A Newtonian problem as a testbed for studying complicated orbital phenomena around Kerr BH's

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An overview

- The Euler problem: The analogue of Kerr, in Newtonian gravity.
- A list of similarities relating the two problems (and some differences).
- How to use this analogy in order to study various relativistic problems that are difficult to handle in the framework of General Relativity. $\rightarrow$ Gain insight about how things might work in Kerr, and yield quantitative estimates.

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The Euler's problem

- A simple Newtonian problem with interesting properties: the gravitational field of two fixed point-masses (~ 1760):

$$
\begin{aligned}
V &= -\frac{G\,m_1}{r_1} - \frac{G\,m_2}{r_2} \\
&= -\frac{G\,m_1}{|\mathbf{r} - a\hat{\mathbf{z}}|} - \frac{G\,m_2}{|\mathbf{r} + a\hat{\mathbf{z}}|}.
\end{aligned}
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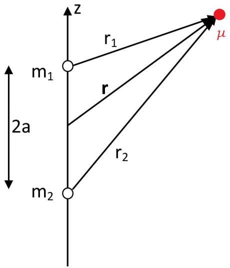
- This problem is better described in spheroidal coordinates:



$$\xi = \frac{r_1 + r_2}{2a}, \quad \eta = \frac{r_1 - r_2}{2a}.$$

$a\,\xi = $ the analogue of r_K

$\eta = $ the analogue of $\cos\theta_K$

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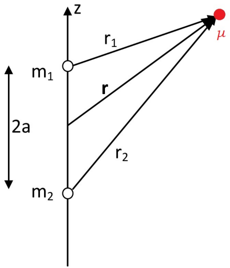

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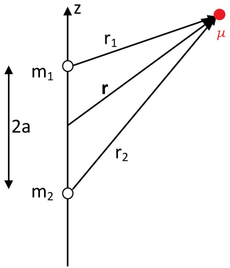

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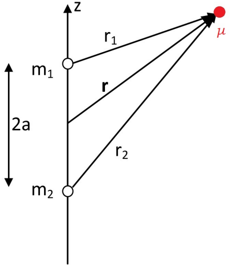

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Similarity #1

- A test-body orbiting the Euler field in Newtonian gravity forms an integrable problem. (Just like the geodesic equations in Kerr.)
  [*Integrals of motion in involution = number of dimensions.*]

$$E \ (\text{energy}),$$
$$L_z \ (z - \text{ang. momentum}),$$
$$Q \ (\text{Carter const for Kerr, no name in Euler}),$$
$$\mu \ (\textit{only} \text{ in Kerr})$$

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- The gravitational field of a Kerr BH is **oblate**, while Euler's potential is prolate by construction! By replacing $\mathbf{a} \to \mathbf{ia}$ though, Euler's problem (although not physically realizable) gives a **real** potential (when $m_1 = m_2$), which is oblate:

$$V = -\frac{M\xi}{a(\xi^2 + \eta^2)},$$

  when $m_1 = m_2 = M/2$.

- As in Kerr, bounded orbits precess around the equatorial plane, due to the non-vanishing quadrupole moment.
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Similarity #3

- The 3rd integral of motion in the Euler problem yields exactly the same form (see next talk) as the Carter constant of Kerr, if we make the following (rational) replacements

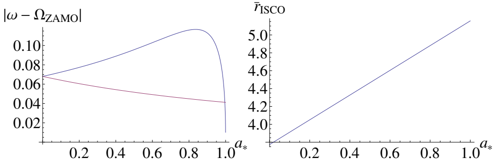
$$\eta \to \cos\theta \ , \ \ p_\eta^2(1-\eta^2) \to p_\theta^2 \ , \ \ E_{\mathrm{N}} \to \frac{1}{2}(1-E^2) \ ,$$

assuming a test-body of mass 1.

- In both cases the 3rd integral is quadratic to momenta and has a similar physical meaning (generalized ang. momentum).

Similarity #4

- There is an innermost stable circular orbit (ISCO) for Euler (as in Kerr).
  Actually there are two ISCO radii for Kerr (for pro- and retro-grade orbits). No such distinction exists for Euler.
- If one considers an equal $|\omega_{\rm phys}|$-related average of $r_{\rm ISCO}$ for Kerr,



  then the $\bar{r}^K_{\rm ISCO}(a)$ has very similar dependence as the $r_{\rm ISCO}$ for Euler.

$$r^E_{\rm ISCO} \equiv a\,\xi_{\rm ISCO} = \sqrt{3}\,a.$$

- Although the Kepler problem ($a = 0$) has no ISCO, the Euler has an ISCO even at $a \to 0$ (but $a \neq 0$), as in Schwarzschild.

