100 Years of Gravitational Lensing

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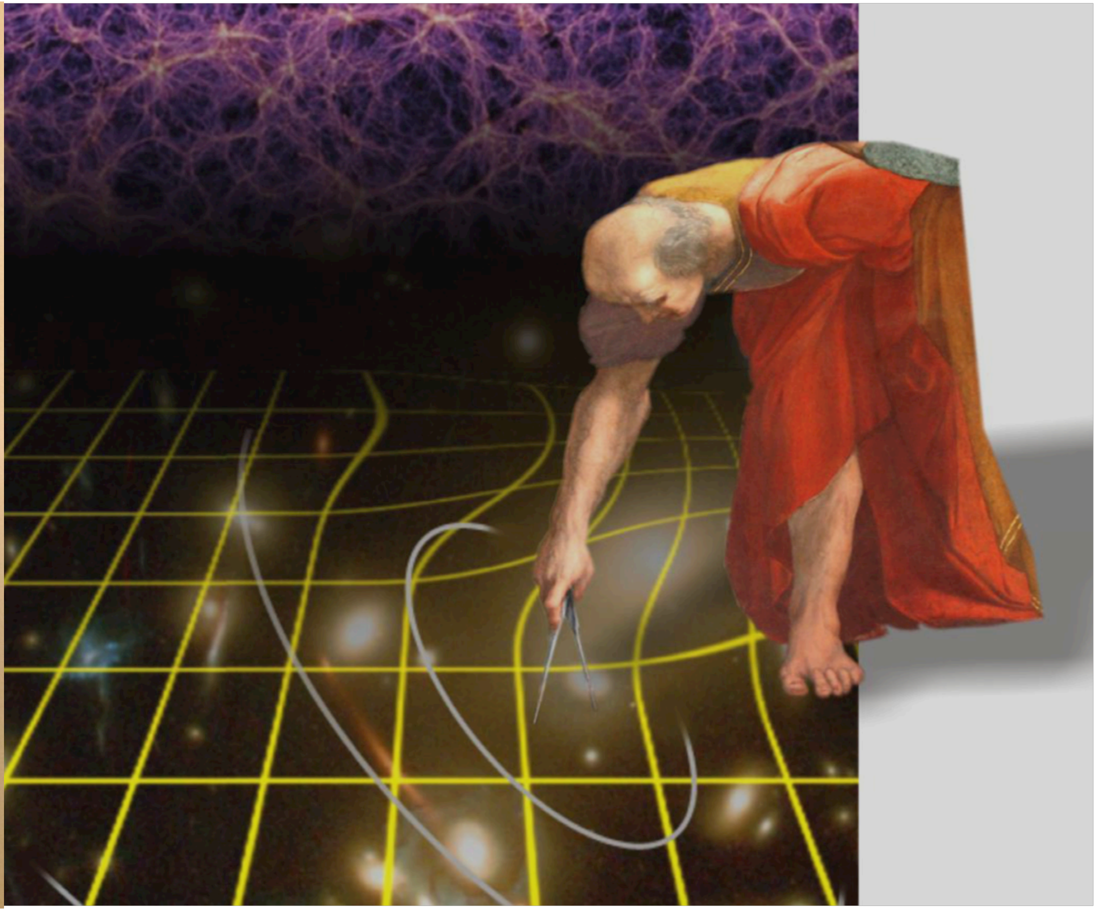
FCT

from Einstein to Euclid

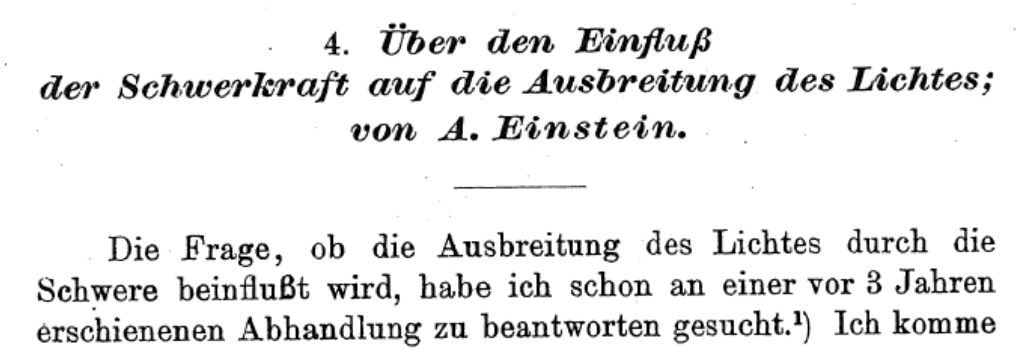
(R. Piccioni - Cosmicomix)



Light deflection in a gravitational field:

the basic Gravitational Lensing effect

Einstein 1911, Annalen der Physik
“On the influence of gravity on the propagation of light ”



4. *Über den Einfluß der Schwerkraft auf die Ausbreitung des Lichtes; von A. Einstein.*

Die Frage, ob die Ausbreitung des Lichtes durch die Schwere beeinflußt wird, habe ich schon an einer vor 3 Jahren erschienenen Abhandlung zu beantworten gesucht.1) Ich komme

Application to the light deflection near the Sun

Discussion of the problem based on the **Principle of Equivalence**

Gravitational field ←→ Acceleration of the reference system

The principle of relativity also applies to systems that are accelerated relative to one another

Bern
1907

> I was sitting in a chair in the patent office in Bern when all of a sudden a thought occurred to me: 'If a person falls freely he will not feel his own weight'. I was startled. This simple thought made a deep impression upon me. It impelled me towards a theory of gravitation.

The happiest thought
(Einstein's apple)

Photon travel time from ceiling to floor $t = h/c$
Floor's velocity increased by $g\,h/c$
Frequency shift $\Delta\nu/\nu = \Delta v/c = gh/c^2$
Time dilation $\Delta t/t = gh/c^2$
Equivalence principle → time dilation $= \Delta\Psi/c^2$

(This was before GR) : Minkowski spacetime in an accelerated frame

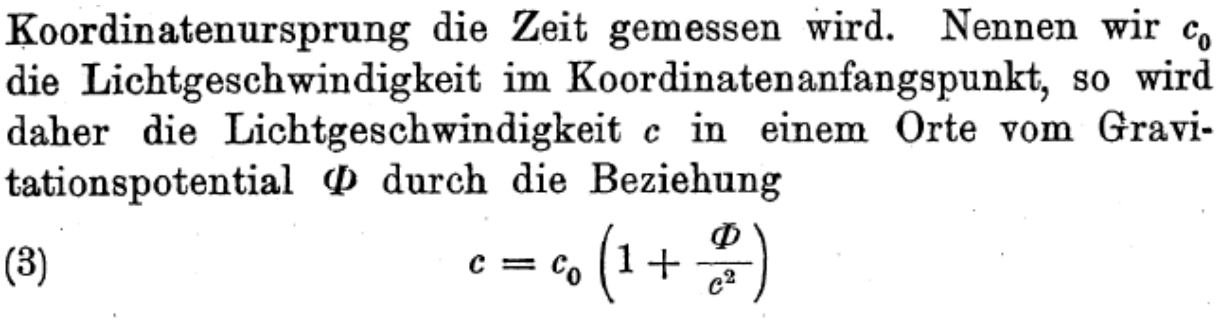
$$ds^2 = -\left(1 + \frac{2\Psi}{c^2}\right) c^2 dt^2 + dx^2$$

Travelling time of a light ray computed from

$$\frac{cdt}{dx} = \left(1 + 2\Psi/c^2\right)^{-1/2} \approx 1 - \frac{\Psi}{c^2}$$

Speed of light decreases in the gravitational field (Ψ < 0), there is an effective refraction index n = c/v > 1

$$\frac{v}{c} \approx 1 + \frac{\Psi}{c^2}$$



Koordinatenursprung die Zeit gemessen wird. Nennen wir c_0 die Lichtgeschwindigkeit im Koordinatenanfangspunkt, so wird daher die Lichtgeschwindigkeit c in einem Orte vom Gravitationspotential Φ durch die Beziehung

$$c = c_0 \left(1 + \frac{\Phi}{c^2}\right) \tag{3}$$

With a non-uniform gravitational potential there is a bending of the light trajectory

$$\frac{(c_1 - c_2)\,dt}{1} = -\frac{\partial c}{\partial n'}\,dt;$$

Fermat's principle:
The path that light takes between 2 points
is the one that takes the least time:

The extremal light path followed from A to B must verify,

$$\delta \int_A^B n(\vec{x})dx = 0 = \delta \int_{\lambda_A}^{\lambda_B} n(\vec{x}(\lambda))\frac{dx}{d\lambda}d\lambda = \delta \int_{\lambda_A}^{\lambda_B} n(\vec{x}(\lambda))|\dot{\vec{x}}|d\lambda = \delta \int_{\lambda_A}^{\lambda_B} L(x,\dot{x};\lambda)d\lambda$$

evolution of \vec x :

$$\frac{d}{d\lambda}\frac{\partial L}{\partial \dot{x}_i} - \frac{\partial L}{\partial x_i} = 0 = \frac{d}{d\lambda}(n(\vec{x})\vec{u}_x) - \vec{\nabla}n = n\dot{\vec{u}}_x + (\vec{\nabla}n.\vec{u}_x)\vec{u}_x - \vec{\nabla}n.$$

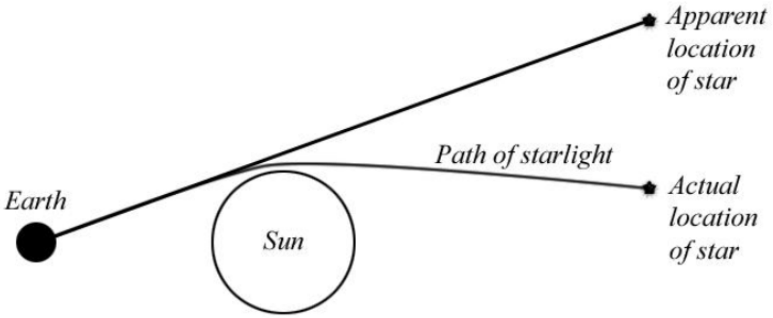
$\vec{u}_x$ is a normalized vector along $\vec{x}$, i.e., tangent to the path.

$$\dot{\vec{u}}_x = \frac{d^2\vec{x}}{d\lambda^2} = \frac{1}{n(\vec{x})}\left(\vec{\nabla}n - (\vec{\nabla}n.\vec{u}_x)\vec{u}_x\right) = \frac{1}{n(\vec{x})}\vec{\nabla}_\perp n(\vec{x}) = (1+\frac{\Psi}{c^2})(-\frac{1}{c^2}\vec{\nabla}_\perp\Psi) \approx -\frac{1}{c^2}\vec{\nabla}_\perp\Psi$$

Total deflection angle:

$$\vec{\alpha} = -\frac{1}{c^2}\int_{\lambda_A}^{\lambda_B} \dot{\vec{u}}_x \, d\lambda = \frac{1}{c^2}\int_{\lambda_A}^{\lambda_B} \vec{\nabla}_\perp \Psi \, d\lambda$$

Light path pulled towards the deflector → image appears to be away from the deflector



$$\alpha = \frac{1}{c^2}\int_{\vartheta=-\frac{\pi}{2}}^{\vartheta=+\frac{\pi}{2}} \frac{k\,M}{r^2}\cos\vartheta \cdot ds = \frac{2\,k\,M}{c^2\,\Delta}$$

At the Sun’s limb, the deviation is 0.87 arcsec

This is the so-called Newtonian deflection.

The earliest known mention of light being deflected by massive objects is the first query in Newton’s Opticks in 1704 :

Query 1: ’Do not Bodies act upon Light at a distance, and by their action bend its Rays; and is not this action strongest at the least distance?”

Einstein’s prediction had been derived by von Soldner in 1801 - Newtonian corpuscular theory of light - deviation of particles under scattering with v=c

(and Cavendish 1784 - unpublished manuscript)

(1915 with GR) : space curvature

$$ds^2 = -\left(1 + \frac{2\Psi}{c^2}\right) c^2 dt^2 + \left(1 - \frac{2\Phi}{c^2}\right) dx^2$$

Travelling time of a light ray computed from

$$\frac{cdt}{dx} = \left(1 + 2\Psi/c^2\right)^{-1/2} \left(1 - 2\Phi/c^2\right)^{1/2} \approx 1 - \frac{\Psi + \Phi}{c^2}$$

In GR: Φ = Ψ → **the factor of 2**

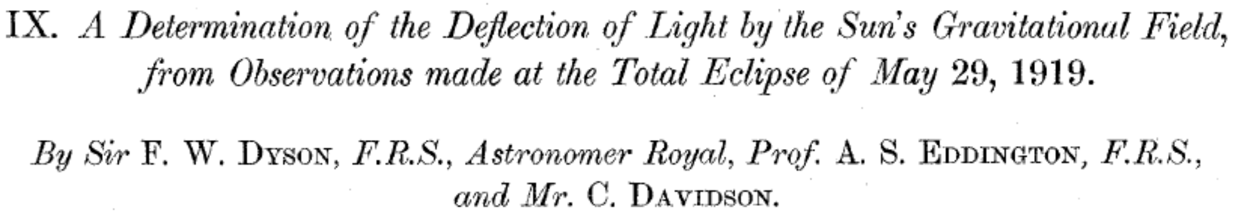
$$\vec{\alpha} = \frac{2}{c^2} \int_{\lambda_A}^{\lambda_B} \vec{\nabla}_\perp \Psi \, d\lambda$$

At the Sun’s limb, the deviation is 1.75 arcsec

The 1919 Eclipse expedition:

Measuring the deflection of light in the Sun's gravitational field

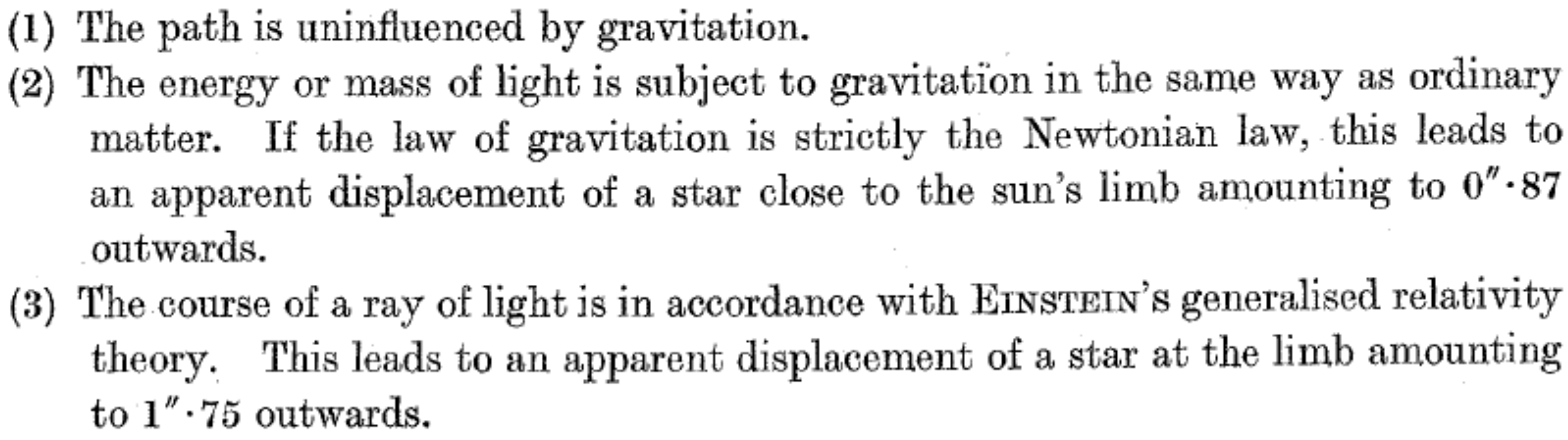
Dyson, Eddington and Davidson 1919, Phil. Trans. Roy. Soc. London



IX. *A Determination of the Deflection of Light by the Sun's Gravitational Field, from Observations made at the Total Eclipse of May* 29, 1919.

