Simple method to reduce the eccentricity of Binary Black Hole simulations with the EinsteinToolkit

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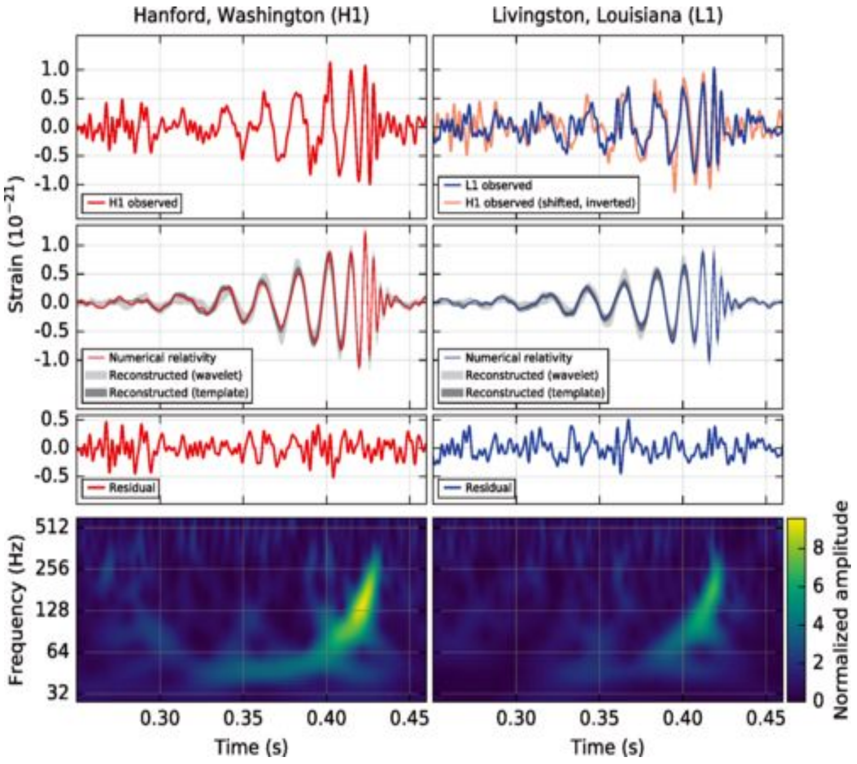
Introduction. Gravitational wave motivation

- Current GW observations from BBHs coalescences are compatible with low eccentricity systems.

- Most of the binary systems are expected to have circularized by the time their GW signals enter the frequency band of the LIGO and Virgo detectors.



Introduction. Framework

- Eccentricity Reduction → within the phenomenological waveform modelling efforts. (Geraint's talk)

- BBH simulations are run with BAM and ET codes to get waveforms.

- Imperfections of NR initial data → waveforms with residual eccentricity.

- **Main Objective:** Establish a systematic procedure to reduce eccentricity.

Initial data for NR simulations of BBH

1) Choose r and $\vec{S}_1, \vec{S}_2$.

2) Choose $\vec{p}/\vec{v}$ of the BH such to result in low eccentricity. (PN approximations)

3) Solve numerically the constraints equations. (usually conformal flatness)

4) Evolve data numerically until eccentricity can be measured reliably from oscillations in $r, \phi, \omega, \phi_{22}, \omega_{22}, \ldots$

5) A correction to the initial parameters is applied, and steps 2-5 (or 1-5) are applied until eccentricity is low enough for applications.

Introduction. Eccentricity estimators

- In numerical simulations, small eccentricity → small residual oscillations with amplitude proportional to the eccentricity are added monotonically changing orbital variables.

- In GR no unique definition of the eccentricity!

- Quantities such as the orbital frequency get eccentricity as

$$\Omega(t) = D(t) + e_{\Omega}(t)\cos(\Omega_r t) \qquad (1)$$

- We define an eccentricity estimator of the form

$$e_{\Omega} = \frac{\Omega(t) - \Omega_{fit}(t)}{2\Omega_{fit}(t)} \qquad (2)$$

Election based on the nearly gauge-independence of the orbital frequency, Ω.

Introduction. Iterative methods

- Iterative schemes $\longrightarrow$ Most efficient way to reduce e_t from numerical simulations.
- Given initial config. $(r_0, p_r^0, p_t^0, \chi_1, \chi_2)$ modify $(p_r^0, p_t^0/r_0)$ to reduce the eccentricity .
- Summary:
  1) Provide initial values for (p_r^0, p_t^0, r_0) data at a given separation/orbital freq.
  2) Measure the eccentricity e_0 from the simulation.
  3) Use e_0 to compute $(\lambda_r, \lambda_t/\delta_r)$.
  4) Apply the correction factors:

$$\begin{aligned} p_r^1 &= \lambda_r^0 p_r^0 \\ p_t^1 &= \lambda_t^0 p_t^0 \end{aligned} \qquad\qquad \begin{aligned} p_r^1 &= \lambda_r^0 p_r^0 , \\ r_1 &= r_0 + \delta r \end{aligned}$$

  5) Run the a simulation with the new parameters and measure the eccentricity e_1 from the new simulation.
  6) If the eccentricity is not low enough go to step 3).

