II Workshop on Black Holes

21-22 December 2009 | Instituto Superior Técnico, Lisboa

Instabilities in higher-dimensional spacetimes

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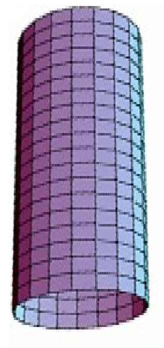
V. C., M. Lemos and M. Marques, Phy. Rev. D, in press (2010).

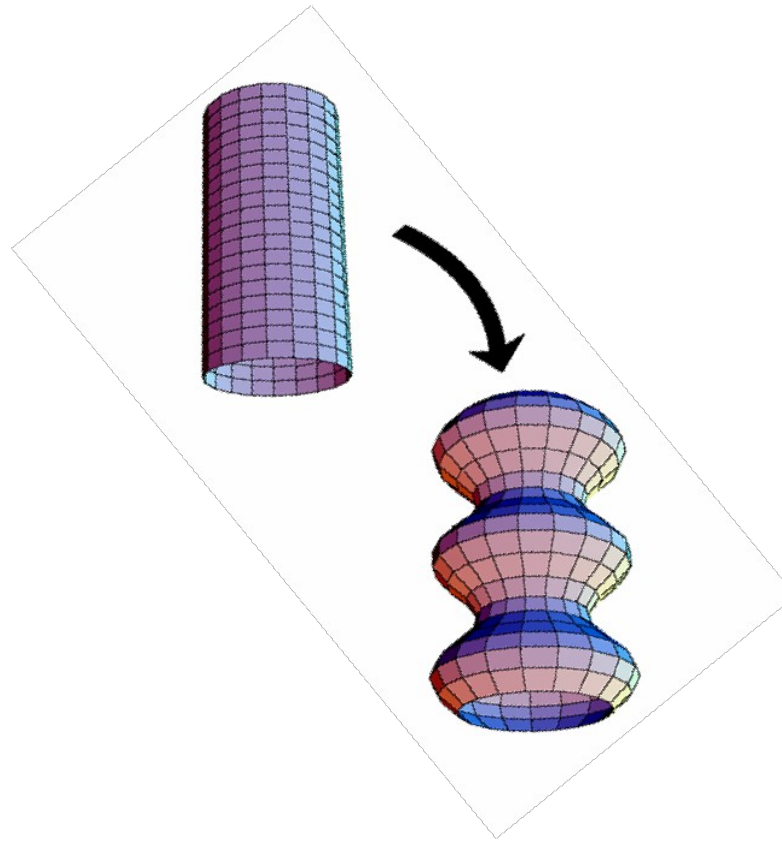
Instabilities of black objects

The Gregory-Laflamme instability

- **Black string:**

$\text{Schwarschild} \times \text{R}$





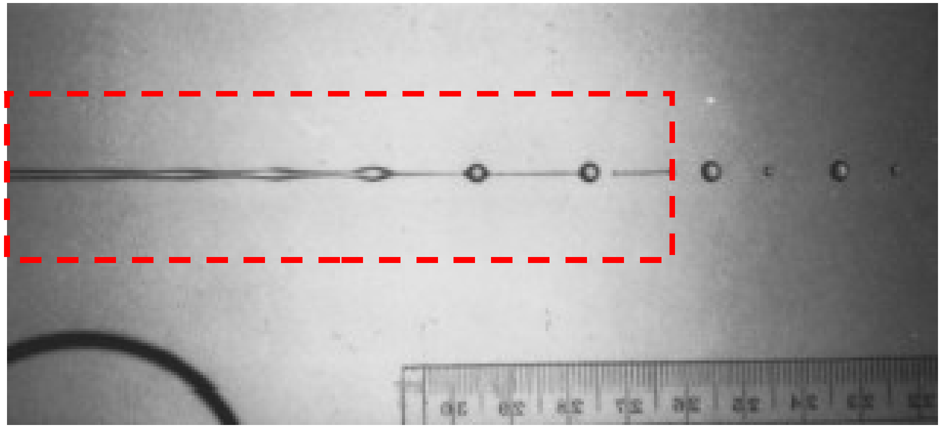
- **Perturb the BS (s-wave)**

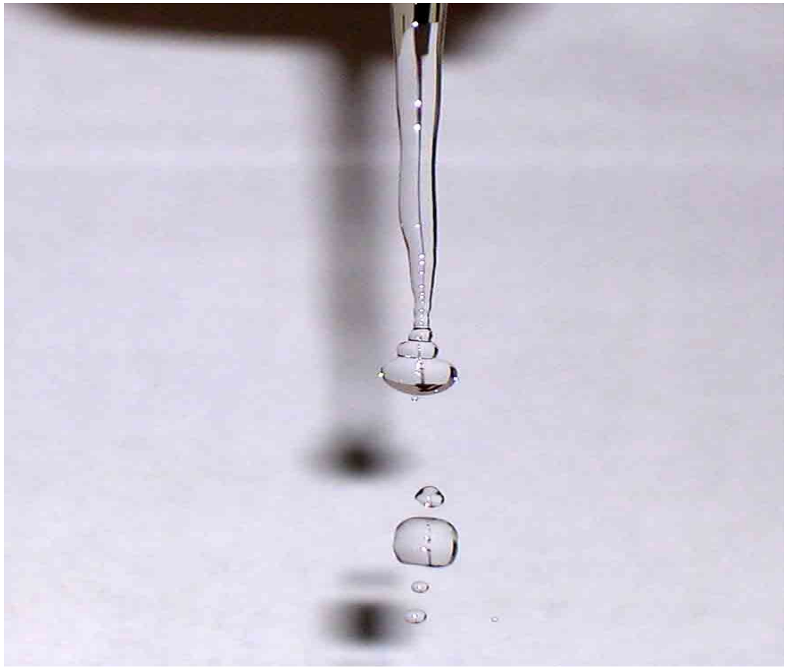
$$\begin{aligned} \text{String} : \quad & S \quad \propto \quad \text{Mass}^2/L \\ \text{Spherical} : \quad & S \quad \propto \quad \text{Mass}^{3/2} \end{aligned}$$