By Sir F. W. DYSON, *F.R.S., Astronomer Royal, Prof.* A. S. EDDINGTON, *F.R.S.,*
and Mr. C. DAVIDSON.

What is the effect produced by a gravitational field on the path of light?



(1) The path is uninfluenced by gravitation.

(2) The energy or mass of light is subject to gravitation in the same way as ordinary matter. If the law of gravitation is strictly the Newtonian law, this leads to an apparent displacement of a star close to the sun's limb amounting to $0''\cdot 87$ outwards.

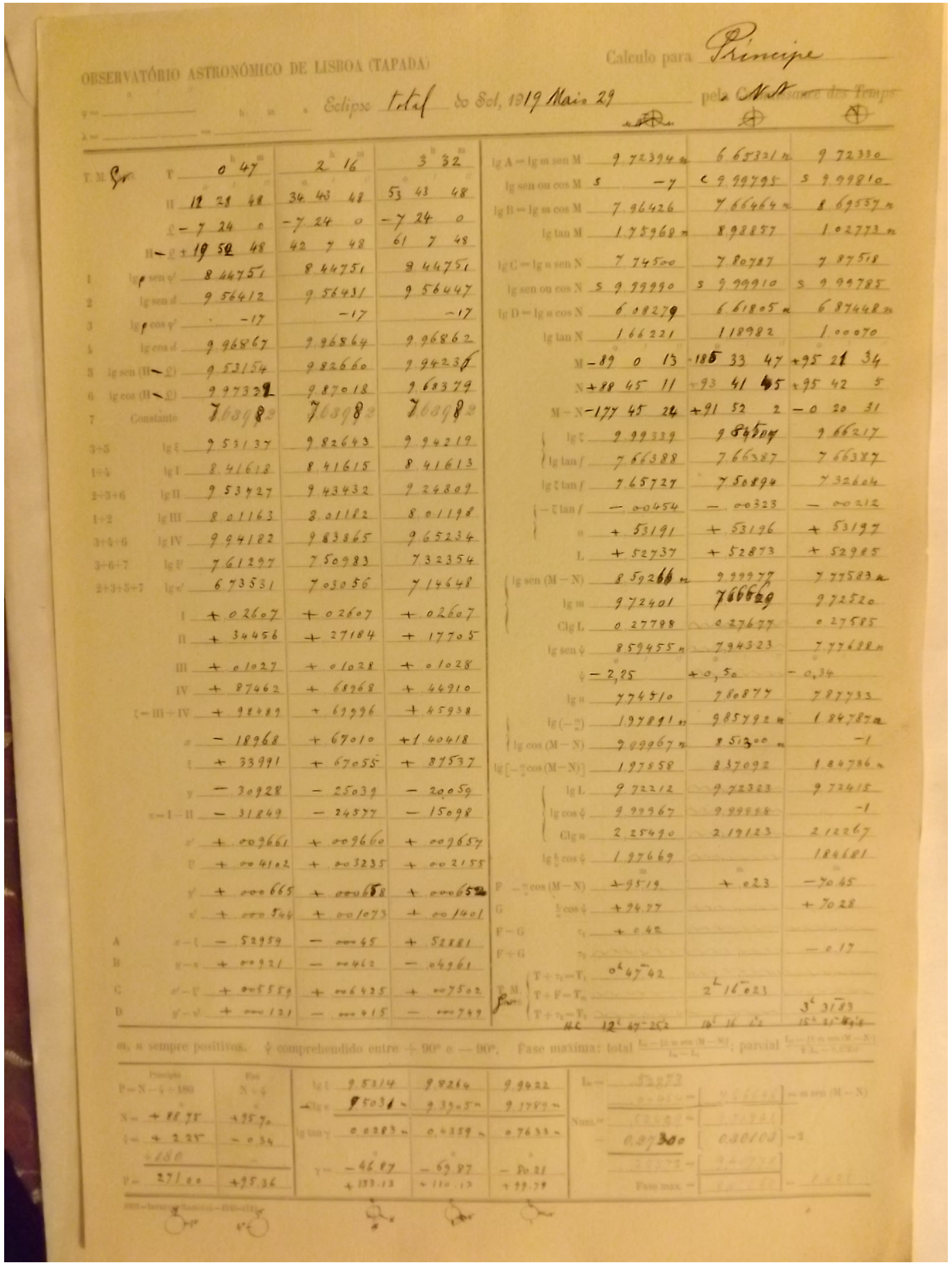
(3) The course of a ray of light is in accordance with EINSTEIN's generalised relativity theory. This leads to an apparent displacement of a star at the limb amounting to $1''\cdot 75$ outwards.

Solar eclipse of May 29, 1919

Good opportunity because of many stars in the background - the Hyades open star cluster

Ephemerides calculations.

Data for Principe made at OAL (Observatório Astronómico de Lisboa) on special eclipse sheets!

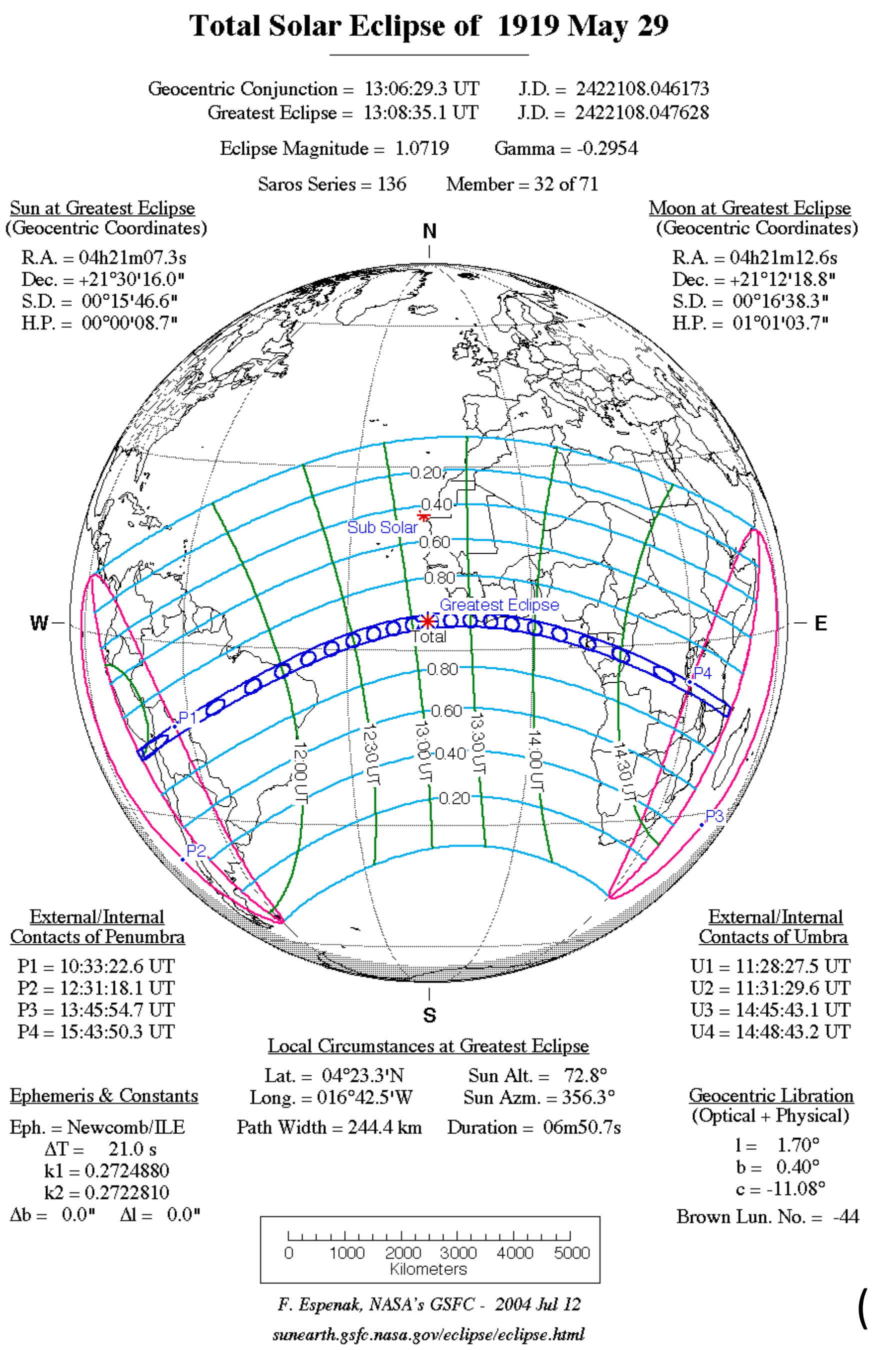


OBSERVATÓRIO ASTRONÓMICO DE LISBOA (TAPADA)

Calculo para Principe

Eclipse total do Sol, 1919 Maio 29 pela ~~Connaissance des Temps~~

(OAL archives)



Total Solar Eclipse of 1919 May 29

Geocentric Conjunction = 13:06:29.3 UT J.D. = 2422108.046173
Greatest Eclipse = 13:08:35.1 UT J.D. = 2422108.047628

Eclipse Magnitude = 1.0719 Gamma = -0.2954

Saros Series = 136 Member = 32 of 71

Sun at Greatest Eclipse
(Geocentric Coordinates)
R.A. = 04h21m07.3s
Dec. = +21°30'16.0"
S.D. = 00°15'46.6"
H.P. = 00°00'08.7"

Moon at Greatest Eclipse
(Geocentric Coordinates)
R.A. = 04h21m12.6s
Dec. = +21°12'18.8"
S.D. = 00°16'38.3"
H.P. = 01°01'03.7"

External/Internal
Contacts of Penumbra
P1 = 10:33:22.6 UT
P2 = 12:31:18.1 UT
P3 = 13:45:54.7 UT
P4 = 15:43:50.3 UT

External/Internal
Contacts of Umbra
U1 = 11:28:27.5 UT
U2 = 11:31:29.6 UT
U3 = 14:45:43.1 UT
U4 = 14:48:43.2 UT

Local Circumstances at Greatest Eclipse
Lat. = 04°23.3'N Sun Alt. = 72.8°
Long. = 016°42.5'W Sun Azm. = 356.3°
Path Width = 244.4 km Duration = 06m50.7s

Ephemeris & Constants
Eph. = Newcomb/ILE
ΔT = 21.0 s
k1 = 0.2724880
k2 = 0.2722810
Δb = 0.0" Δl = 0.0"

Geocentric Libration
(Optical + Physical)
l = 1.70°
b = 0.40°
c = -11.08°
Brown Lun. No. = -44

F. Espenak, NASA's GSFC - 2004 Jul 12
sunearth.gsfc.nasa.gov/eclipse/eclipse.html

Magnitude (diameter) : 1.07

Maximum duration: 6m50s

2 groups:

Príncipe (São Tomé e Príncipe, Portugal):
Eddington and Cottingham

Sobral (Ceará, Brasil):
Davidson and Crommelin

(NASA)

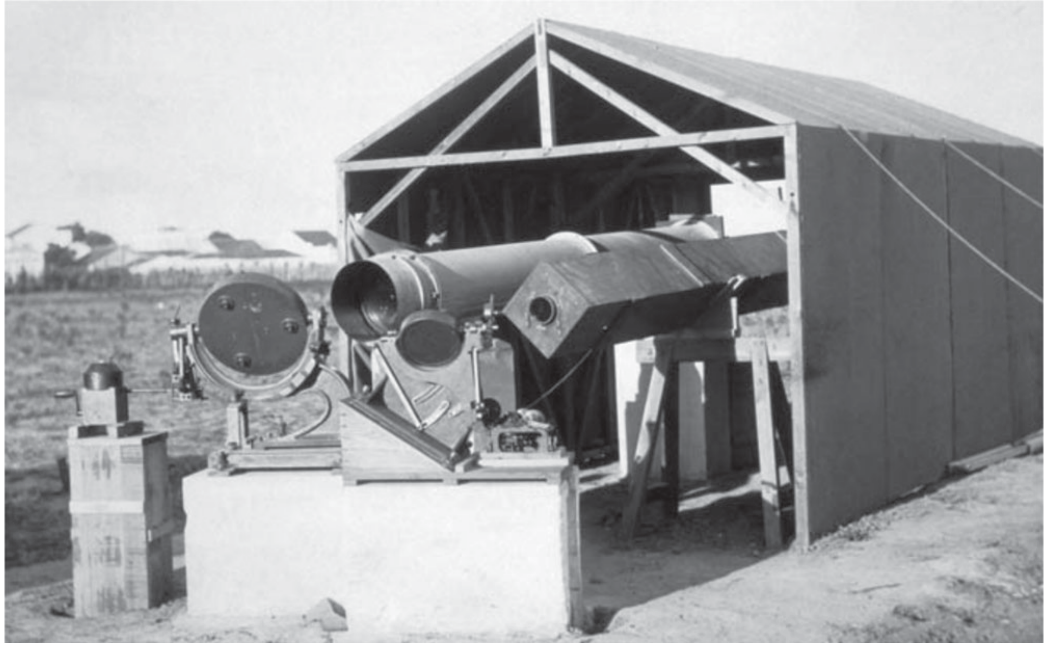
Instruments

Big optical refractor telescopes:

- 13 inch aperture - focal length = 3.43m
- 4 inch aperture

used with16-inch and 8-inch coelostats (mirrors) attached to clocks - easier to follow the rotation of the sky by moving the mirrors

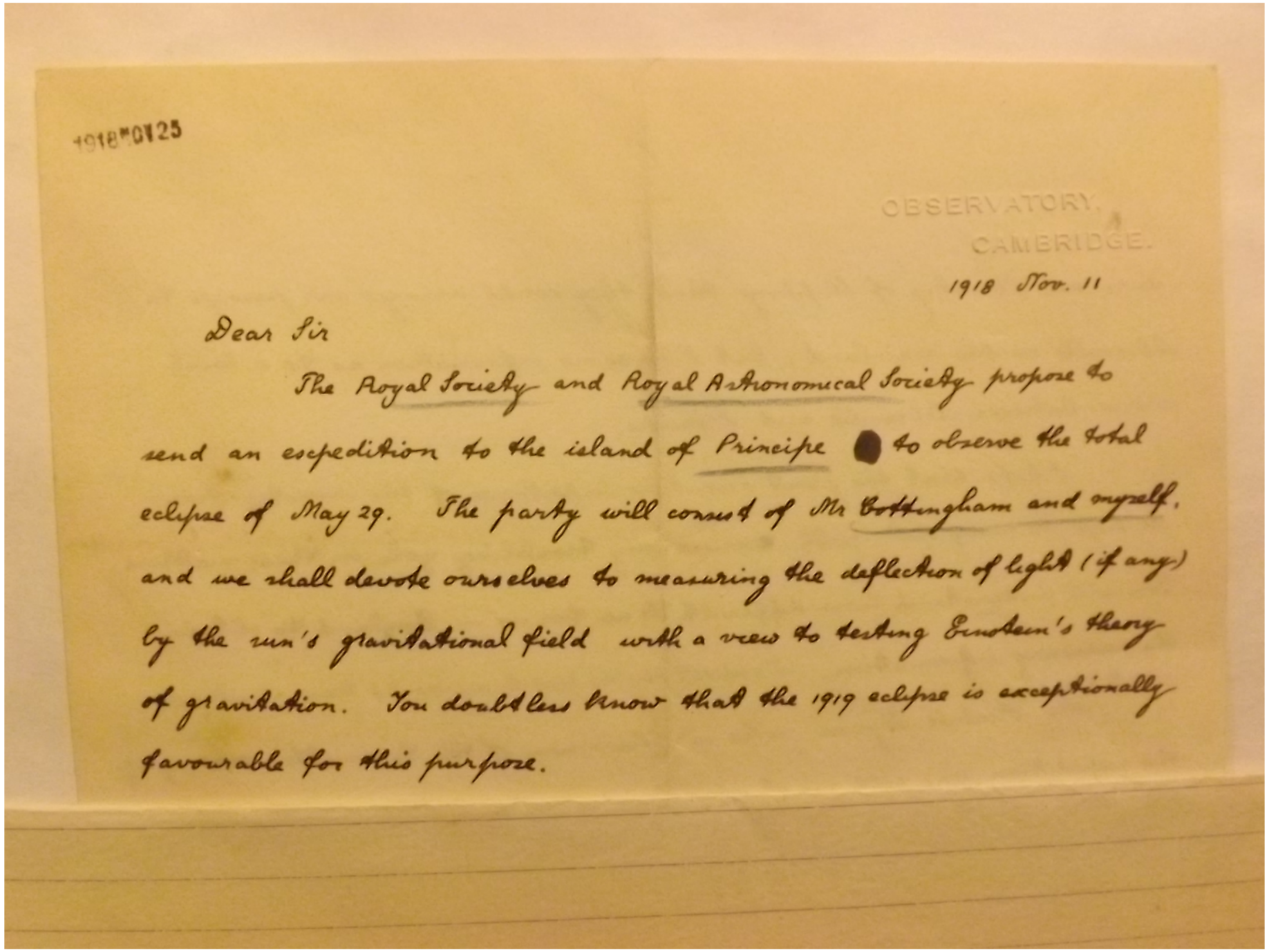
2 tons of luggage!