Eccentricity measurement (I)

- A fit to the data (NR or PN) is necessary to compute $e_\Omega(t)$. *(Mathematica NonlinearModelFit function)*

- We use an Ansatz based on the TaylorT3 approximant [Buonanno et al. Phys.Rev.D80,084043(2009)]

$$\theta = [\eta|T_{merg}t_0 - t|/5]^{-1/8}$$
$$A = \frac{a_1\theta^3}{16\pi}\left(1 + b_2\theta^2 + b_3\theta^3 + b_4\theta^4 + b_5\theta^5 + b_6\theta^6\right)$$
$$ansatz = A + a_6\cos(\Omega_0\omega_1 t + t_1)$$

  - T_{merg} is an scale of the merger time.
  - $b_2, b_3, b_4, b_5, b_6 \rightarrow$ analytical coefficients from TaylorT3.
  - $t_0, a_1, a_6, \omega_1, t_1$ are unknown coefficients to fit.
  - Ω_0 is the 3.5PN orbital frequency for quasi-circular orbits.

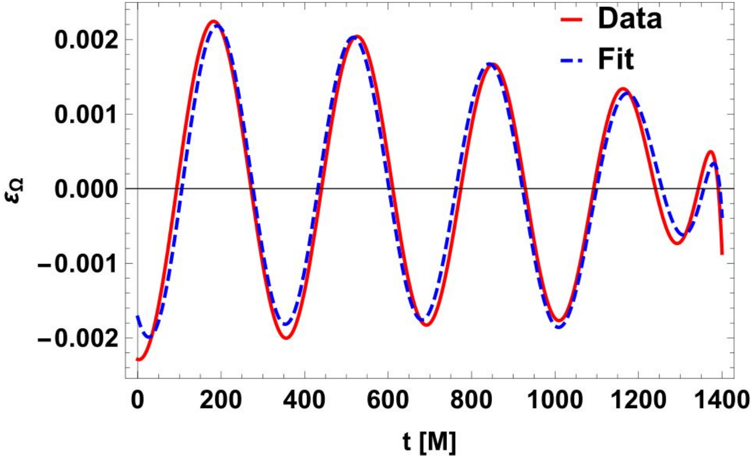
Eccentricity measurement (II)

- The ansatz can be identified with the residual between the data and our model based on 3.5PN approximant.

$$\mathcal{R}(t) = \Omega(t) - \Omega_0$$

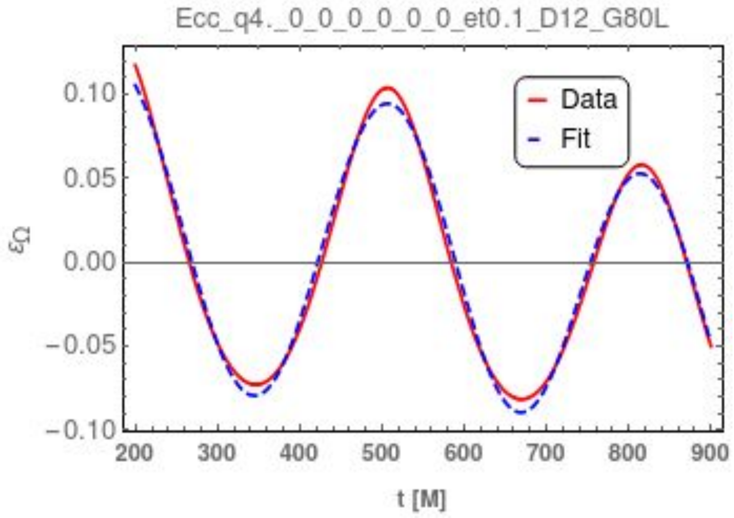
- Once the model is fitted to data, e_Ω can be calculated from the amplitude of the oscillations,

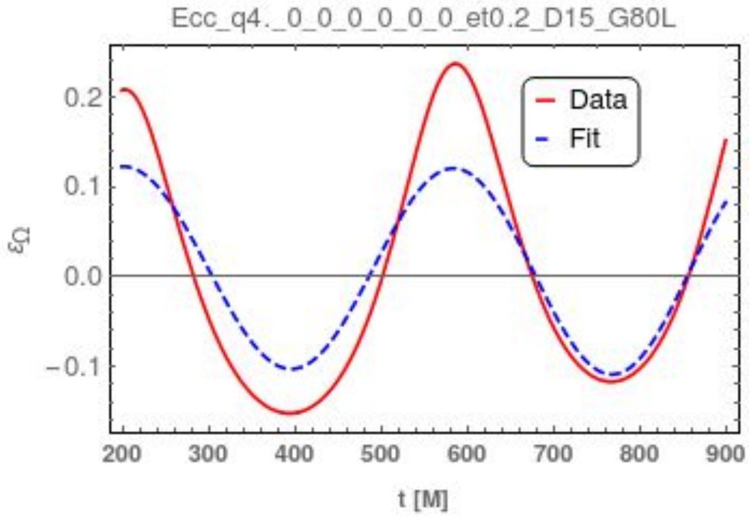
$$e_\Omega = \frac{1}{2}\frac{a_6}{\Omega_0}, \qquad \delta e_\Omega = \frac{1}{2}\frac{\delta a_6}{\Omega_0}.$$



Limit eccentricity measurement

- Eccentricity measurement tested to work up to $e \sim 0.1$
- For higher eccentricities ansatz must be a sum of harmonics.





PN Initial data. Introduction

First step: Provide low eccentric initial values for (r_0, p_r^0, p_t^0) .