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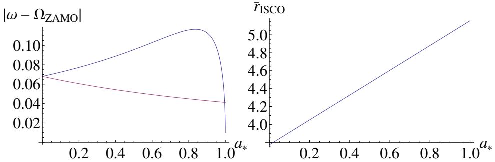

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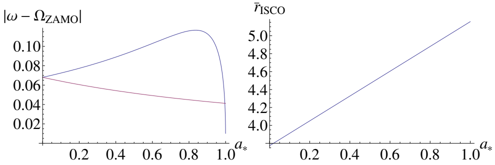

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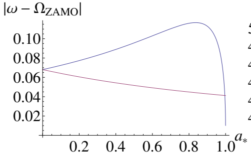

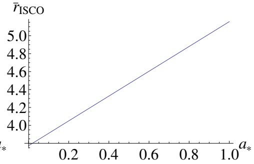

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Similarity #5

- The invariant frequencies related to bounded geodesic orbits (rotation rate, perihelion precession, azimuthal precession) in Kerr are what can be measured (through e.g.,GW).
- The Euler problem is characterized by 3 corresponding frequencies. These frequencies are expressed by similar but not the same functions (see next talk). [The V_θ potential is the same, while the V_r potentials are similar quartic functions.]
- The frequencies share common qualitative characteristics:
(i) $\omega_r/\omega_\theta/\omega_\phi \to 1:1:1$ for $r \to \infty$,
(ii) there is a separatrix ($\omega_r = 0$), separating eternal (bounded) orbits from plunging orbits,
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Similarity #6

- Both Kerr and Euler possess pairs of orbits characterized by the same triplet of frequencies (see next talk):

$$\omega_r^{(1)} = \omega_r^{(2)} \quad , \quad \omega_\theta^{(1)} = \omega_\theta^{(2)} \quad , \quad \omega_\phi^{(1)} = \omega_\phi^{(2)} ,$$

  see *Warburton, Barack, Sago (2013)* for Kerr.

- There is not always a 1-1 correspondence between frequencies and orbital characteristics.

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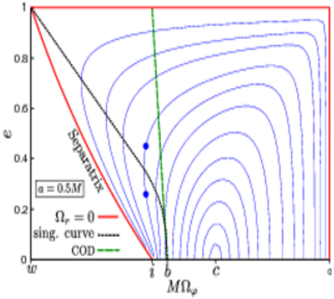

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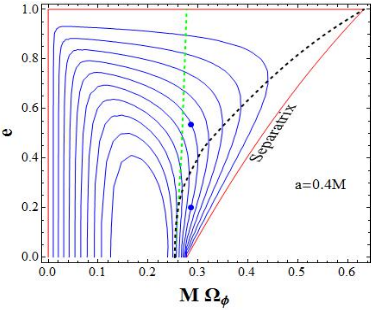

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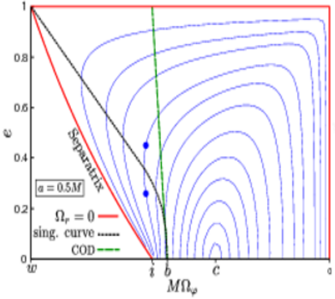

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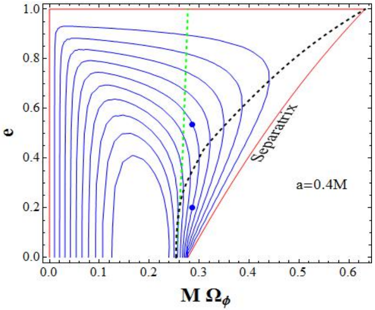

Similarity #7 C. Will (2010)

- The relativistic (Geroch-Hansen) multipole moments of Kerr are

$$M_{2k} = M(-a^2)^k \ , \ S_{2k+1} = Ma(-a^2)^k .$$

  They are related by the iterative equation $M_{2k} = -a^2 M_{2k-2}$ (same for S_k's).

- Will showed that in Newtonian gravity the only axially symmetric, and reflection symmetric, potential that posesses a Carter-like constant (bilinear with respect to momenta) is the (prolate) Euler potential which has almost the same spectrum of mass moments:

$$M_{2k} = M(a^2)^k \quad \text{(prolate)}$$
$$\text{but with } a \to i\,a$$
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- Kerr metric provides a separable set of differential equations when analysing propagation of perturbations.
- Looking for a Newtonian axially-symmetric potential that renders the wave-equation $\Box\Psi = -\kappa V \Psi$ separable, one obtains two such solutions:
  i. In spherical coordinates: the monopole solution.
  ii. In spheroidal coordinates: the Euler's problem.
- For $\kappa = 4\omega^2/c^4$ (in ii) the corresponding azimuthal differential equations are (1) identical, and (2) the radial o.d.e's are quite similar, compared with the corresponding scalar perturbations in Kerr. (The radial o.d.e. is formally the same with Kerr's, while the function multiplying the term $R(r)$ differs from the corresponding Kerr term only beyond the second to leading term at large radii.)