Logistics: connection with OAL, Lisbon

From Nov 11, 1918 - armistice day - Eddington started an exchange of letters with Frederico Oom (sub-director of OAL), asking information about

- lodgings in Principe, local works, materials, possible sites
- transportation to Principe



1918 NOV 25

OBSERVATORY,
CAMBRIDGE.

1918 Nov. 11

Dear Sir

The Royal Society and Royal Astronomical Society propose to send an expedition to the island of Principe to observe the total eclipse of May 29. The party will consist of Mr Cottingham and myself, and we shall devote ourselves to measuring the deflection of light (if any) by the sun's gravitational field with a view to testing Einstein's theory of gravitation. You doubtless know that the 1919 eclipse is exceptionally favourable for this purpose.

Nov 11

(OAL archives)

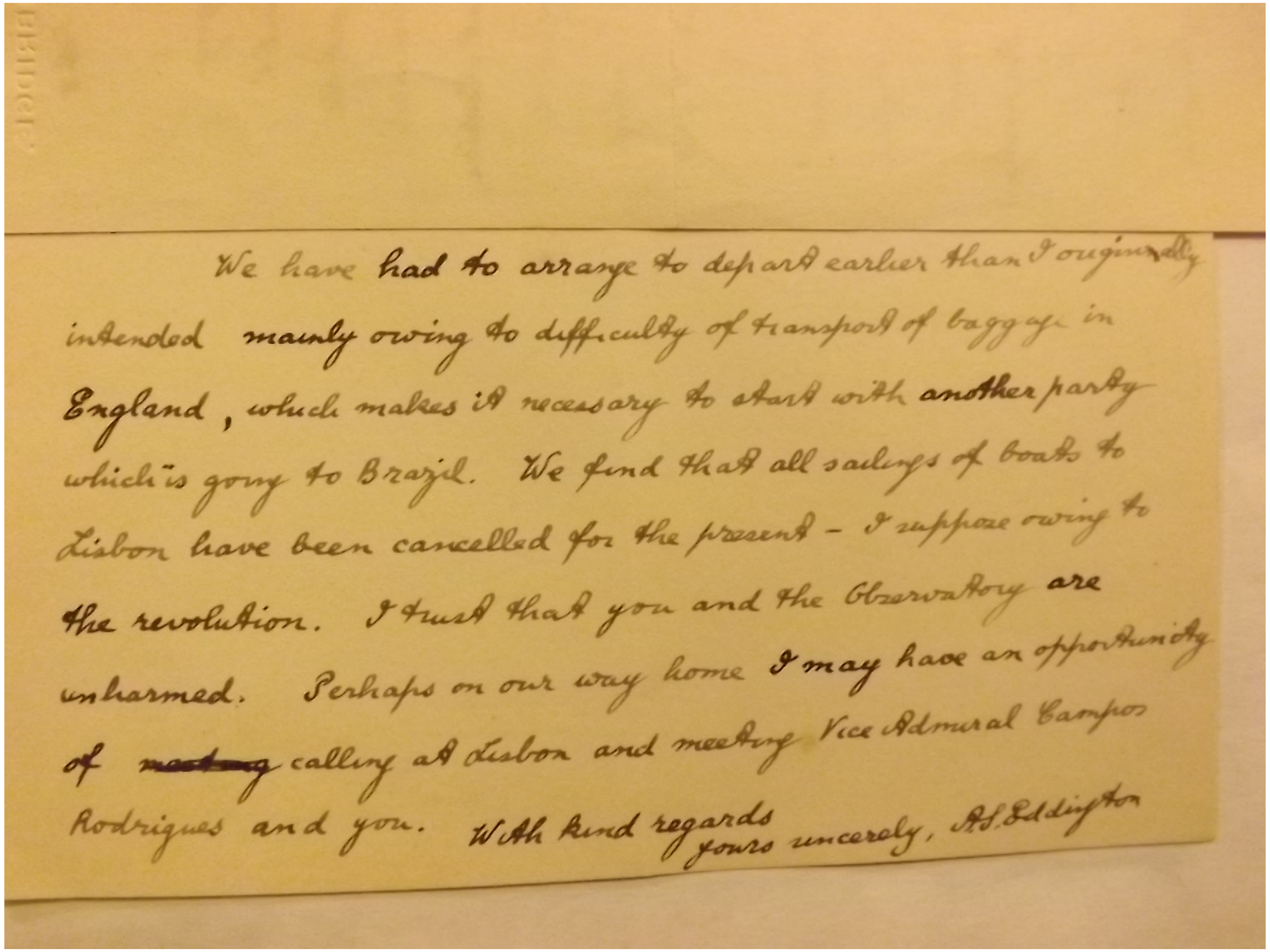
Change of plans

The journey should have been:

Liverpool - Lisbon
Lisbon - Principe

but ships to Lisbon were cancelled due to the revolution!

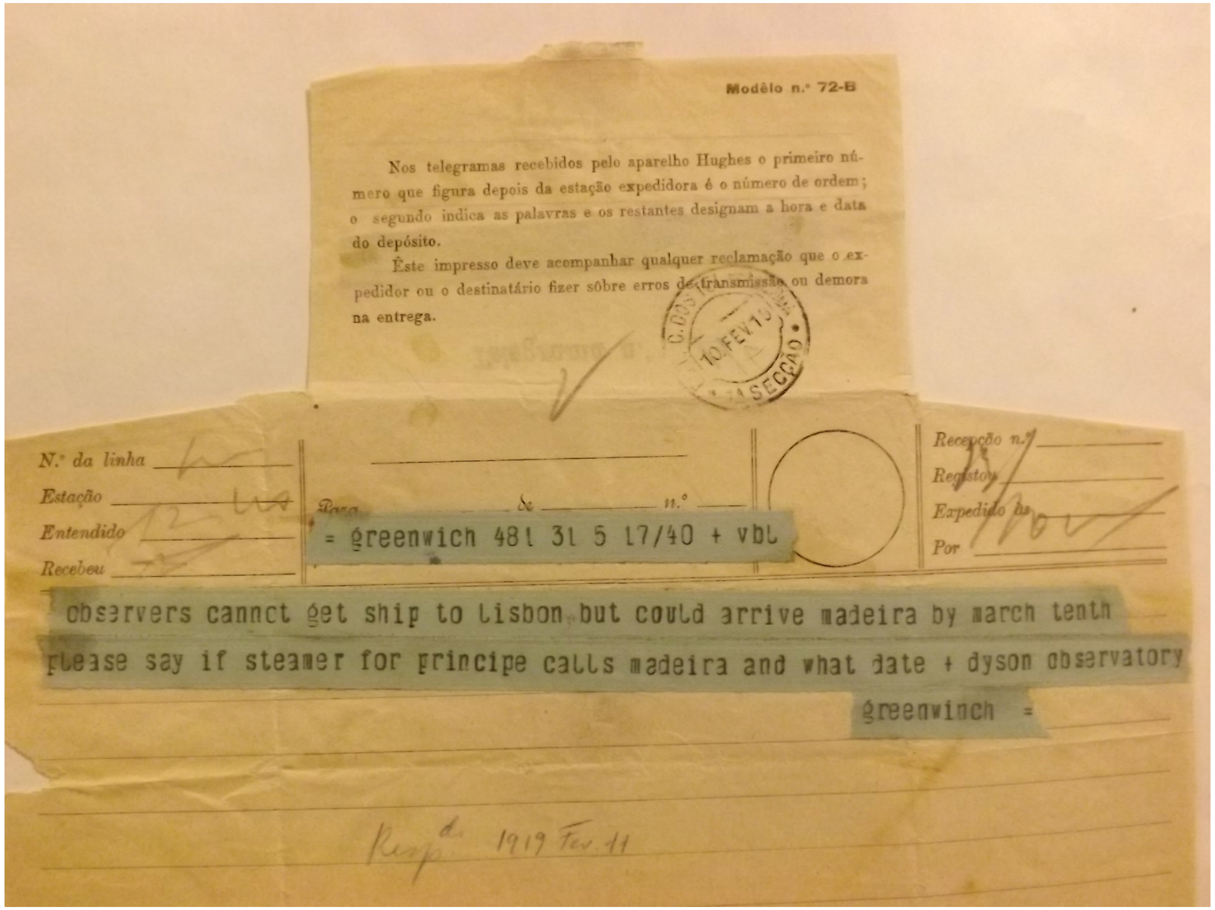
Paiva Couceiro -
monarquia do norte,
jan -feb 1919 ?



We have had to arrange to depart earlier than I originally intended mainly owing to difficulty of transport of baggage in England, which makes it necessary to start with another party which is going to Brazil. We find that all sailings of boats to Lisbon have been cancelled for the present – I suppose owing to the revolution. I trust that you and the Observatory are unharmed. Perhaps on our way home I may have an opportunity of ~~meeting~~ calling at Lisbon and meeting Vice Admiral Campos Rodrigues and you. With kind regards

yours sincerely, A.S. Eddington

Feb 8



Modêlo n.º 72-B

Nos telegramas recebidos pelo aparelho Hughes o primeiro número que figura depois da estação expedidora é o número de ordem; o segundo indica as palavras e os restantes designam a hora e data do depósito.

Êste impresso deve acompanhar qualquer reclamação que o expedidor ou o destinatário fizer sôbre erros de transmissão ou demora na entrega.

10 FEV 19

SECÇÃO

N.º da linha
Estação
Entendido
Recebeu

Para ______ de ______ n.º ______

= greenwich 481 31 5 17/40 + vbl

Recepção n.º
Registo
Expedido ás
Por

observers cannct get ship to Lisbon but could arrive madeira by march tenth please say if steamer for principe calls madeira and what date + dyson observatory greenwich =

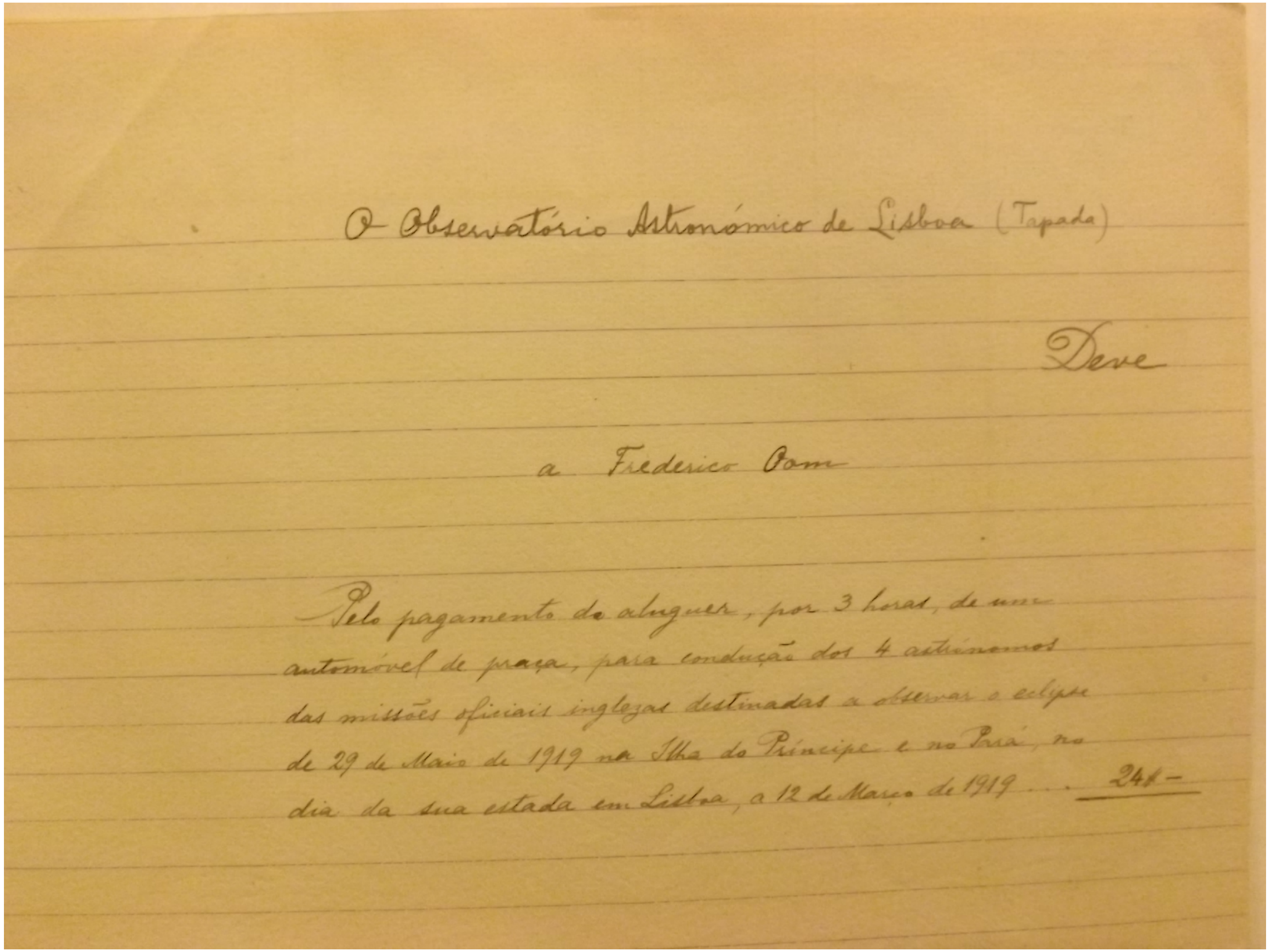
Resp.d 1919 Fev. 11

telegram from Dyson

The 2 groups will travel together to Madeira and from there to Príncipe and Pará

But the ship stopped in Lisbon after all,
and they could spend a few hours in Lisbon, on march 12.