R. Gregory and R. Laflamme, Phys.Rev.Lett.70:2837,1993

Instabilities of black objects

The Rayleigh-Plateau instability (Plateau 1849, Rayleigh 1878)





$$\mathrm{GL}: \quad Rk_{\mathrm{crit}} \sim \sqrt{D}$$
$$\mathrm{RP}: \quad Rk_{\mathrm{crit}} \sim \sqrt{D}$$

V.C. and O.J. C. Dias, Phys.Rev.Lett.96:181601,2006

Instabilities of black objects

Ultra-spinning instabilities (R. Monteiro and J. Santos' talks)

Emparan and Myers, JHEP 0309:025,2003; O. J. C. Dias et al, arXiv:0907.2248

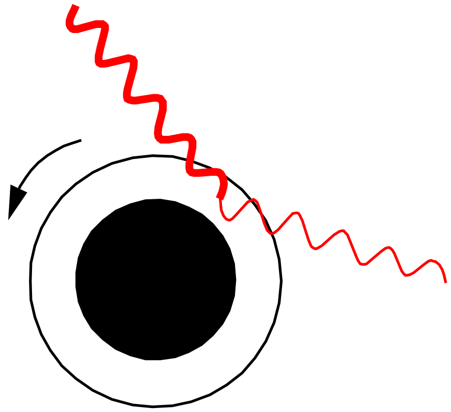
Instabilities of black objects

Super-radiant instabilities (J. Rosa's talk)

Zel'dovich 1971; Damour et al (1976); VC and O.J. C.Dias PRD70:084011,2004.