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Exploiting the similarities. A

- The self force in Kerr is not well known except at some special cases. (Great improvement has been done over the year; see Leor's paper series).
- The exact knowledge of self force might be crucial (1) to detect an EMRI signal (by matched filtering), and (2) to extract astrophysical information about the source.
- There is especially one case where the very knowledge of self-force might be crucial for detection. When the orbit crosses a resonance (commensurate ratio of freqs) the orbit does **not** sweep a whole region in phase space. Thus the perturbative methods used to compute the adiabatic changes of the "constants" of motion might be highly misleading (see *Flanagan and Hinderer (2008 & 2010)*).

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- We will evolve the Euler problem near resonance, under an artificial force that has qualitative similarities with what one expects for the Kerr case: [symmetric with respect to the equatorial plane, no ϕ-dependence, having the same $r \to \infty$ dependence (from quadrupole formula)]. E.g. the dessipative force could be

$$\mathbf{F}_{\mathrm{GR}} \propto -F_{\mathrm{G}} \left(\frac{u}{c}\right)^4 \frac{\mathbf{u}}{c}.$$

- By comparing with the evolution from the average losses, we could estimate the amount of deviation of the EMRI evolution when we the evolution is based on adiabatic approximations, and see if we could miss signals.

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Exploiting the similarities B

- One of the ways suggested to look for non-Kerr objects was based on the exact integrability of Kerr geodesics. Thus a slightly different generic geometry should correspond to a slightly non-integrable system. This will leave its imprints in a passage of an orbit near a ''strong" resonance.
- The "plateau effect", that is, the locking of the ratio of frequencies at a comensurate ratio for a finite time (due to KAM theorem) will reveal the non-Kerrness.
- Previous study of this phenomenon for Kerr was based on the wrong adiabatic evolution of the orbit (through the quadrupole formula). *TA, Gerakopoulos, Contopoulos (2009)*

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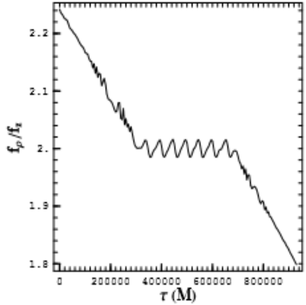
- 

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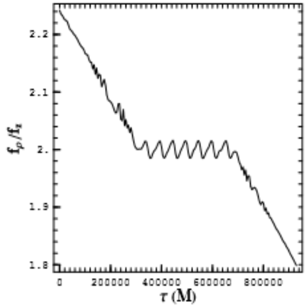
- 

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- If the oblate Euler problem is endowed with an extra small mass μ at the origin, the problem is no more integrable. The ratio μ/M could be used as a non-integrability parameter.
- The spectrum of multipoles is not that of Kerr anymore.
- Our plan is to investigate numerically the evolution of an orbit near resonance of this deviating Euler problem and compare the duration of the plateau effect due to self-force with that due to average losses of constants.

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  "As far as the resonance condition $\omega_r = 2k\omega_\theta$ is never met, the circular orbit will remain circular. Otherwise the radial oscillations will grow."
- Resonance condition is probably never met in Kerr (not rigorous answer yet). *We plan to investigate this thoroughly.*
- The oblate Euler problem **has** orbits that meet the resonance condition.
- We integrate that numerically, under some artificial self-force

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- We integrate that numerically, under some artificial self-force

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- We have integrated numerically orbits, under some artificial self-force mimicking the gravitational self-force.
- If we are far from resonance, radial (ξ) oscillations exist, but do not grow, due to θ oscillations of the self-force.
- When we evolve "resonant orbits", the radial oscillations actually grow but not indefinitely. There is something like a beating effect.
- We believe there are 2 reasons for that:
(i) As the orbit evolves the resonance is moving around.
(ii) The self-force acts as a dissipative mechanism for the radial oscillations.
- The radial oscillations work approximately as:

$$\ddot{\xi} + 2\gamma_\xi \dot{\xi} + \omega_\xi^2 \xi = F_{0\theta} \cos(2\omega_\theta t)$$

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