Apêndice A

Formulário de Física IV-Electromagnetismo

(LEFT-LEA-LMAC / 2ºSemestre 2002/2003)

A.1 Electromagnetismo

A.1.1 Força de Lorentz

$\mathbf{F} = q\vec{\mathbf{E}} + q\,(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

Carga Elementar $e = 1.602 \times 10^{-19}\ C$

A.1.2 Electrostática

$$\vec{\mathbf{E}} = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\,\vec{\mathbf{e}}_r$$

$$\frac{1}{4\pi\epsilon_o} = 10^{-7}c^2 \approx 9 \times 10^9\ (\tfrac{Nm^2}{C^2})$$

$$\nabla \times \vec{\mathbf{E}} = 0$$

$$\vec{\mathbf{E}} = -\nabla\phi \qquad (\phi = \text{Potencial Electrostático})$$

$$\nabla \cdot \vec{\mathbf{D}} = \rho$$

$$\nabla^2\phi = -\frac{\rho}{\epsilon_o} \qquad \text{(no vazio)}$$

$$\vec{\mathbf{D}} = \epsilon_o\,\vec{\mathbf{E}} + \vec{\mathbf{P}} = \epsilon\,\vec{\mathbf{E}}$$

$$\phi(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_o}\iiint \frac{\rho(\vec{\mathbf{r}}\,')}{r}\,dV' \qquad (r = |\vec{\mathbf{r}} - \vec{\mathbf{r}}\,'|) \qquad \text{(no vazio)}$$

$$\nabla \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_o}\,(\rho + \rho_{pol})$$

$$\nabla \cdot \vec{\mathbf{P}} = -\rho_{pol} \quad ; \quad \vec{\mathbf{n}} \cdot \vec{\mathbf{P}} = \sigma_{pol}$$

A.1.3 Correntes Eléctricas

$$I = \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{S}}$$

$$\frac{\partial\rho}{\partial t} = -\nabla \cdot \vec{\mathbf{J}} \qquad \text{(Equação da Continuidade)}$$

$$\vec{\mathbf{J}} = \rho_+\vec{\mathbf{v}}_{d_+} + \rho_-\vec{\mathbf{v}}_{d_-}$$

$$\vec{\mathbf{J}} = \sigma\,\vec{\mathbf{E}} \qquad (\sigma:\ \text{condutividade eléctrica})$$

A.1.4 Força Electromotriz e Energia Eléctrica

$$\epsilon_{fem} = \int_1^2 \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{r}} = -\Delta\mathcal{V}_{12} \quad \text{(Queda de Potencial)}$$

$$W_{el.} = \frac{1}{2}\iiint \rho\,\phi\,dV = \iiint \frac{1}{2}\vec{\mathbf{D}} \cdot \vec{\mathbf{E}}\,dV$$

A.1.5 Magnetoestática

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_o}{4\pi} \iiint \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}\,') \times \vec{\mathbf{e}}_r}{r^2}\, dV\,' = \frac{\mu_o}{4\pi} I \int \frac{d\vec{\mathbf{l}}\,' \times \vec{\mathbf{e}}_r}{r^2}$$

$$\mu_o = 4\pi \times 10^{-7} \left(\tfrac{N}{A^2}\right)$$

$$\nabla \cdot \vec{\mathbf{B}} = 0$$

$$\vec{\mathbf{B}} = \nabla \times \vec{\mathbf{A}} \qquad (\vec{\mathbf{A}} = \text{Potencial Vector})$$

$$\vec{\mathbf{B}} = \mu_o\, (\vec{\mathbf{H}} + \vec{\mathbf{M}}) = \mu\, \vec{\mathbf{H}} \qquad (\vec{\mathbf{M}} = \text{Magnetização})$$

$$\vec{\mathbf{A}} = \frac{\mu_o}{4\pi} \iiint \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}\,')}{r}\, dV\,' = \frac{\mu_o}{4\pi} I \int \frac{d\vec{\mathbf{l}}\,'}{r}$$

$$\nabla \times \vec{\mathbf{B}} = \mu_o\, (\vec{\mathbf{J}} + \vec{\mathbf{J}}_{mag}) = \mu\, \vec{\mathbf{J}}$$

$$\nabla^2 \vec{\mathbf{A}} = -\mu_o\, \vec{\mathbf{J}}$$

$$\vec{\mathbf{J}}_{mag} = \nabla \times \vec{\mathbf{M}} \qquad \text{(Corrente de Magnetização)}$$

$$\vec{\mathbf{K}}_{mag} = \vec{\mathbf{M}} \times \vec{\mathbf{n}} \qquad \text{(Corrente Superficial Mag.)}$$

A.1.6 Fluxo Magnético, Coeficientes de Indução e Energia Magnética

$$\Phi = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = \oint \vec{\mathbf{A}} \cdot d\vec{\mathbf{l}}\,'$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{r}} = \mu_o\, I$$

$$\Phi_\alpha = L_\alpha I_\alpha + \sum_{\beta \neq \alpha} M_{\alpha\beta} I_\beta$$

$$W_{mag} = \frac{1}{2} \iiint \vec{\mathbf{J}} \cdot \vec{\mathbf{A}}\, dV = \iiint \frac{1}{2} \vec{\mathbf{B}} \cdot \vec{\mathbf{H}}\, dV$$