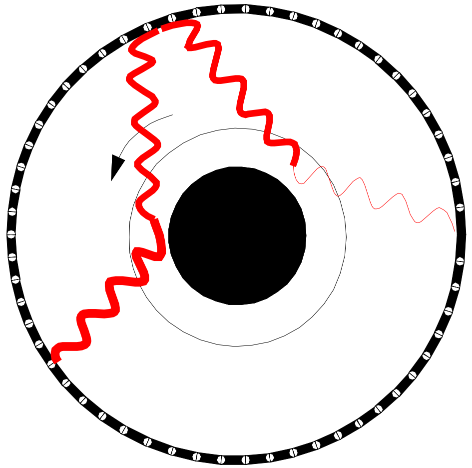
→ **Superradiant scattering <u>on a rotating BH</u>**

Waves incident upon a <u>Kerr black hole</u> are amplified by superradiant scattering if $\omega < m\Omega$



<u>Insert a *mirror* around the black hole:</u>

Make a black hole bomb!



(Cardoso, Dias, Lemos,Yoshida, 2004)

AdS acts as box: small Kerr-AdS BHs are unstable (Cardoso and Dias, 2004)

Instabilities of black objects

Instabilities in Einstein-Gauss-Bonnet theories

$$S_{GB} = \int d^D x \sqrt{-g} \frac{\alpha}{(D-3)(D-4)} \left(R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2 \right)$$

$$\begin{aligned} ds^2 &= -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2_{D-2} \\ f(r) &= 1 + \frac{r^2}{2\alpha} - \frac{r^2}{2\alpha}\sqrt{1 + \frac{8\alpha\mu}{r^{D-1}}} \end{aligned}$$

Unstable for some range of parameters in D=5,6

Instabilities in Chern-Simons theories (L. Gualtieri's talk)

Gleiser and Dotti, Phys.Rev.D72:124002,2005.

Instabilities of charged BHs in de Sitter backgrounds

$$
\begin{aligned}
ds^2 &= -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_n^2 \\
f(r) &= 1 - \lambda r^2 - \frac{2M}{r^{n-1}} + \frac{Q^2}{r^{2n-2}} \\
E &= q/r^n
\end{aligned}
$$

$$
\begin{aligned}
& r_a\,(\text{Cauchy}),\ r_b = 1\,(\text{Event}),\ r_c\,(\text{Cosmological}) \\
2M &= 1 + Q^2 - \lambda \\
\lambda &= \frac{r_c^{-4-n}(r_c^{n+2} - r_c^3)(r_c^{n+2} - Q^2 r_c^3)}{r_c^{n+2} - r_c} \\
Q_{\text{ext}}^2 &= \frac{r_c^n\left(-2r_c + (n+1)r_c^n - (n-1)r_c^{n+2}\right)}{-r_c\left(r_c(n+1) - 2nr_c^n + (n-1)r_c^{2n+1}\right)}
\end{aligned}
$$

Konoplya and Zhidenko, Phys.Rev.Lett.103:161101,2009

Instabilities of charged BHs in de Sitter backgrounds

$$\frac{d^2}{dr_*^2}\Psi_\pm + \left(\omega^2 - V_{S\pm}\right)\Psi_\pm = 0\,,\ V_{S\pm} = \frac{fU_\pm}{64r^2H_\pm^2}\,,\ x \equiv \frac{2M}{r^{n-1}}$$