In Lisbon



O Observatório Astronómico de Lisboa (Tapada)

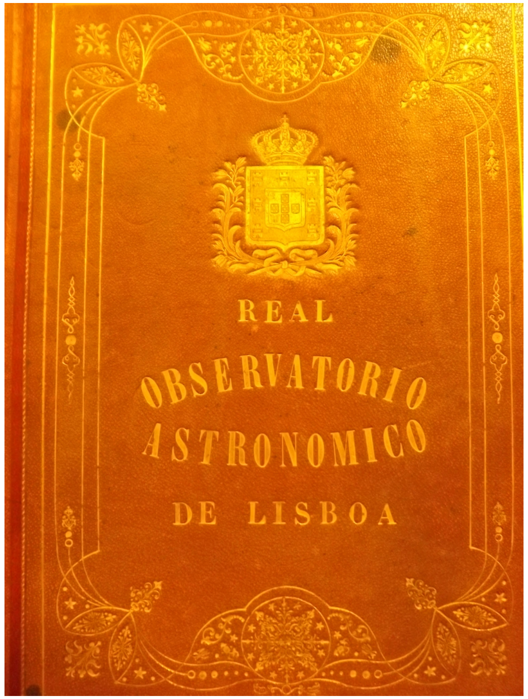
Deve

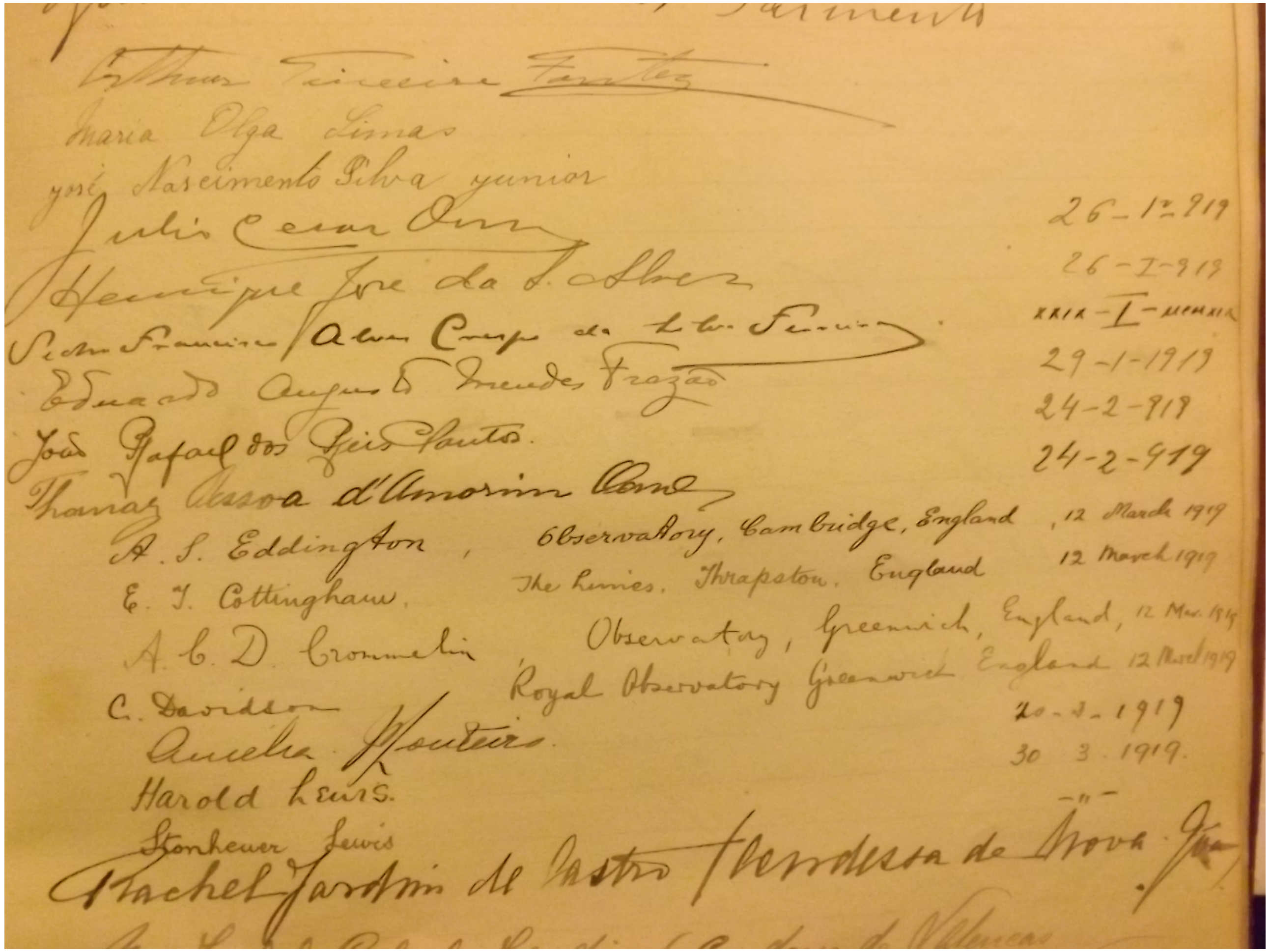
a Frederico Pom

Pelo pagamento do aluguer, por 3 horas, de um automóvel de praça, para condução dos 4 astrónomos das missões oficiais inglezas destinadas a observar o eclipse de 29 de Maio de 1919 na Ilha do Príncipe e no Pará, no dia da sua estada em Lisboa, a 12 de Março de 1919 . . . 24$-

reimboursement form!

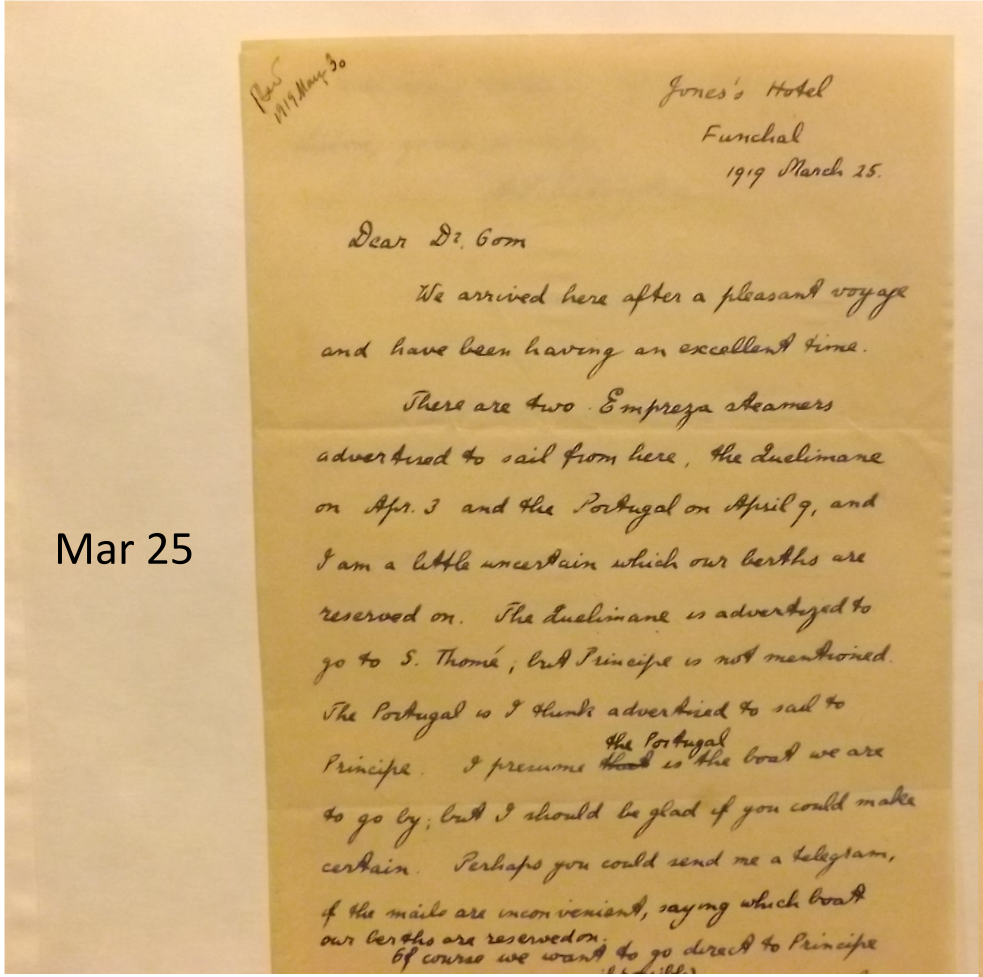
Visitor's book




Arthur Teixeira Fontes
Maria Olga Limas
José Nascimento Silva junior
Julio Cesar [illegible] 26-1°-919
Henrique José da S. Alves 26-I-919
Pedro Francisco | Alves Crespo da Silva Ferreira XXIX-I-MCMXIX
Eduardo Augusto Mendes Frazão 29-1-1919
João Rafael dos Reis Santos. 24-2-919
Thomaz Bessa d'Amorim [illegible] 24-2-919
A. S. Eddington, Observatory, Cambridge, England, 12 March 1919
E. T. Cottingham, The Limes, Thrapston, England 12 March 1919
A. C. D. Crommelin, Observatory, Greenwich, England, 12 Mar. 1919
C. Davidson Royal Observatory Greenwich England 12 March 1919
Amelia Monteiro. 20-3-1919
Harold Lewis. 30 3. 1919.
Stonehewer Lewis —"—
Rachel Jardim de Castro Henderson de Nova [illegible]

More requests from **Madeira**

Mar 25



Recd 1919 May 30

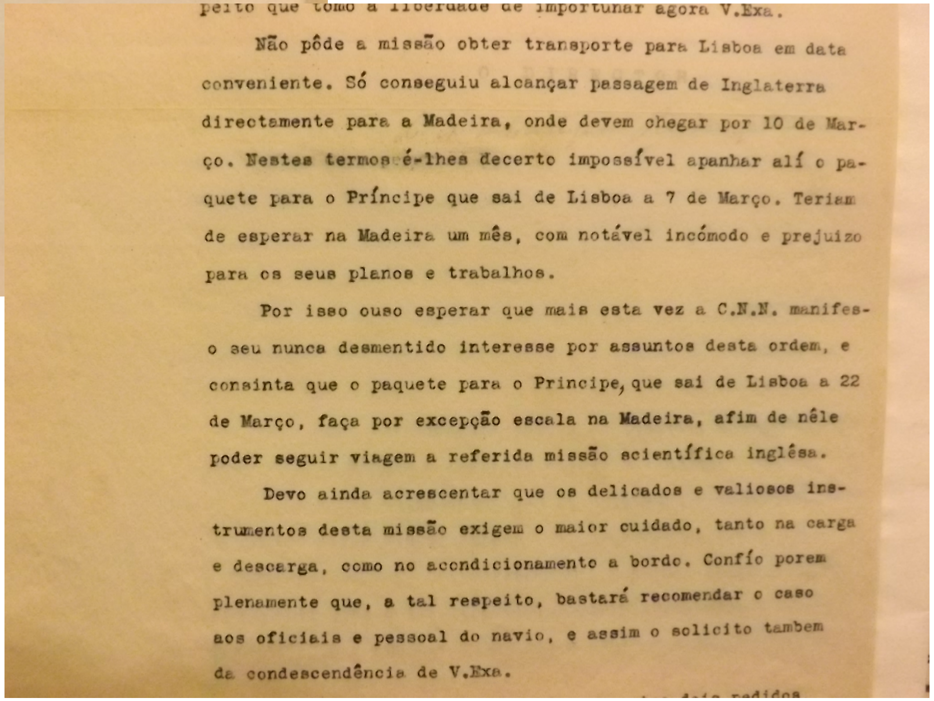
Jones's Hotel
Funchal
1919 March 25.

Dear Dr. Oom

We arrived here after a pleasant voyage and have been having an excellent time.

There are two Empreza steamers advertised to sail from here, the Quelimane on Apr. 3 and the Portugal on April 9, and I am a little uncertain which our berths are reserved on. The Quelimane is advertized to go to S. Thomé; but Principe is not mentioned. The Portugal is I think advertised to sail to Principe. I presume ~~that~~ the Portugal is the boat we are to go by; but I should be glad if you could make certain. Perhaps you could send me a telegram, if the mails are inconvienient, saying which boat our berths are reserved on.
Of course we want to go direct to Principe

A large number of letters from F. Oom to Centro Colonial and Companhia Nacional de Navegação



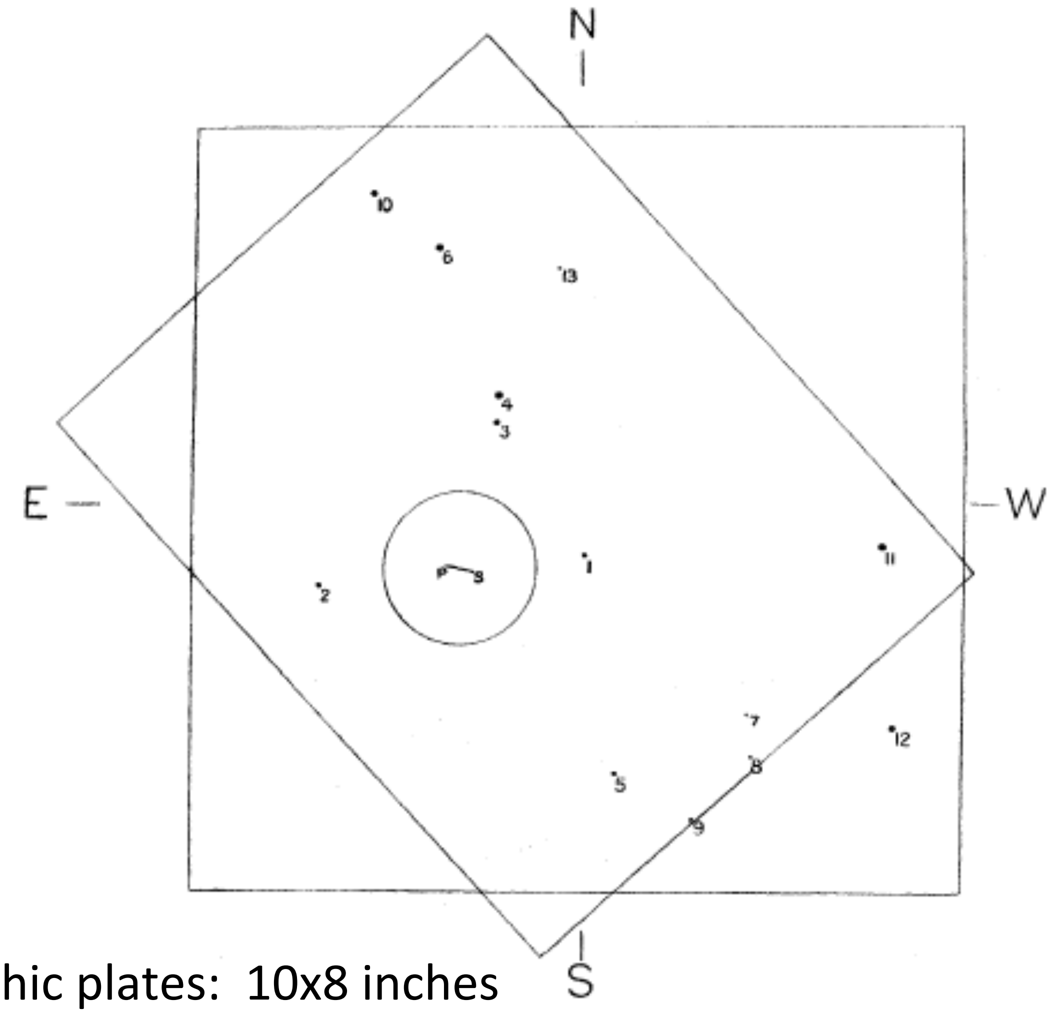
peito que tomo a liberdade de importunar agora V.Exa.