- Use PN theory (slow motion, weak field regime) to derive analytical formulas for $p_r(r, \eta, \chi_1, \chi_2)$ and $p_t(r, \eta, \chi_1, \chi_2)$. $\eta = m_1 m_2/(m_1+m_2)^2$
- From PN Hamiltonians in ADMTT gauge up to 3.5PN, we compute analytical expressions for Ω, p_r, p_t.

$$H = H_{orb} + H_{SO} + H_{SS} + H_{SSS},$$

$$p_r = 0, \qquad \left(\frac{\partial H}{\partial r}\right)_{p_r=0} = 0.$$

- Once computed $p_t(r)$, then, one can compute the orbital frequency as

$$\Omega = \left(\frac{\partial H}{\partial p_\phi}\right)_{p_r=0}.$$

- The computation of p_r requires the gravitational wave energy flux,

$$-\frac{dE_{GW}}{dt} = \frac{dM}{dt} + \frac{dH_{circ}}{dt}.$$

PN initial data. Postcircular Initial data

- Using the chain rule

$$\frac{dH_{circ}}{dt} = \left(\frac{dr}{dt}\right)\left(\frac{dH_{circ}}{dr}\right) \rightarrow \left(\frac{dr}{dt}\right) \approx \frac{-\frac{dE_{GW}}{dt}}{\left(\frac{dH_{circ}}{dr}\right)}.$$

- The procedure to obtain a postcircular (PC) expression for p_r can be summarized in the following algorithm:

1. Compute the circular expression for $p_t(r)$.
2. Use the expression for $p_t(r)$ and $p_r = 0$ to compute dH_{circ}/dr .
3. Combine dH_{circ}/dr with the GW flux for the QC orbits, dE_{GW}/dt, to obtain dr/dt.
4. Use Hamilton's equations to compute $dr/dt = \partial H/\partial p_r$. Taylor expand at first order in p_r of the RHS and isolate p_r as a function of dr/dt.
5. From step 4 compute an expression of p_r using the value of dr/dt calculated in step 3.

PN initial data. Postpostcircular Initial data

- Extensively used to construct initial data for EOB dynamics.
- It describes a prescription to compute p_t using the PC information.
- Start with the Hamilton's equation for the radial momentum

$$\frac{dp_r}{dt} = -\frac{\partial H}{\partial r}.$$

- LHS of can be approximated using the chain rule and the PC solution to

$$\frac{dp_r}{dt} = \frac{dp_r}{dr}\frac{dr}{dt} \approx \frac{dp_r^{PC}}{dr}\frac{dr}{dt} = \frac{dp_r^{PC}}{dr}\frac{\partial H}{\partial P_r}.$$

- Combining these equations we get:

$$-\left[\frac{\partial H}{\partial r}\right]_{p_r=p_r^{PC}} \approx \left(\frac{dp_r^{PC}}{dr}\right)\left[\frac{\partial H}{\partial p_r}\right]_{p_r=p_r^{PC}}.$$

  - Given $p_r, r, m_1, m_2, \vec{\chi}_1, \vec{\chi}_2$; one can solve for p_t using a root finding method.

PN correction factors. Introduction (I)

- When PC and PPC initial data produce a simulation with non-negligible eccentricity $\longrightarrow$ iterative step is needed.
- Initial linear momenta corrections: $(p_t, p_r) \rightarrow (\lambda_t p_t, \lambda_r p_r)$.
- One can also choose: $(r, p_r) \rightarrow (r + \delta r, \lambda_r p_r)$.
- We provide analytical expressions to compute $\lambda_t, \lambda_r, \delta r$.
- Computations based on the Quasi-Keplerian 1PN equations of motion.

$$
\begin{aligned}
n_t(t - t_0) &= u - e_t \sin u, \\
(\phi - \phi_0) &= (1 + k) A_{e_\phi}(u), \\
A_{e_\phi}(u) &= 2 \arctan\left[\left(\frac{1 + e_\phi}{1 - e_\phi}\right)^{1/2}\right], \\
r &= a_r(1 - e_r \cos u),
\end{aligned}
$$

- e_t, e_r, e_ϕ are the temporal, radial and angular eccentricities, n_t is called the mean anomaly, *u* is the true anomaly and *k* is the fractional periastron advance per orbit.

PN correction factors. Introduction (II)

- From QK eqs. at 1PN the radial coordinate, r, and the orbital frequency, $\Omega = \dot{\phi}$ up to linear order in eccentricity are:

$$r = a_r(1 - e_r \cos[\Omega_r t]),$$

$$\Omega \equiv \dot{\phi} = \Omega_\phi \left(1 + (e_\phi + e_t) \cos[\Omega_r t]\right) .$$

- We define an eccentricity estimator for the orbital frequency as

$$e_\Omega = \frac{\Omega(t) - \Omega(e{=}0)}{2\Omega(e{=}0)}$$

Introducing eq. for Ω in eq. for e_Ω, one gets

$$e_\Omega = \frac{e_\phi + e_t}{2}.$$

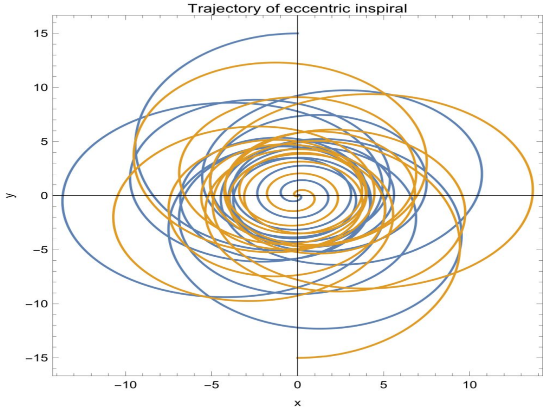
PN correction factors: 1D problem

- From the 1PN QK solution of the equations of motion one can obtain an estimate of the correction factor for p_t .

$$\lambda_t = 1 + \frac{e_\Omega}{2} - \frac{e_\Omega}{2rc^2}(\eta + 2).$$

- Details of the computation in the upcoming paper (Ramos et al 2018).
- 1D computation limiting. Not able to reduce the eccentricity over some threshold.