A.1.7 Campos Variáveis no Tempo

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\nabla \times \vec{\mathbf{H}} = \vec{\mathbf{J}}_C + \vec{\mathbf{J}}_D \equiv \vec{\mathbf{J}}_C + \frac{\partial \vec{\mathbf{D}}}{\partial t}$$

$$\vec{\mathbf{E}} = -\nabla\phi - \frac{\partial \vec{\mathbf{A}}}{\partial t}$$

$$\vec{\mathbf{B}} = \nabla \times \vec{\mathbf{A}}$$

A.1.8 Força Magnetomotriz e Energia no Campo

$$\epsilon_{fmm} = -\frac{d\Phi}{dt} = \oint (\vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot d\vec{\mathbf{l}} - \iint \frac{\partial \vec{\mathbf{B}}}{\partial t} \cdot d\vec{\mathbf{S}} = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

$$\vec{\mathbf{S}} = \vec{\mathbf{E}} \times \vec{\mathbf{H}} \qquad \text{(Vector de Poynting)}$$

$$W_{em} = \iiint \frac{1}{2} (\vec{\mathbf{E}} \cdot \vec{\mathbf{D}} + \vec{\mathbf{H}} \cdot \vec{\mathbf{B}})\, dV = \iiint \omega_{em} dV$$

$$-\frac{\partial \omega_{em}}{\partial t} = \vec{\mathbf{E}} \cdot \vec{\mathbf{J}} + \nabla \cdot \vec{\mathbf{S}}$$

A.1.9 Ondas Electromagnéticas

$$\nabla^2 \vec{\mathbf{E}} = \frac{1}{c^2} \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$$

$$\nabla^2 \vec{\mathbf{B}} = \frac{1}{c^2} \frac{\partial^2 \vec{\mathbf{B}}}{\partial t^2}$$

$$c = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}} \approx 3 \times 10^5\, \frac{Km}{s}$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{E}}_o\, e^{i\,(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)}$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{k}} \cdot \vec{\mathbf{B}} = 0$$

$$\vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega \vec{\mathbf{B}}$$

A.1.10 Osciladores

$$\ddot{x} = -\omega_o^2\, x \qquad \Rightarrow \qquad x(t) = c_1\, e^{i\omega_o t} + c_2\, e^{-i\omega_o t}$$

$$\ddot{x} = -\omega_o^2 x - 2\zeta\dot{x} \qquad \Rightarrow \qquad x_a(t) = e^{-\zeta t}\,(c_1\, e^{i\omega_a t} + c_2\, e^{-i\omega_a t}) \qquad (\omega_a = \sqrt{\omega_o^2 - \zeta^2})$$

$$\ddot{x} = -\omega_o^2 x - 2\zeta\dot{x} + A_o\, e^{i\,\omega t} \qquad \Rightarrow \qquad x_f(t) = x_a(t) + \frac{A_o}{{\omega_o}^2 - \omega^2 + 2\,i\,\zeta\,\omega} e^{i\,t\,\omega}$$

$$< \psi > = \underset{\Delta T\to\infty}{\to} Limit\ \frac{1}{\Delta T}\int_0^{\Delta T} \psi(t)dt \qquad < cos^2(\omega t) > = < sin^2(\omega t) > = \frac{1}{2} \qquad < \dot{x}^2 > = \frac{1}{2}\,\omega^2 x_o^2$$

A.2 Matemática

A.2.1 Identidades Algébricas

$$\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = (\vec{\mathbf{a}} \cdot \vec{\mathbf{c}})\vec{\mathbf{b}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{c}} \qquad \vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \cdot \vec{\mathbf{c}} \qquad (\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{d}}) = (\vec{\mathbf{a}} \cdot \vec{\mathbf{c}})(\vec{\mathbf{b}} \cdot \vec{\mathbf{d}}) - (\vec{\mathbf{a}} \cdot \vec{\mathbf{d}})(\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})$$

A.2.2 Identidades Diferenciais

$$\nabla \cdot (\psi\vec{\mathbf{a}}) = (\nabla\psi)\vec{\mathbf{a}} + \psi\nabla \cdot \vec{\mathbf{a}} \qquad \iint_{\partial V} \vec{\mathbf{a}} \cdot d\vec{\mathbf{S}} = \iiint_V \nabla \cdot \vec{\mathbf{a}}\, dV$$

$$\nabla \times (\psi\, \vec{\mathbf{a}}) = (\nabla\psi) \times \vec{\mathbf{a}} + \psi\, \nabla \times \vec{\mathbf{a}} \qquad \iint_{\partial V} \vec{\mathbf{a}} \times d\vec{\mathbf{S}} = -\iiint_V \nabla \times \vec{\mathbf{a}}\, dV$$

$$\nabla \cdot \left(\vec{\mathbf{a}} \times \vec{\mathbf{b}}\right) = \vec{\mathbf{b}} \cdot (\nabla \times \vec{\mathbf{a}}) - \vec{\mathbf{a}} \cdot \left(\nabla \times \vec{\mathbf{b}}\right) \qquad \oint_{\partial S} \vec{\mathbf{a}} \cdot d\vec{\mathbf{l}} = \iint_S (\nabla \times \vec{\mathbf{a}}) \cdot d\vec{\mathbf{S}}$$

$$\oint_{\partial S} \psi\, d\vec{\mathbf{l}} = -\iint_S \nabla\psi \times d\vec{\mathbf{S}}$$

A.2.3 Números Complexos

$$z = x + i\,y = |z|e^{i\theta} \qquad \overline{z} = x - i\,y = |z|e^{-i\theta}$$

$$|z|^2 = x^2 + y^2 = z\,\overline{z} \qquad z^{-1} = \frac{\overline{z}}{|z|^2} = \frac{1}{|z|}e^{-i\theta}$$

$$e^{i\theta} = cos(\theta) + i\,sin(\theta) \qquad \theta = Arg(z) \qquad \tan(\theta) = \frac{y}{x} \qquad \cos(\theta) = \frac{x}{|z|}$$