Não pôde a missão obter transporte para Lisboa em data conveniente. Só conseguiu alcançar passagem de Inglaterra directamente para a Madeira, onde devem chegar por 10 de Março. Nestes termos é-lhes decerto impossível apanhar alí o paquete para o Príncipe que sai de Lisboa a 7 de Março. Teriam de esperar na Madeira um mês, com notável incómodo e prejuizo para os seus planos e trabalhos.

Por isso ouso esperar que mais esta vez a C.N.N. manifeste o seu nunca desmentido interesse por assuntos desta ordem, e consinta que o paquete para o Principe, que sai de Lisboa a 22 de Março, faça por excepção escala na Madeira, afim de nêle poder seguir viagem a referida missão scientífica inglêsa.

Devo ainda acrescentar que os delicados e valiosos instrumentos desta missão exigem o maior cuidado, tanto na carga e descarga, como no acondicionamento a bordo. Confío porem plenamente que, a tal respeito, bastará recomendar o caso aos oficiais e pessoal do navio, e assim o solicito tambem da condescendência de V.Exa.

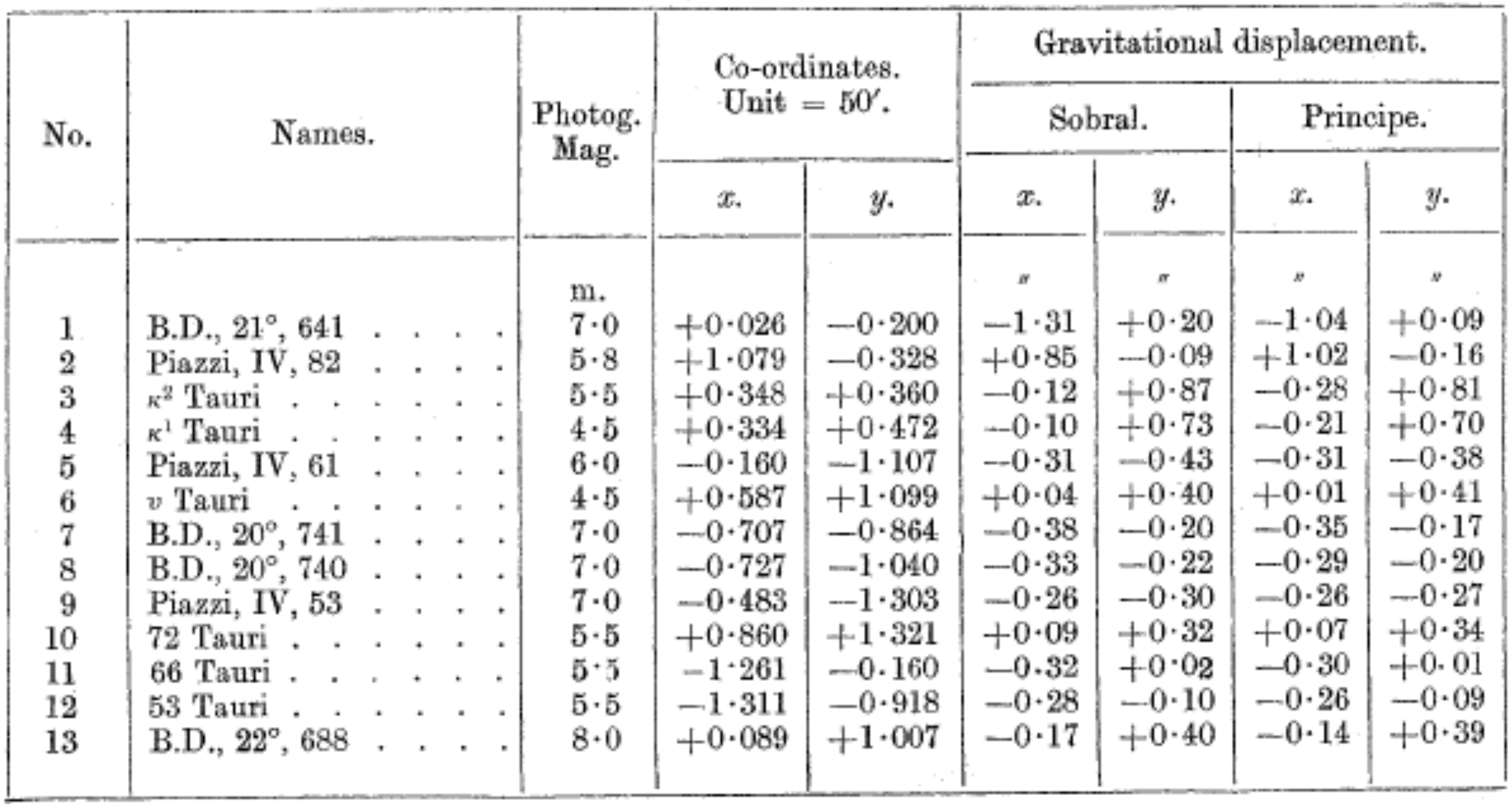
The observations



Large photographic plates: 10x8 inches and 16X16 cm

Wide field: FoV ~ 6 deg^2

Earth translation - in 2 hours the Sun changes with respect to the ‘fixed stars’



No.	Names.	Photog. Mag.	Co-ordinates. Unit = 50′.		Gravitational displacement.			
					Sobral.		Principe.	
			x.	*y*.	*x*.	*y*.	*x*.	*y*.
		m.			″	″	″	″
1	B.D., 21°, 641	7·0	+0·026	−0·200	−1·31	+0·20	−1·04	+0·09
2	Piazzi, IV, 82	5·8	+1·079	−0·328	+0·85	−0·09	+1·02	−0·16
3	κ^2 Tauri	5·5	+0·348	+0·360	−0·12	+0·87	−0·28	+0·81
4	κ^1 Tauri	4·5	+0·334	+0·472	−0·10	+0·73	−0·21	+0·70
5	Piazzi, IV, 61	6·0	−0·160	−1·107	−0·31	−0·43	−0·31	−0·38
6	υ Tauri	4·5	+0·587	+1·099	+0·04	+0·40	+0·01	+0·41
7	B.D., 20°, 741	7·0	−0·707	−0·864	−0·38	−0·20	−0·35	−0·17
8	B.D., 20°, 740	7·0	−0·727	−1·040	−0·33	−0·22	−0·29	−0·20
9	Piazzi, IV, 53	7·0	−0·483	−1·303	−0·26	−0·30	−0·26	−0·27
10	72 Tauri	5·5	+0·860	+1·321	+0·09	+0·32	+0·07	+0·34
11	66 Tauri	5·5	−1·261	−0·160	−0·32	+0·02	−0·30	+0·01
12	53 Tauri	5·5	−1·311	−0·918	−0·28	−0·10	−0·26	−0·09
13	B.D., 22°, 688	8·0	+0·089	+1·007	−0·17	+0·40	−0·14	+0·39

Stars positions in the field (no Sun) and expected image displacement

Outline of the observations

Take several exposures of the field during the eclipse (eclipse plates), hoping to get as many stars as possible

Take several exposures of the same field (comparison plates) without the Sun preferably at the same height and temperature:

> need to wait a few weeks, the Sun moves away ~30 degrees per month (2 hours in the sky).
> If eclipse at 8am, in 1 month the field is visible at dawn

Take one reference exposure, same as the comparisons (scale plate)

Parallax is not a problem (Hyades at 40 pc) << 1 arcsec ~ deflection

Seeing ~1 arcsec ?

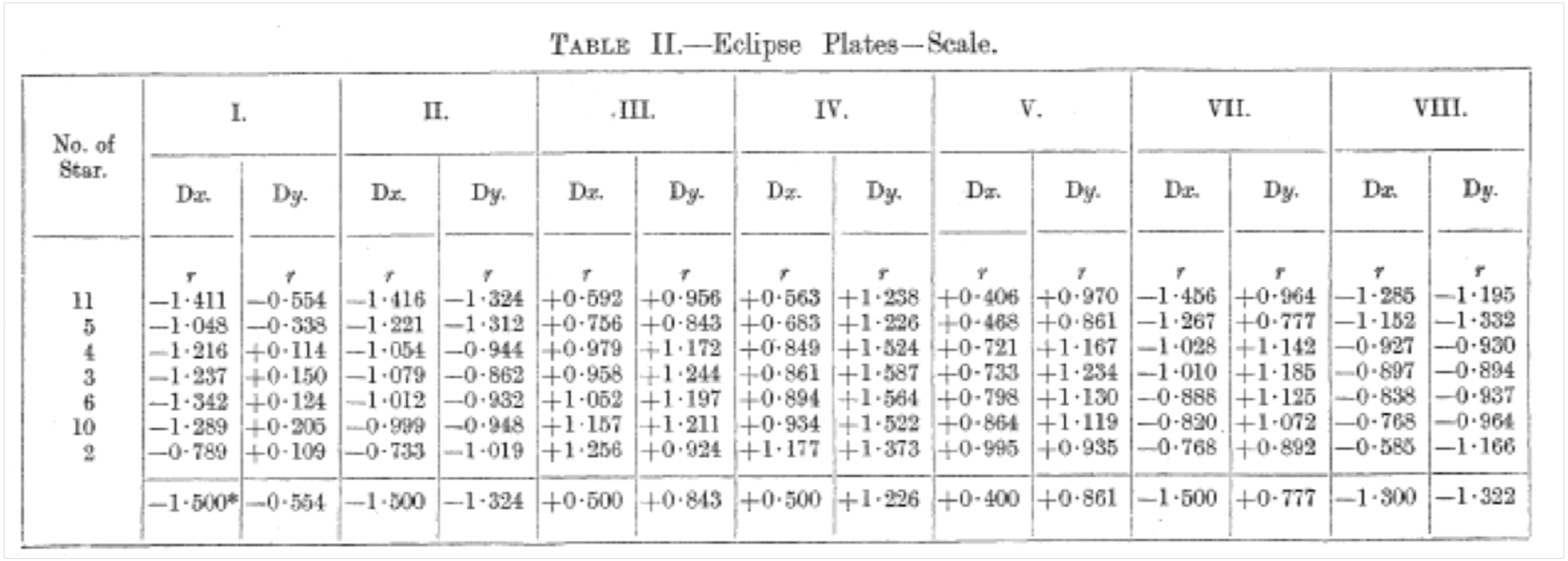
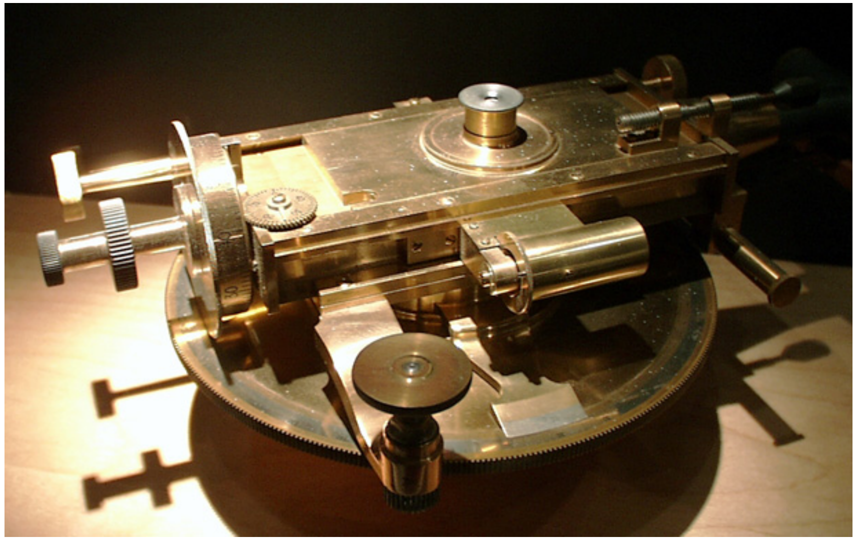


TABLE II.—Eclipse Plates—Scale.

No. of Star.	I.		II.		III.		IV.		V.		VII.		VIII.	
	Dx.	Dy.	Dx.	Dy.	Dx.	Dy.	Dx.	Dy.	Dx.	Dy.	Dx.	Dy.	Dx.	Dy.
	r	r	r	r	r	r	r	r	r	r	r	r	r	r
11	−1·411	−0·554	−1·416	−1·324	+0·592	+0·956	+0·563	+1·238	+0·406	+0·970	−1·456	+0·964	−1·285	−1·195
5	−1·048	−0·338	−1·221	−1·312	+0·756	+0·843	+0·683	+1·226	+0·468	+0·861	−1·267	+0·777	−1·152	−1·332
4	−1·216	+0·114	−1·054	−0·944	+0·979	+1·172	+0·849	+1·524	+0·721	+1·167	−1·028	+1·142	−0·927	−0·930
3	−1·237	+0·150	−1·079	−0·862	+0·958	+1·244	+0·861	+1·587	+0·733	+1·234	−1·010	+1·185	−0·897	−0·894
6	−1·342	+0·124	−1·012	−0·932	+1·052	+1·197	+0·894	+1·564	+0·798	+1·130	−0·888	+1·125	−0·838	−0·937
10	−1·289	+0·205	−0·999	−0·948	+1·157	+1·211	+0·934	+1·522	+0·864	+1·119	−0·820	+1·072	−0·768	−0·964
2	−0·789	+0·109	−0·733	−1·019	+1·256	+0·924	+1·177	+1·373	+0·995	+0·935	−0·768	+0·892	−0·585	−1·166
	−1·500*	−0·554	−1·500	−1·324	+0·500	+0·843	+0·500	+1·226	+0·400	+0·861	−1·500	+0·777	−1·300	−1·322

Example from Sobral. Overlay of I - VIII eclipse plates with the scale plate.

The positions are measured in revolutions of the micrometer (r = 6.25 arcsec), in the reference of the scale plate.