- This equation can be used to generate eccentric NR simulation given a desired eccentricity (at leading order).



PN correction factors: 2D problem (I)

- 2D problem (λ_t, λ_r) or $(\delta r, \lambda_r)$: follow another procedure.
- **Main goal:** fit the data using an ansatz made of **2 components**: a **non-eccentric** part + an **oscillatory** one.

- Hence, the relative oscillations in Ω can be fitted using an ansatz like

$$\mathcal{R}_\Omega = A + B\cos(\Omega_r t + \Psi),$$

where Ω_r is the frequency of the radial oscillations, and A, B, Ψ are coefficients to be determined.

- We have derived analytical expressions for $\lambda_t, \lambda_r, \delta_r$ in terms of the amplitude, B, and the phase, Ψ of the ansatz.

PN correction factors: 2D problem (II)

Method: Compute analytically $\mathcal{R}_\Omega$, i.e., the difference between the configuration perturbing p_r and p_t and the unperturbed configuration with $e = 0$.

- As a result one obtains (details in an upcoming paper),

$$\lambda_t = 1 + \left[\frac{B}{4\Omega_0} - \frac{B(3\eta+1)}{8r_0 c^2 \Omega_0}\right] \cos\Psi,$$

$$\lambda_r = 1 + \frac{B\eta}{2r_0^{1/2}\Omega_0 |p_r^0|}\left[1 + \frac{1}{r_0 c^2}\right] \sin\Psi.$$

$$\delta r = \frac{Br_0}{2\Omega_0} - \frac{3B(3+\eta)}{4c^2\Omega_0}$$

where $\eta = m_1 m_2/(m_1+m_2)^2$, Ω_0 is the orbital frequency of QC orbits and p_r^0 is the initial radial momentum.

Results. Application to PN (I)

- Solve numerically 3.5PN Hamilton equations in ADMTT gauge.

$$\frac{d\mathbf{X}}{dt} = \frac{\partial H}{\partial \mathbf{P}}, \qquad \frac{d\mathbf{P}}{dt} = -\frac{\partial H}{\partial \mathbf{X}} + \mathbf{F}\,.$$

with **X** and **P** the position and the momentum vectors, in the CM frame, H the Hamiltonian and **F** is the radiation reaction force.

- To compute the orbital frequency from the simulations we use

$$\Omega = |\vec{\Omega}| = \frac{|\vec{r} \times \vec{v}|}{r^2},$$

where $\vec{r}$ is the position vector in the CM frame, $r = |\vec{r}|$, and $\vec{v}$ is the velocity vector.

Results. Application to PN (II)

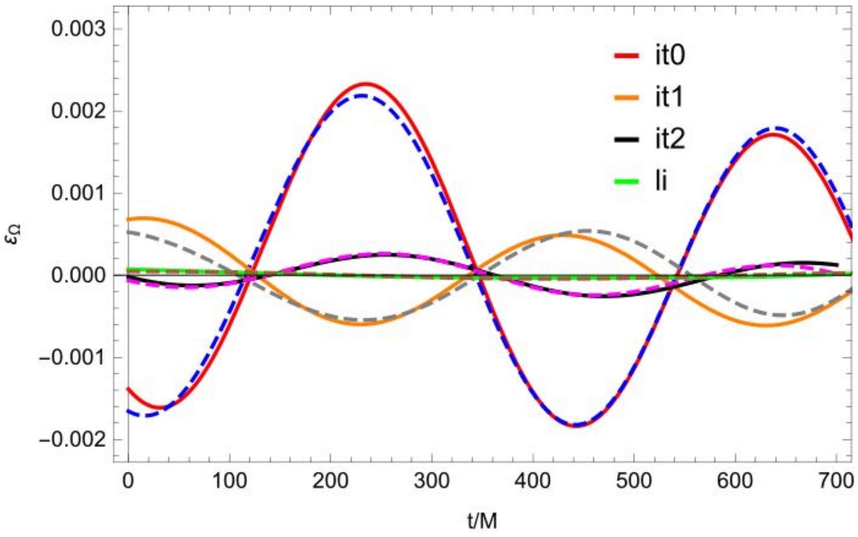
- Test configuration: $q = 4, \chi_{1z} = 0.8, \chi_{2z} = -0.8, D = 12M$
- Sign correction **+1/-1** depending behaviour residual at origin.

Post-Circular correcting for (λ_t, λ_r)					
Iteration	$(\varepsilon_\Omega \pm \delta\varepsilon_\Omega) \cdot 10^{-3}$	$10 \cdot p_t$	$p_r \cdot 10^3$	λ_t	λ_r
0	1.973 ± 0.006	0.56477	0.238712	1.00085	1.19247
1	0.561 ± 0.015	0.56529	0.284657	0.99974	0.94794
2	0.221 ± 0.007	0.56516	0.271206		
Post-Post-Circular correcting for (λ_t, λ_r)					
Iteration	$(\varepsilon_\Omega \pm \delta\varepsilon_\Omega) \cdot 10^{-3}$	$10 \cdot p_t$	$p_r \cdot 10^3$	λ_t	λ_r
0	0.833 ± 0.005	0.56517	0.238712	1.00013	1.19737
1	0.567 ± 0.003	0.56525	0.285827	0.99974	0.96201
2	0.197 ± 0.005	0.56510	0.274971		
Post-Circular correcting for $(\delta r, \lambda_r)$					
Iteration	$(\varepsilon_\Omega \pm \delta\varepsilon_\Omega) \cdot 10^{-3}$	D	$p_r \cdot 10^3$	δr	λ_r
0	1.973 ± 0.006	12.0	0.238712	0.01432	1.19247
1	0.718 ± 0.004	12.0143	0.284657	0.00445	0.999083
2	0.230 ± 0.003	12.0099	0.284396		

