approach: here, the transformation of the Y is not affected by the Weyl background.

Thus, $\qquad Y_{\text{sugra}} \neq Y_{\text{top}}$!

New Variables for the Hesse Potential

Proposal:

Reexpress the Hesse variables (ϕ, χ) in terms of new variables $\tilde{Y}$:

$$
\begin{aligned}
Y^I + \bar{Y}^{\bar{I}} &= \phi^I = \tilde{Y}^I + \bar{\tilde{Y}}^{\bar{I}} \quad , \\
F_I + \bar{F}_{\bar{I}} &= \chi_I = F_I^{(0)}(\tilde{Y}) + \bar{F}_{\bar{I}}^{(0)}(\bar{\tilde{Y}}) \quad .
\end{aligned}
$$

- The new variables $\tilde{Y}$ transform precisely as the topological string variables. Natural to identify $\tilde{Y} = Y_{\rm top}$.
- $\tilde{Y} = Y + \Delta Y(Y, \Upsilon)$, iteratively in Υ, complicated expressions.
- Attractor equations: $Y = Y(q, p) \quad \rightarrow \quad \tilde{Y} = \tilde{Y}(q, p)$

The Hesse Potential and the Topological String

Conjecture:

the Hesse potential computes the topological free energy (TFE),

$$H(\phi, \chi) \sim \mathrm{Re} \left(\sum_{g=0}^{\infty} \Upsilon^g \, F^{(g)}(\tilde{Y}) \right)$$

Passes various checks.

Outlook:

Legendre of H is entropy. Relation with microstate counting?

$$\sum_{q,p} d(q,p) \, \mathrm{e}^{q\cdot\phi - p\cdot\chi} \sim \mathrm{e}^{H(\phi,\chi)} \sim \mathrm{e}^{\mathrm{TFE}}$$