The positions Dx, Dy in the eclipse plates are modeled:

$$ax + by + c + \alpha E_x = Dx$$

$$dx + ey + f + \alpha E_y = Dy$$

x, y - position in the reference (scale) plate

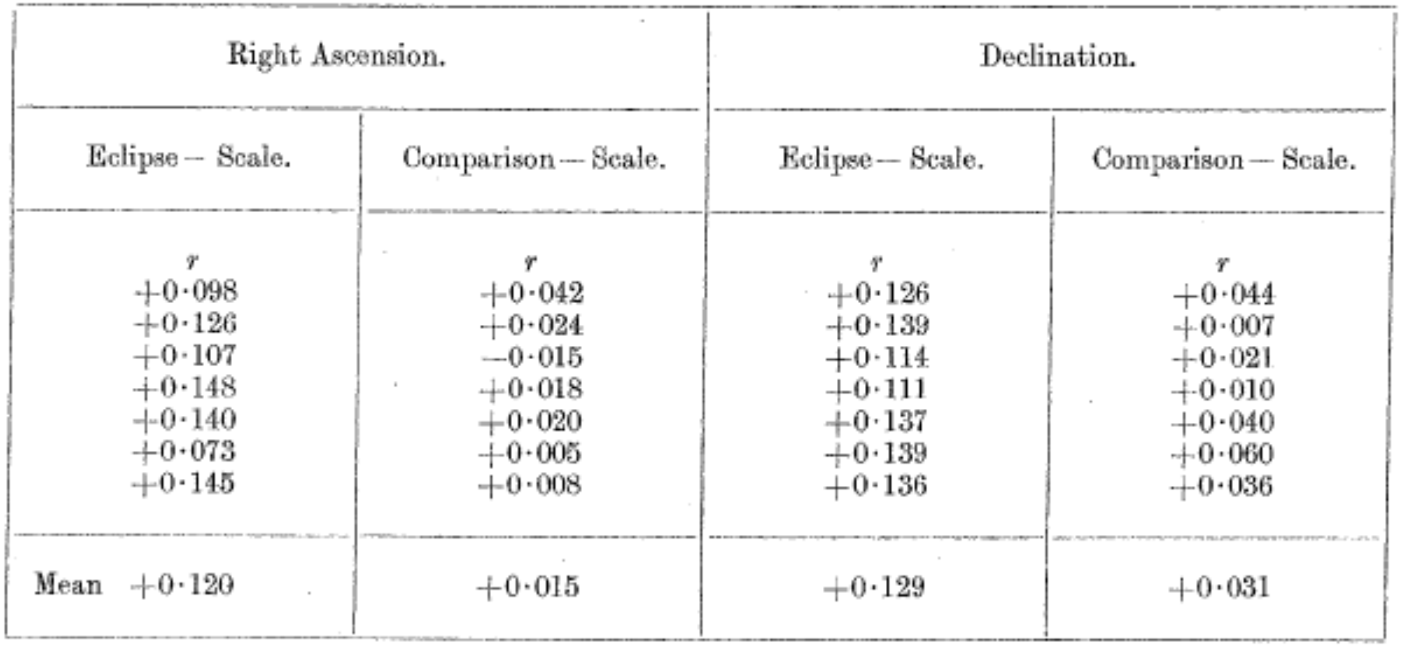
a, b, c - nuisance parameters for scaling (different focus from changing conditions), orientation, shift between plates

Ex,Ey - gravitational deflection at (x,y) / alpha

alpha - gravitational deflection at a reference point (50’ from the Sun center)

Write the system of equations for all stars in one plate
(assuming a,b,c,alpha constant in the plate)

Sobral results for alpha: (from 1 instrument, the other had problem in the focus)



Right Ascension.		Declination.	
Eclipse – Scale.	Comparison – Scale.	Eclipse – Scale.	Comparison – Scale.
r	r	r	r
+0·098	+0·042	+0·126	+0·044
+0·126	+0·024	+0·139	+0·007
+0·107	−0·015	+0·114	+0·021
+0·148	+0·018	+0·111	+0·010
+0·140	+0·020	+0·137	+0·040
+0·073	+0·005	+0·139	+0·060
+0·145	+0·008	+0·136	+0·036
Mean +0·120	+0·015	+0·129	+0·031

alpha is averaged across the plates.

Comparison - Scale plates do not have zero alpha → subtract to eliminate the scale plate

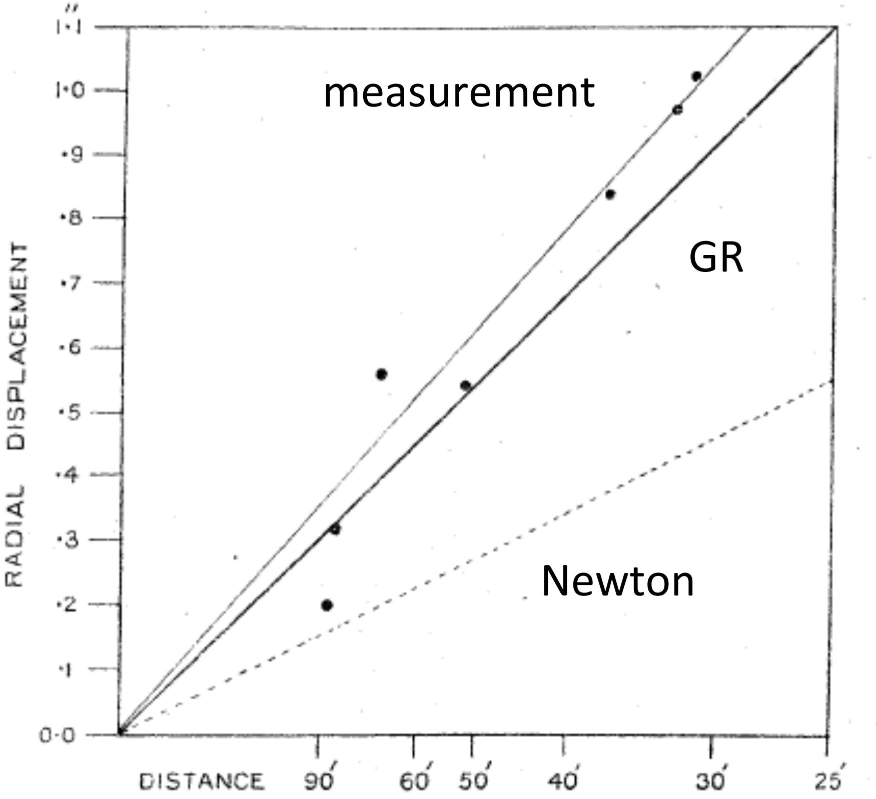
Weighted average between RA and DEC :

$$\alpha = +0^{r}\cdot 100 = +0''\cdot 625.$$

Deflection at the limb $1''.98 \pm 0''.12$

Error bar is just from the dispersion between plates

Inserting the parameter values (nuisance and alpha),
deflection on the individual stars are computed (no error bars):



Príncipe

Cloudy weather, only 2 eclipse plates had reasonable number of stars

Eclipse at noon (local time) would need to wait several months to take comparison plates.
Not possible also due to uncertainty of return ship (due to strikes)

Comparison plates of the Hyades were taken at Oxford

Check plates of another star field were taken at Principe and at Oxford.

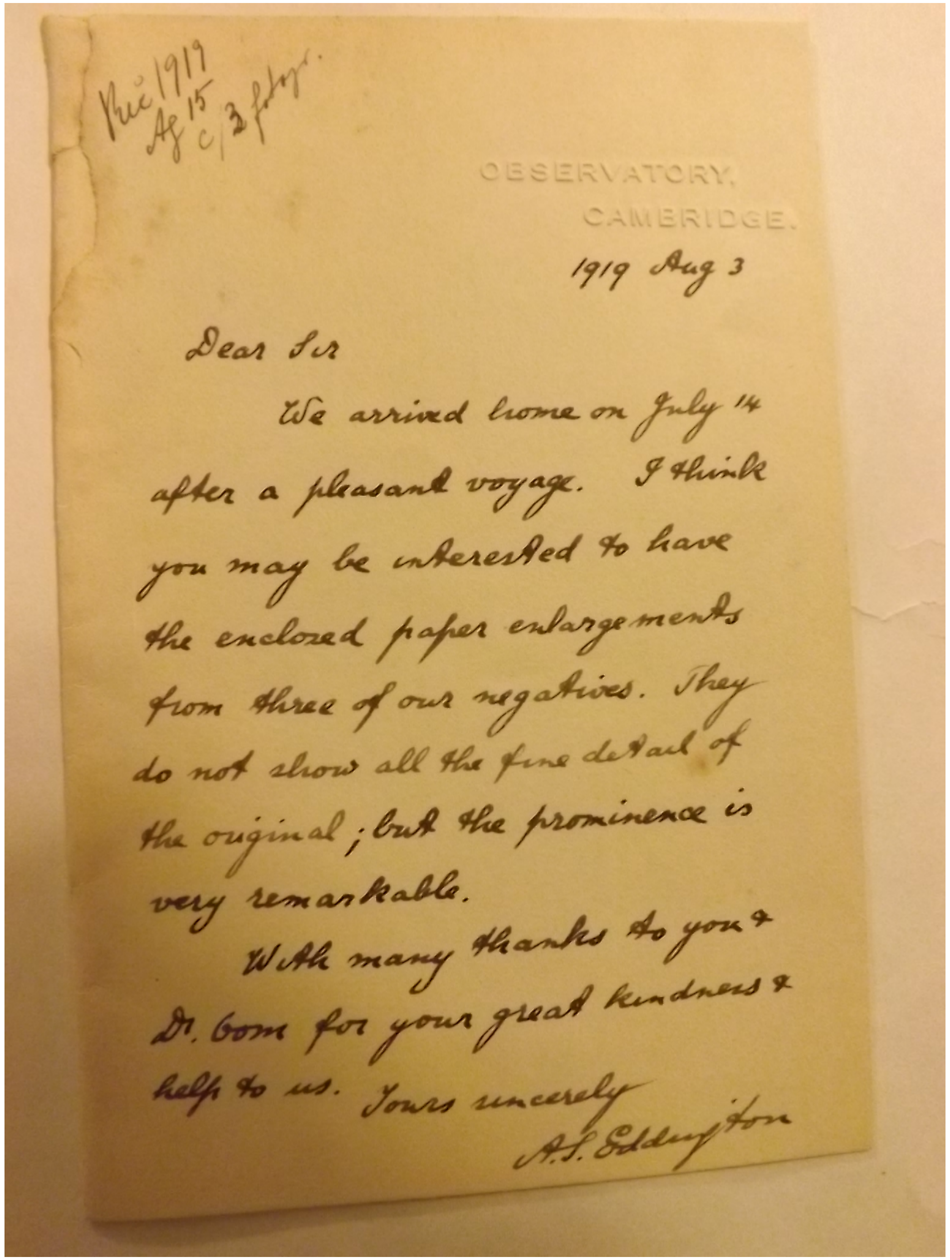
(Always with the same instrument!)

Check plates were compared to determine the nuisance parameters.
Those were then replaced in the analysis of the eclipse plates → all eclipse data could be used to get alpha.

The result at limb is: $1''\cdot 61 \pm 0''\cdot 30.$

Returning home

Eddington sends
3 prints to OAL



Recº 1919
Ag 15
c/3 fotogs.

OBSERVATORY,
CAMBRIDGE.

1919 Aug 3

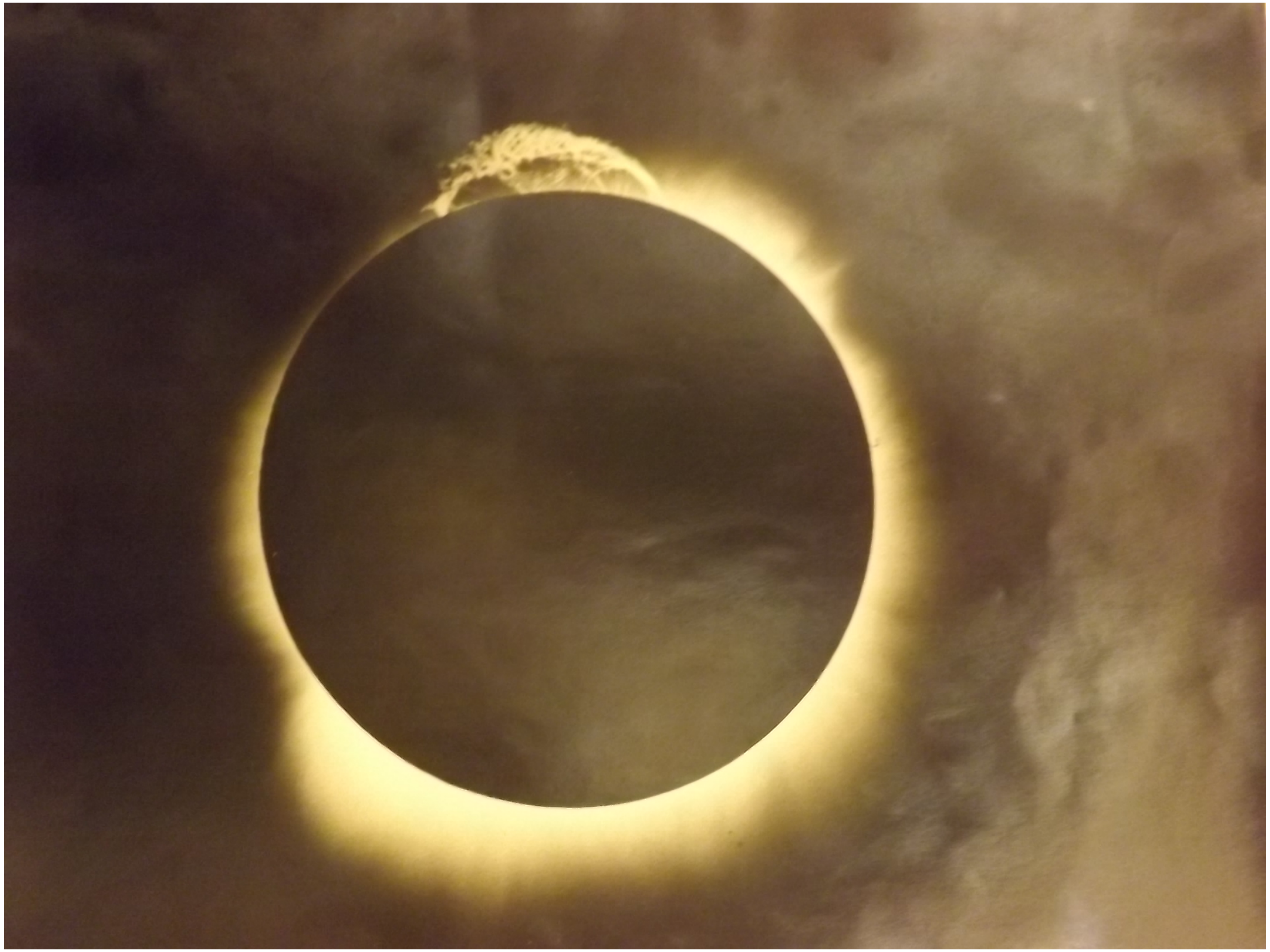
Dear Sir

We arrived home on July 14 after a pleasant voyage. I think you may be interested to have the enclosed paper enlargements from three of our negatives. They do not show all the fine detail of the original; but the prominence is very remarkable.