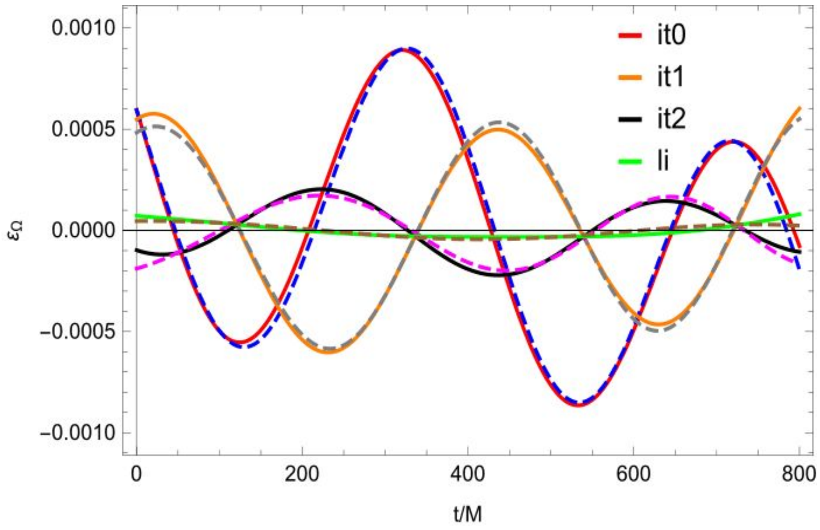
Results. Application to PN (III)

$$q = 4, \chi_{1z} = 0.8, \chi_{2z} = -0.8, D = 12M$$

Post-circular



Post-Post-circular



- Long integration (D=30M) best result, but unpractical for precessing systems.
- Procedure also tested in precessing PN with similar results.

Results. NR set up

- We use ET and BAM (Sascha's talk) codes.
- Both implement a moving puncture version of the BSSN formulation of the Einstein equations.

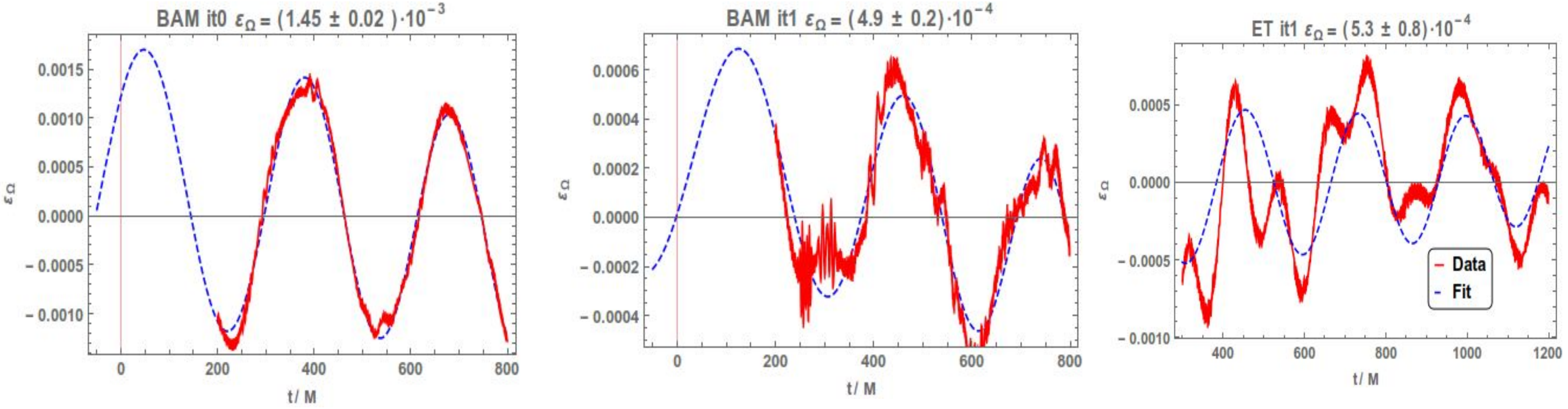
- Initial burst of junk radiation $\rightarrow$ neglect data until $\sim 200M$.

- For ET we use the multipatch Llama thorn: regular cartesian near zone and adapted spherical grids covering the wave zone.

- To avoid gauge oscillations $\rightarrow$ η parameter appearing in the Γ -driver shift condition to 0.25 for simulations used to reduce the eccentricity and $\eta = 1$ for the production ones.

Results. NR example

- $\text{Ex.} \;: q = 2; \chi_1 = \{0., 0., 0.\}; \chi_2 = \{0.353553, 0.353553, 0.5\}; d_0 = 10.8M$
- We use PC initial data. Low resolution N=64 and $\eta = 0.25$



Results. NR initial iteration (I)

- NR and PN different coordinates $\rightarrow$ They agree up to 2PN in ADMTT gauge *(Tichy et al 2002)*.

- Test what works better in NR PC or PPC for 12 configurations.

- All BAM simulations at low resolution (N=64).