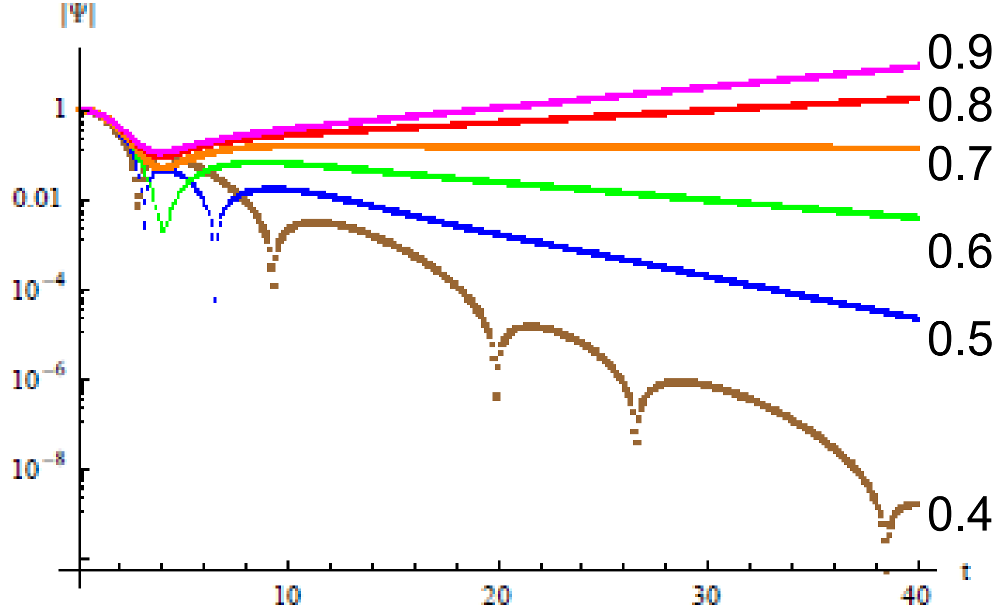
$$\begin{aligned}
U_+ \;=\;& \left[-4n^3(n+2)(n+1)^2\delta^2x^2 - 48n^2(n+1)(n-2)\delta x\right.\\
&\left.-16(n-2)(n-4)\right]y - \delta^3n^3(3n-2)(n+1)^4(1+m\delta)x^4\\
&+4\delta^2n^2(n+1)^2\left\{(n+1)(3n-2)m\delta + 4n^2 + n - 2\right\}x^3\\
&+4\delta(n+1)\left\{(n-2)(n-4)(n+1)(m+n^2K)\delta - 7n^3 + 7n^2 - 14n + 8\right\}x^2\\
&+\left\{16(n+1)\left(-4m + 3n^2(n-2)K\right)\delta - 16(3n-2)(n-2)\right\}x\\
&+64m + 16n(n+2)K,\\
U_- \;=\;& \left[-4n^3(n+2)(n+1)^2(1+m\delta)^2x^2 + 48n^2(n+1)(n-2)m(1+m\delta)x\right.\\
&\left.-16(n-2)(n-4)m^2\right]y - n^3(3n-2)(n+1)^4\delta(1+m\delta)^3x^4\\
&-4n^2(n+1)^2(1+m\delta)^2\left\{(n+1)(3n-2)m\delta - n^2\right\}x^3\\
&+4(n+1)(1+m\delta)\left\{m(n-2)(n-4)(n+1)(m+n^2K)\delta\right.\\
&\quad\left.+4n(2n^2-3n+4)m + n^2(n-2)(n-4)(n+1)K\right\}x^2\\
&-16m\left\{(n+1)m\left(-4m + 3n^2(n-2)K\right)\delta\right.\\
&\left.+3n(n-4)m + 3n^2(n+1)(n-2)K\right\}x + 64m^3 + 16n(n+2)m^2K
\end{aligned}$$