With many thanks to you & Dr. Oom for your great kindness & help to us. Yours sincerely

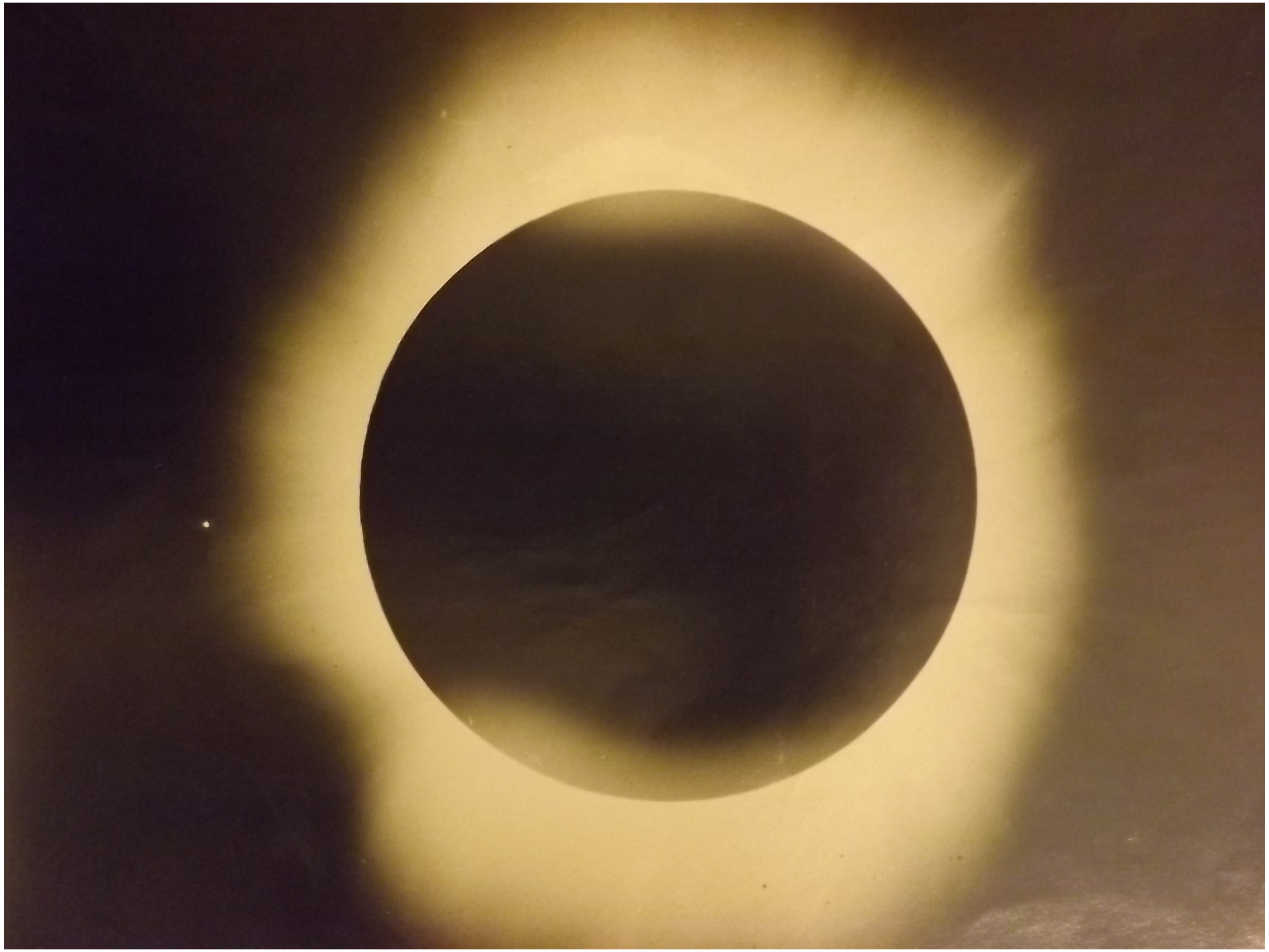
A.S. Eddington

Aug 3



(OAL archives)

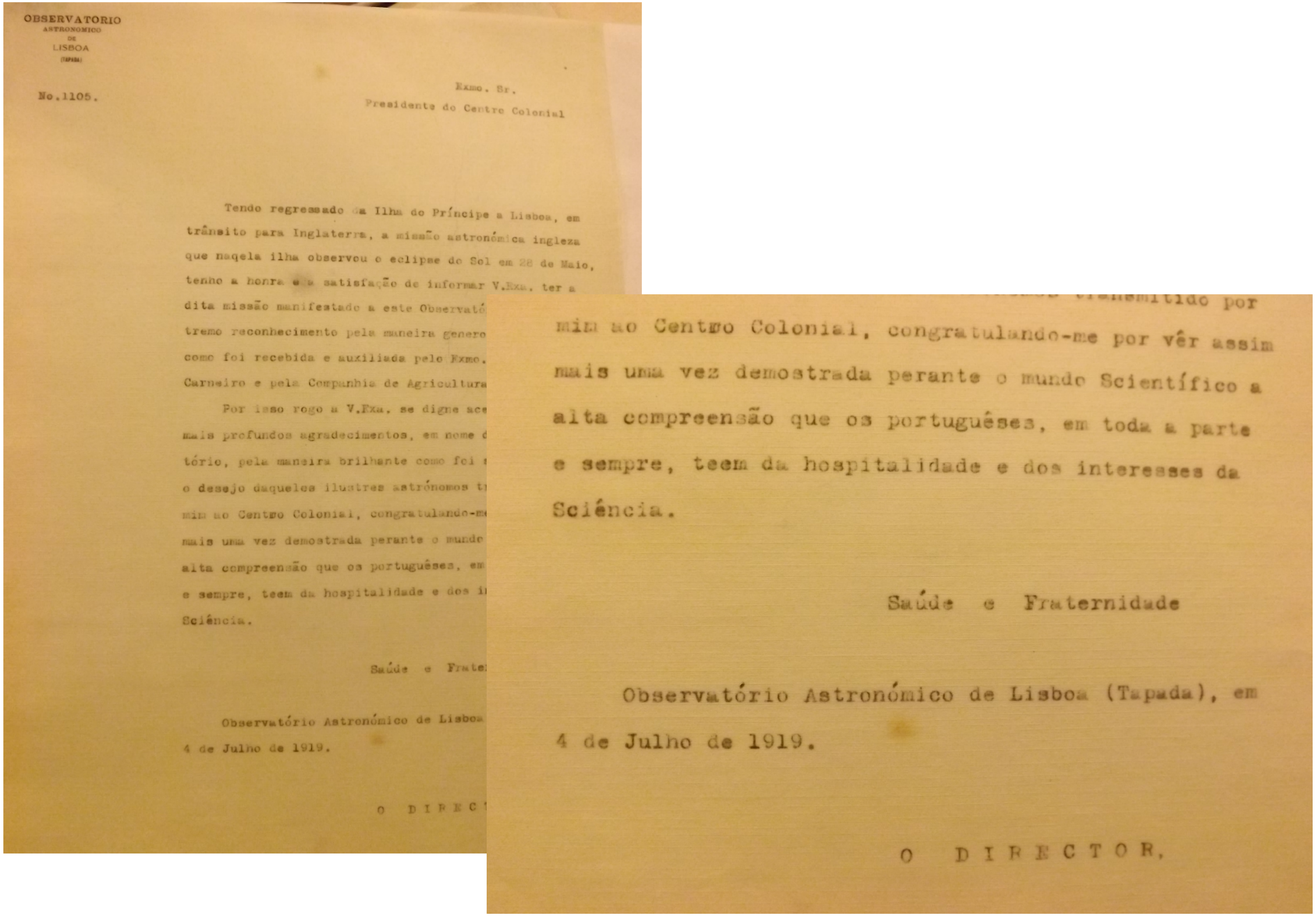
Enlargement of plate R, t_exp = 20s , through thin clouds



(OAL archives)

Enlargement of plate X, t_exp = 3s , clear skies, 1 of the 2 plates used in the analysis

OAL: a job well done



OBSERVATORIO
ASTRONOMICO
DE
LISBOA
(TAPADA)

No.1105.

Exmo. Sr.
Presidente do Centre Colonial

Tendo regressado da Ilha do Príncipe a Lisboa, em
trânsito para Inglaterra, a missão astronómica ingleza
que naqela ilha observou o eclipse do Sol em 28 de Maio,
tenho a honra e a satisfação de informar V.Exa. ter a
dita missão manifestado a este Observató
tremo reconhecimento pela maneira genero
como foi recebida e auxiliada pelo Exmo.
Carneiro e pela Companhia de Agricultura

Por isso rogo a V.Exa. se digne ace
mais profundos agradecimentos, em nome d
tório, pela maneira brilhante como foi
o desejo daqueles ilustres astrónomos t
mim ao Centmo Colonial, congratulando-m
mais uma vez demostrada perante o mundo
alta compreensão que os portuguêses, em
e sempre, teem da hospitalidade e dos i
Sciência.

Saúde e Frate

Observatório Astronómico de Lisboa
4 de Julho de 1919.

O DIREC

transmitido por
mim ao Centmo Colonial, congratulando-me por vêr assim
mais uma vez demostrada perante o mundo Scientífico a
alta compreensão que os portuguêses, em toda a parte
e sempre, teem da hospitalidade e dos interesses da
Sciência.

Saúde e Fraternidade

Observatório Astronómico de Lisboa (Tapada), em
4 de Julho de 1919.

O DIRECTOR.

Gravitational Lensing:

the dark ages - a tidal effect impossible to observe

There is much more to lensing than the light deflection from a point source by a spherical mass:

Change of position of images

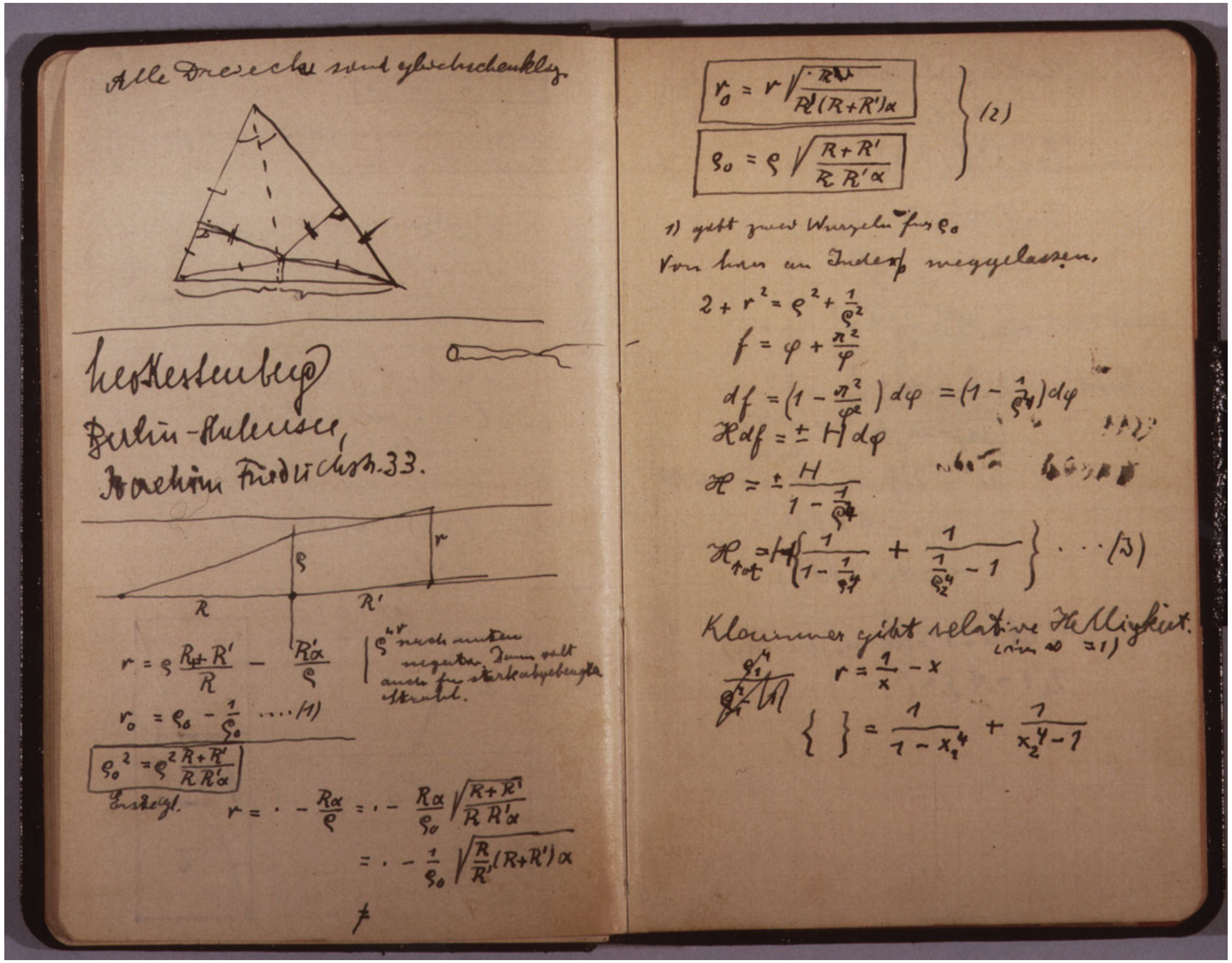
Distortion of extended sources

Multiplication of images

Magnification: increase of size + conservation of brightness →
increase of flux (natural telescope)

Einstein 1912, Notebook

Unpublished work, described in Renn, Sauer, Stachel 1997, Science
"The origin of gravitational lensing"



Alle Dreiecke sind gleichschenklig.

[illegible]
Berlin-Halensee,
Joachim Friedrichstr. 33.

$$r = \rho\frac{R+R'}{R} - \frac{R'\alpha}{\rho}$$

ρ u. r nach unten negativ. Dann gilt auch für stark abgebeugten Strahl.

$$r_0 = \rho_0 - \frac{1}{\rho_0} \quad \ldots (1)$$

$$\boxed{\rho_0^2 = \rho^2\frac{R+R'}{RR'\alpha}}$$

Erste Gl.

$$r = \cdot - \frac{R\alpha}{\rho} = \cdot - \frac{R\alpha}{\rho_0}\sqrt{\frac{R+R'}{RR'\alpha}}$$

$$= \cdot - \frac{1}{\rho_0}\sqrt{\frac{R}{R'}(R+R')\alpha}$$

≠

$$\left.\begin{aligned} &\boxed{r_0 = r\sqrt{\frac{\cdot R}{R'(R+R')\alpha}}} \\ &\boxed{\rho_0 = \rho\sqrt{\frac{R+R'}{RR'\alpha}}} \end{aligned}\right\} (2)$$

1) gibt zwei Wurzeln für ρ_0
Von hier an Index $_0$ weggelassen.

$$2 + r^2 = \rho^2 + \frac{1}{\rho^2}$$

$$f = \varphi + \frac{\pi^2}{\varphi}$$

$$df = \left(1 - \frac{\pi^2}{\varphi^2}\right)d\varphi = \left(1 - \frac{1}{\rho^4}\right)d\varphi$$

$$\mathcal{H}\,df = \pm H\,d\varphi$$

$$\mathcal{H} = \pm\frac{H}{1 - \frac{1}{\rho^4}}$$

$$\mathcal{H}_{tot} = H\left\{\frac{1}{1 - \frac{1}{\rho_1^4}} + \frac{1}{\frac{1}{\rho_2^4} - 1}\right\} \cdots (3)$$

Klammer gibt relative Helligkeit. (in ∞ = 1)

$$r = \frac{1}{x} - x$$

$$\{\ \} = \frac{1}{1 - x_1^4} + \frac{1}{x_2^4 - 1}$$

Lens equation

Double images

Magnification factor

with the factor of 2 missing

No astrophysical interest

It seems it was more like a curiosity, since it was thought there was no great chance of observing this phenomenon:

small displacements

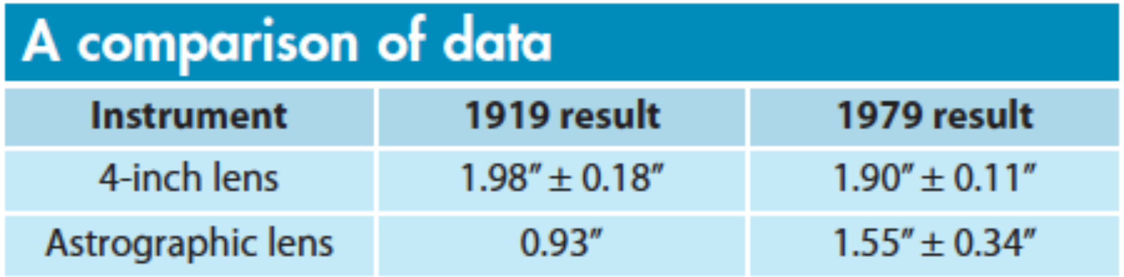
lensed image would be overwhelmed by the brightness of the star (lens)

Even the deflection at the Sun’s limb was very difficult to measure.

Some later eclipse expeditions could not reproduce Eddington’s result, others did.

Controversies about the measurement.