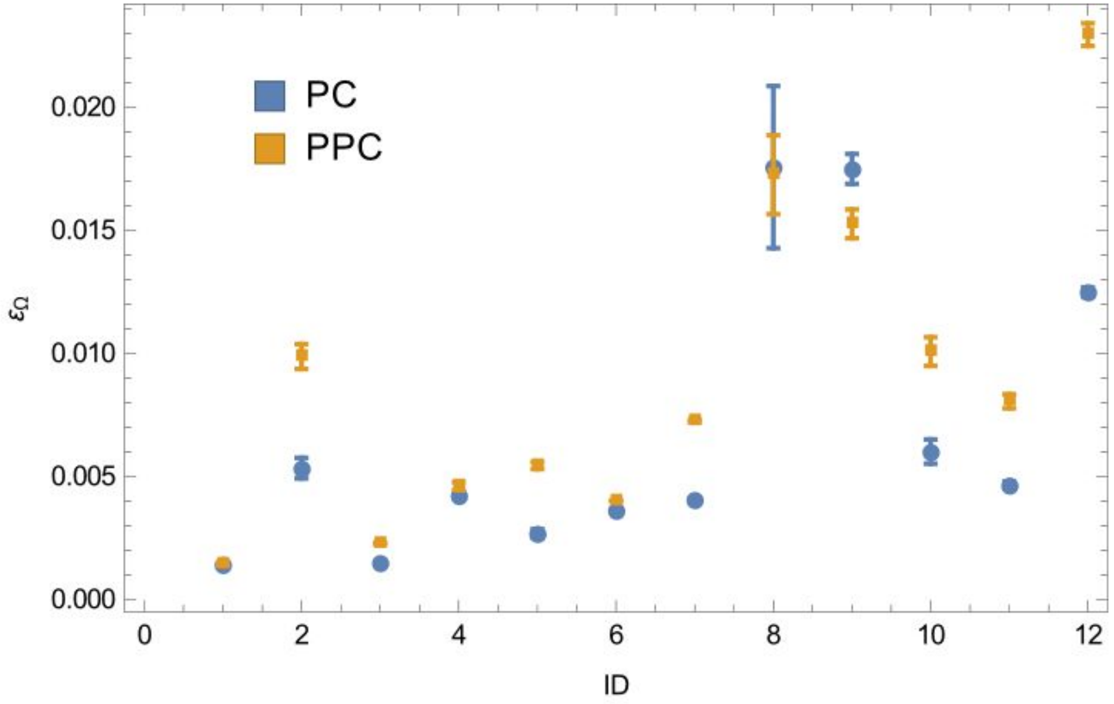
Results. NR initial iteration (II)

ID	Approx.	q	$\vec{\chi}_1$	$\vec{\chi}_2$	D/M	$(\varepsilon_\Omega \pm \delta\varepsilon_\Omega) \cdot 10^3$
1	PC	1	$(0,0,0)$	$(0,0,0)$	11	1.42 ± 0.02
1	PPC	1	$(0,0,0)$	$(0,0,0)$	11	1.43 ± 0.04
2	PC	1	$(0,0,-0.5)$	$(0,0,-0.5)$	11	5.3 ± 0.4
2	PPC	1	$(0,0,-0.5)$	$(0,0,-0.5)$	11	9.8 ± 0.5
3	PC	1	$(0,0,0.5)$	$(0,0,-0.5)$	11	1.5 ± 0.05
3	PPC	1	$(0,0,0.5)$	$(0,0,-0.5)$	11	2.27 ± 0.04
4	PC	2	$(0,0,-0.75)$	$(0,0,-0.75)$	12.6	4.22 ± 0.07
4	PPC	2	$(0,0,-0.75)$	$(0,0,-0.75)$	12.6	4.61 ± 0.16
5	PC	2	$(0,0,0)$	$\vec{\alpha}$	10.8	2.68 ± 0.17
5	PPC	2	$(0,0,0)$	$\vec{\alpha}$	10.8	5.43 ± 0.13
6	PC	2	$(0,0,0)$	$\vec{\beta}$	10.8	3.61 ± 0.017
6	PPC	2	$(0,0,0)$	$\vec{\beta}$	10.8	4.003 ± 0.018
7	PC	4	$(0,0,-0.8)$	$(0,0,0.8)$	11	4.05 ± 0.07
7	PPC	4	$(0,0,-0.8)$	$(0,0,0.8)$	11	7.25 ± 0.06
8	PC	4	$(0,0,-0.8)$	$(0,0,-0.8)$	11	17.9 ± 1.5
8	PPC	4	$(0,0,-0.8)$	$(0,0,-0.8)$	11	17.5 ± 1.5
9	PC	4	$(0,0,0.8)$	$(0,0,-0.8)$	11	17.4 ± 0.6
9	PPC	4	$(0,0,0.8)$	$(0,0,-0.8)$	11	15.3 ± 0.5
10	PC	4	$(0,0,0.8)$	$(0,0,0.8)$	11	5.5 ± 0.5
10	PPC	4	$(0,0,0.8)$	$(0,0,0.8)$	11	9.9 ± 0.6
11	PC	8	$(0,0,0.5)$	$(0,0,-0.5)$	11	4.64 ± 0.14
11	PPC	8	$(0,0,0.5)$	$(0,0,-0.5)$	11	8.0 ± 0.2
12	PC	8	$(0,0,-0.5)$	$(0,0,-0.5)$	11	12.49 ± 0.18
12	PPC	8	$(0,0,-0.5)$	$(0,0,-0.5)$	11	22.9 ± 0.4

$$\vec{\alpha} = (0.3535, -0.3535, -0.5), \quad \vec{\beta} = (0.3535, -0.3535, 0.5)$$

Results. NR initial iteration (III)

- One observes that PC initial data work better in NR than PPC. (except for ID9)