KI, Prog.Theor.Phys.111:29-73,2004

Instabilities of charged BHs in de Sitter backgrounds

$$D = 11(n = 9), \quad 1/r_c = 0.8$$

$$Q/Q_{\text{ext}} =$$

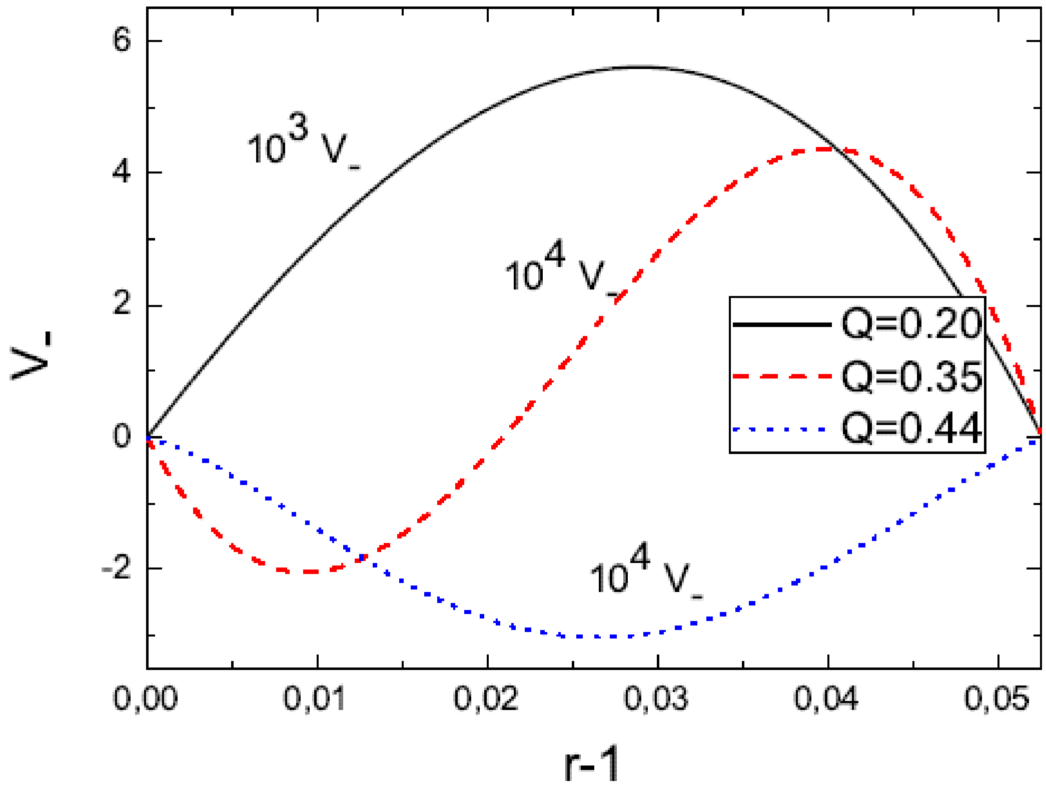


For sufficiently large Q and D>6, BHs are unstable

Konoplya and Zhidenko, Prog.Theor.Phys.111:29-73,2004

Instabilities of charged BHs in de Sitter backgrounds

$$D = 8(n = 6), \quad 1/r_c = 0.95$$



VC, M. Lemos and M. Marques, Phys.Rev. D (in press)

Instabilities of charged BHs in de Sitter backgrounds

$$\frac{d^2\Phi(\omega,r)}{dr_*^2}+\left[\omega^2-\frac{V_0}{\cosh\left(\kappa_b r_*\right)^2}\right]\Phi(\omega,r)=0$$

$$\omega=\kappa_b\left[-\left(j+\frac{1}{2}\right)i+\sqrt{\frac{V_0}{\kappa_b^2}-\frac{1}{4}}\right]\,,\qquad j=0,1,\ldots\,.$$

	D					
	7	8	9	10	11	$D\to\infty$
$Q/Q^{N}_{\rm ext}$	0.913	0.774	0.683	0.617	0.567	$\sqrt{2/D}$
$Q/Q^{Num}_{\rm ext}$	0.94	0.78	0.68	0.61	0.55	—

VC, M. Lemos and M. Marques, Phys.Rev. D (in press)

Summary

- Black objects are fascinating from a dynamical viewpoint
- Stability/Instability is an important criterium for relevance
- Stability of charged and/or spinning BHs poorly understood
- Interesting topic for further research

Thank you