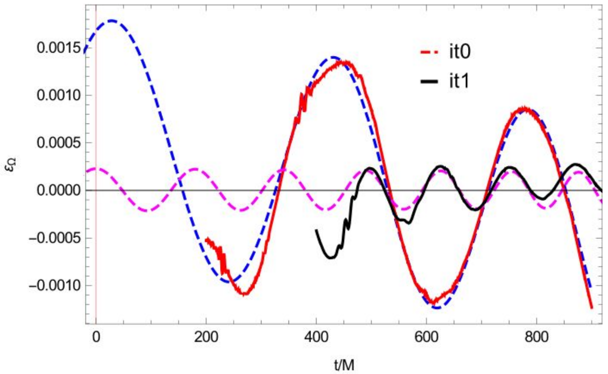


Results. Reduced NR simulations (I)

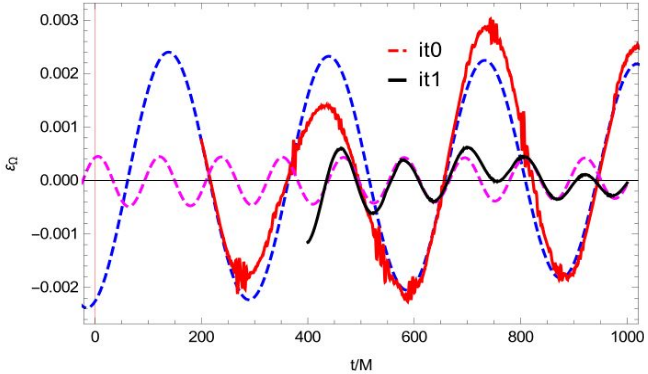
ID	Iteration	Code	N	η	q	$\vec{\chi}_1$	$\vec{\chi}_2$	D/M	$10 \cdot p_t$	$10^3 \cdot p_r$	λ_t	λ_r	$(\varepsilon_\Omega \pm \delta\varepsilon_\Omega) \cdot 10^3$
1	0	BAM	64	0.25	1	$(0,0,0)$	$(0,0,0)$	12	0.850941	0.53833	0.9997	0.8695	1.42 ± 0.02
	1	BAM	96	0.25	1	$(0,0,0)$	$(0,0,0)$	12	0.850686	0.468113			0.22 ± 0.02
13	0	BAM	64	0.25	1	$(0,0,-0.5)$	$(0,0,0.5)$	11	0.901836	0.722706	0.9998	0.8581	1.24 ± 0.03
	1	BAM	64	0.25	1	$(0,0,-0.5)$	$(0,0,0.5)$	11	0.901688	0.620187			0.27 ± 0.02
14	0	BAM	64	0.25	1	$(0,0,0.5)$	$(0,0,0.5)$	11	0.874251	0.601797	0.999237	0.9346	1.64 ± 0.03
	1	BAM	64	0.25	1	$(0,0,0.5)$	$(0,0,0.5)$	11	0.873583	0.562465			0.39 ± 0.03
15	0	ET	64	0.24	1.5	$(0,0,-0.6)$	$(0,0,0.6)$	10.8	0.868557	0.699185	0.999737	0.9168	1.12 ± 0.05
	1	ET	80	0.24	1.5	$(0,0,-0.6)$	$(0,0,0.6)$	10.8	0.856941	0.641051			0.84 ± 0.165
16	0	ET	64	0.2314	1.75	$(0,0,0.6)$	$(0,0,-0.6)$	10.8	0.856941	0.685199	0.999643	0.8525	1.52 ± 0.08
	1	ET	80	0.2314	1.75	$(0,0,0.6)$	$(0,0,-0.6)$	10.8	0.856636	0.584173			0.43 ± 0.07
17	0	ET	64	0.2314	1.75	$(0,0,-0.6)$	$(0,0,0.6)$	10.8	0.834827	0.649957	0.999903	0.8941	1.12 ± 0.14
	1	ET	80	0.2314	1.75	$(0,0,-0.6)$	$(0,0,0.6)$	10.8	0.834746	0.581178			0.66 ± 0.13
18	0	BAM	64	0.2222	2	$(0,0,0.75)$	$(0,0,0.75)$	11.1117	0.760924	0.450647	0.999937	0.6566	2.38 ± 0.07
	1	BAM	96	0.2222	2	$(0,0,0.75)$	$(0,0,0.75)$	11.1117	0.760876	0.295898			0.47 ± 0.05
19	0	BAM	80	0.1875	3	$(0,0,0)$	$(0,0,0)$	10	0.72377	0.575703	0.999914	0.8629	1.41 ± 0.07
	1	BAM	64	0.1875	3	$(0,0,0)$	$(0,0,0)$	10	0.723708	0.496774			0.29 ± 0.24
7	0	BAM	64	0.16	4	$(0,0,-0.8)$	$(0,0,0.8)$	11	0.559207	0.336564	0.998501	0.7341	4.05 ± 0.07
	1	BAM	64	0.16	4	$(0,0,-0.8)$	$(0,0,0.8)$	11	0.558369	0.24708			0.45 ± 0.4
20	0	BAM	64	0.0987	8	$(0,0,0.5)$	$(0,0,0.5)$	11	0.102969	0.345755	1.00066	1.3512	2.2 ± 0.4
	1	BAM	64	0.0987	8	$(0,0,0.5)$	$(0,0,0.5)$	11	0.139132	0.345985			0.45 ± 0.4
21	0	BAM	64	0.2222	2	$(0,0,0)$	$(0.4949,0.4949,0)$	10.8	0.811783	0.649957	0.999788	0.9802	6.4 ± 1.7
	1	ET	80	0.2222	2	$(0,0,0)$	$(0.4949,0.4949,0)$	10.8	0.811611	0.581178			0.40 ± 0.05
22	0	BAM	64	0.2222	2	$(0,0,0)$	$(0.1767,0.1767,0)$	10.8	0.812379	0.610965	0.999534	0.9009	1.46 ± 0.02
	1	ET	80	0.2222	2	$(0,0,0)$	$(0.1767,0.1767,0)$	10.8	0.812001	0.550427			0.54 ± 0.05
23	0	ET	64	0.2222	2	$(0,0,0)$	$(-0.1767,0.1767,0.5)$	10.8	0.793749	0.53149	0.99994	0.881549	1.88 ± 0.01
	1	ET	80	0.2222	2	$(0,0,0)$	$(-0.1767,0.1767,0.5)$	10.8	0.793701	0.468535			0.28 ± 0.05
24	0	ET	64	0.2222	2	$(0,0,0)$	$(-0.3535,0.3535,0.5)$	10.8	0.7935	0.531374	0.999772	0.843376	2.13 ± 0.03
	1	ET	80	0.2222	2	$(0,0,0)$	$(-0.3535,0.3535,0.5)$	10.8	0.79332	0.448148			0.48 ± 0.05
25	0	ET	64	0.2222	2	$(0,0,0)$	$(-0.3535,0.3535,0.)$	10.8	0.812118	0.611108	0.999848	0.895657	1.78 ± 0.07
	1	ET	80	0.2222	2	$(0,0,0)$	$(-0.3535,0.3535,0.)$	10.8	0.811994	0.547343			0.69 ± 0.07

Results. Reduced NR simulations (II)

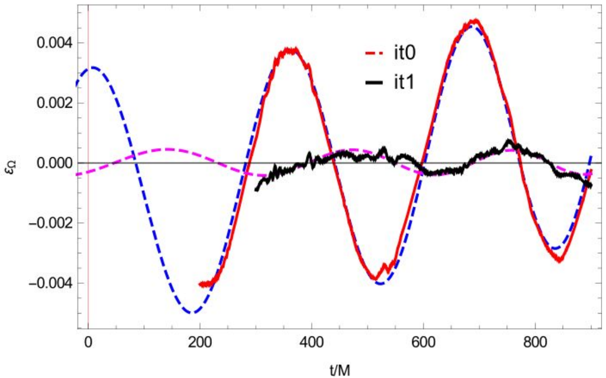
$q1.\_\_(0,0,0)\_\_(0,0,0)\_\_D12$



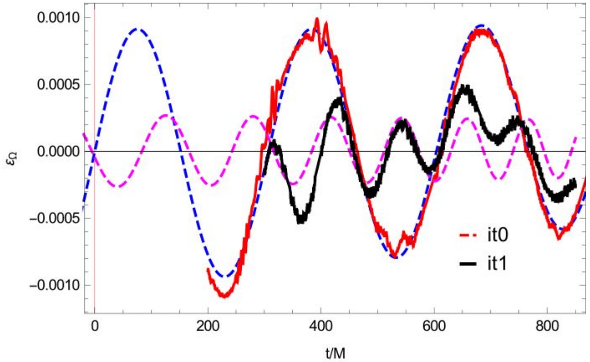
$q2\_\_(0,0,0.75)\_\_(0,0,0.75)\_\_D11.1117$



$q4\_\_(0,0,-0.8)\_\_(0,0,0.8)\_\_D11$



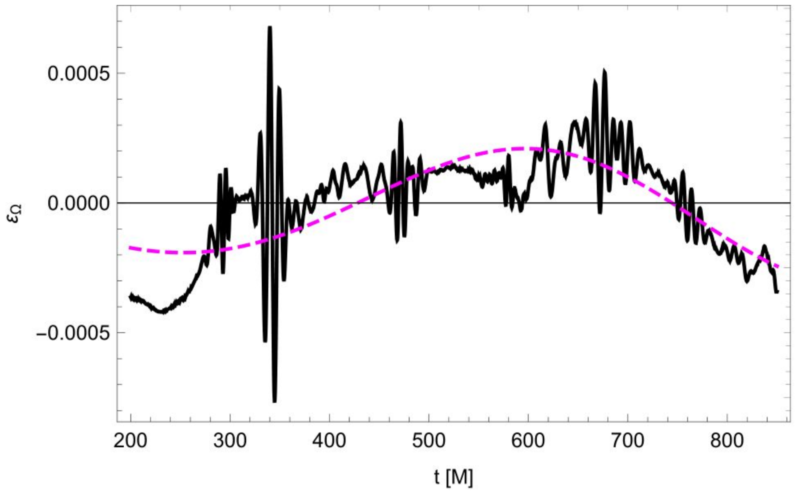
$q2\_\_(0,0,0)\_\_(-0.17,0.17,0.5)\_\_D10.8$



Results. NR separation correction (I)

- We test also the separation correction in NR.
- For the configuration ID2, $q = 1, \quad \chi_{1z} = -0.5, \quad \chi_{2z} = 0.5, \quad D = 11M$

Iteration	Code	N	δr	λ_r	$(\varepsilon_\Omega \pm \delta\varepsilon_\Omega) \cdot 10^{-3}$
0	BAM	64			1.24 ± 0.03
1	BAM	64	-0.0023	0.8581	0.2 ± 0.2



Conclusions

- PN based method to reduce eccentricity from numerical simulations.
- Method tested for two different moving puncture binary black hole evolution codes (ET and BAM).
- Method tested for different mass ratios and spin orientations.
- The method provides eccentricities of $O(10^{-4})$ in one iteration.
- **Limitations:** short evolutions unable an accurate eccentricity measurement, the higher the initial eccentricity the more iteration will be needed.

- Open questions: could the method work for spectral codes? could the method work for neutron star simulations?