But reanalyses of the same data gave similar results.
Einstein centenary reanalysis (1979):



A comparison of data

Instrument	1919 result	1979 result
4-inch lens	1.98″ ± 0.18″	1.90″ ± 0.11″
Astrographic lens	0.93″	1.55″ ± 0.34″

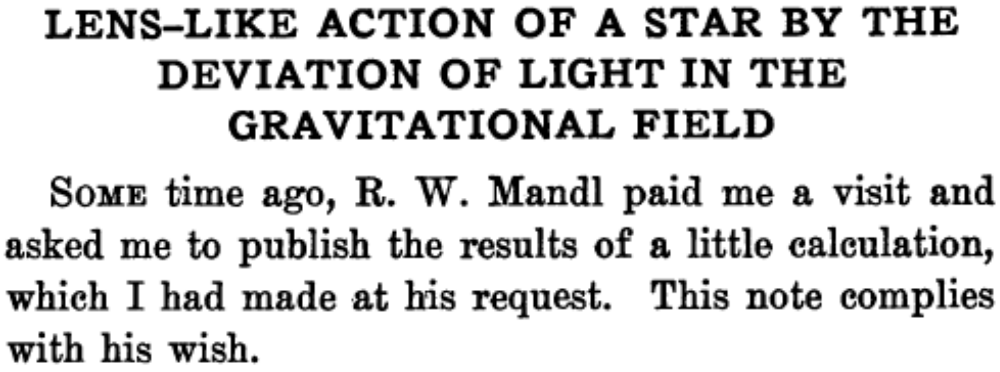
1920 - 1960 Very few publications on Gravitational Lensing

Eddington 1920

Chowlson 1924

Einstein 1936, Science

“Lens-like action of a star by the deviation of light in the gravitational field”

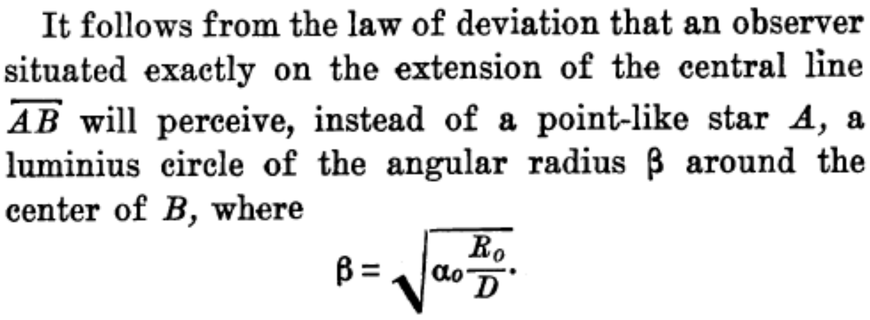


LENS–LIKE ACTION OF A STAR BY THE DEVIATION OF LIGHT IN THE GRAVITATIONAL FIELD

Some time ago, R. W. Mandl paid me a visit and asked me to publish the results of a little calculation, which I had made at his request. This note complies with his wish.

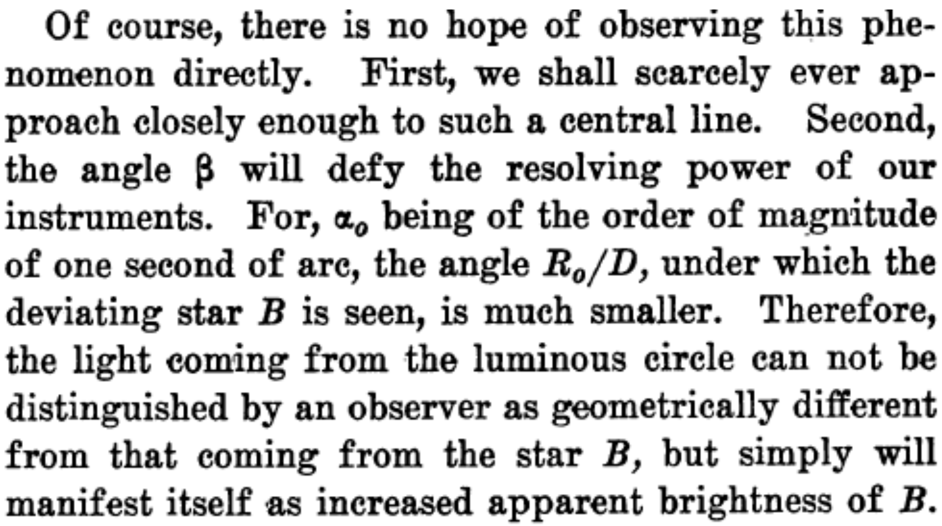
New calculations (with the factor of 2) for:

The **Einstein ring**



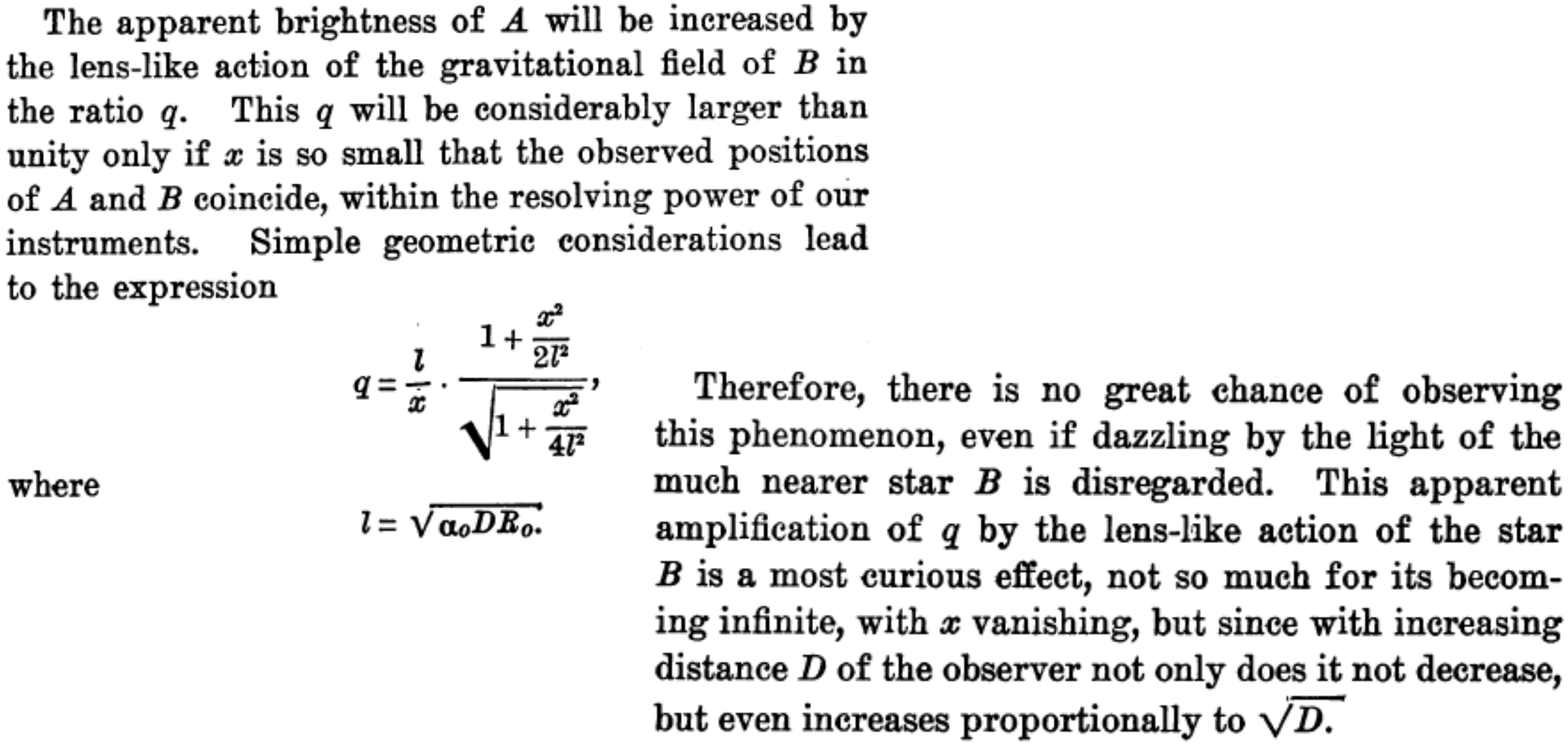
It follows from the law of deviation that an observer situated exactly on the extension of the central line $\overline{AB}$ will perceive, instead of a point-like star A, a luminius circle of the angular radius β around the center of B, where

$$\beta = \sqrt{\alpha_0 \frac{R_0}{D}}.$$



Of course, there is no hope of observing this phenomenon directly. First, we shall scarcely ever approach closely enough to such a central line. Second, the angle β will defy the resolving power of our instruments. For, α_0 being of the order of magnitude of one second of arc, the angle R_0/D, under which the deviating star B is seen, is much smaller. Therefore, the light coming from the luminous circle can not be distinguished by an observer as geometrically different from that coming from the star B, but simply will manifest itself as increased apparent brightness of B.

Magnification



The apparent brightness of A will be increased by the lens-like action of the gravitational field of B in the ratio q. This q will be considerably larger than unity only if x is so small that the observed positions of A and B coincide, within the resolving power of our instruments. Simple geometric considerations lead to the expression

$$q = \frac{l}{x} \cdot \frac{1 + \frac{x^2}{2l^2}}{\sqrt{1 + \frac{x^2}{4l^2}}},$$

where

$$l = \sqrt{\alpha_0 D R_0}.$$

Therefore, there is no great chance of observing this phenomenon, even if dazzling by the light of the much nearer star B is disregarded. This apparent amplification of q by the lens-like action of the star B is a most curious effect, not so much for its becoming infinite, with x vanishing, but since with increasing distance D of the observer not only does it not decrease, but even increases proportionally to $\sqrt{D}$.

The increase of magnification with the distance turned out to be key for the future of gravitational lensing: to go **extragalactic**

Zwicky 1937: galaxies could act as gravitational lenses, and also be used to detect dark matter

Gravitational Lensing:

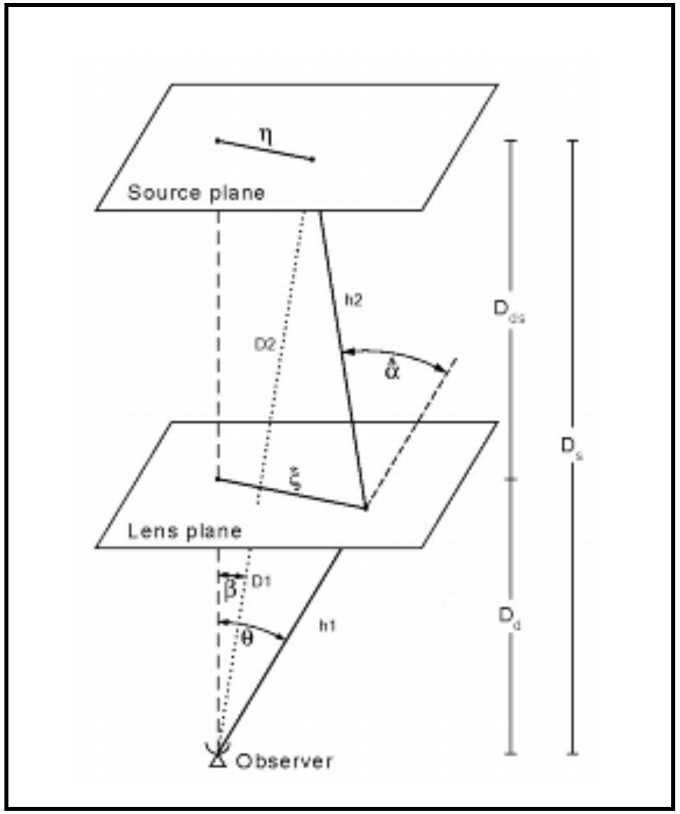
the 1960's renaissance

New theoretical developments

Klimov 1963: galaxy-galaxy lensing

Liebes 1964: star-star lensing

Refsdal 1964:
cosmological applications;
evolution of a bundle of
geodesics (Sachs);
time-delay



optical axis

source position

impact
parameter

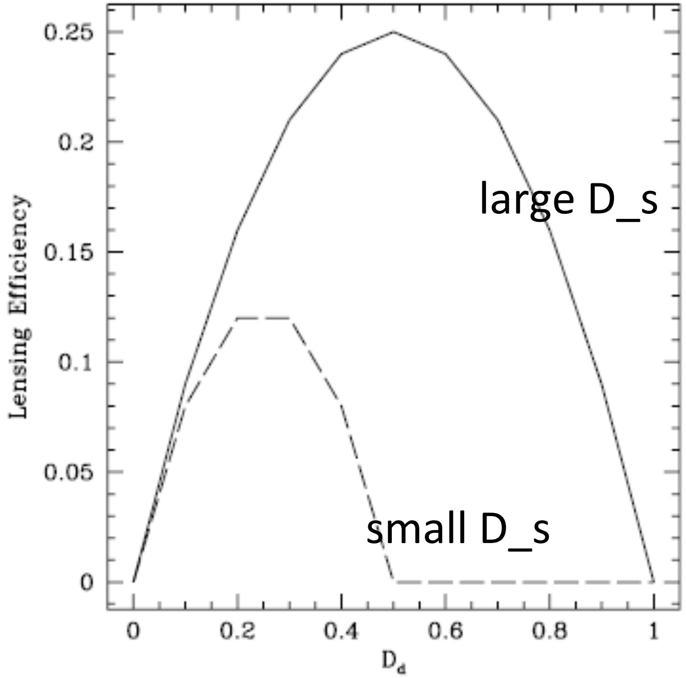
deflection
angle

image position

The lens equation

$$D_s\vec{\theta} = D_s\vec{\beta} + 2D_{ds}\frac{\hat{\vec{\alpha}}}{2}$$

(vector addition on the source plane)



The lens equation is a mapping between source and image planes. The central quantity of gravitational lensing is the vectorial field $\vec{\alpha}\,(\vec{\theta})$. It contains the dependence on the gravitational field and deflection potential.

The **deflection field** contains the physics of the lens, and the gravity model

$$\boxed{\vec{\alpha} = -\frac{2}{c^2}\int_{\lambda_A}^{\lambda_B} \dot{\vec{u}}_x \, d\lambda = \frac{2}{c^2}\int_{\lambda_A}^{\lambda_B} \vec{\nabla}_\perp \Phi \, d\lambda,}$$

$$\vec{x}(\vec{\theta},w) = f_K(w)\vec{\theta} - \frac{2}{c^2}\int_0^w dw'\, f_K(w-w')\left[\vec{\nabla}_\perp\Phi(\vec{x}(\vec{\theta},w'),w') - \vec{\nabla}_\perp\Phi(0,w')\right] \Leftrightarrow$$

$$\Leftrightarrow \beta_i(\vec{\theta},w) = \theta_i - \frac{2}{c^2}\int_0^w dw'\, \frac{f_K(w-w')}{f_K(w)} f_K(w') \left[\Phi_{,i}\left(\vec{x}(\vec{\theta},w'),w'\right) - \Phi_{,i}\left(0,w'\right)\right].$$

integration over
cosmological distances

It depends on the 2D projection of the mass density of the lens on the lens plane.

Image distortion

Extended sources are differentially distorted by the tidal field of the lens (spatial derivatives of alpha)

$$\beta(\theta) = \beta(\theta_0) + A(\theta_0).(\theta - \theta_0)$$ + higher-orders

Amplification matrix

$$A_{ij}(\theta) = \frac{\partial \beta_i}{\partial \theta_j} = \left(\delta_{ij} - \frac{\partial \alpha_i}{\partial \theta_j}\right)$$

symmetrical traceless
antisymmetrical
trace

$$A = \begin{bmatrix} \sigma_1 & \sigma_2 \\ \sigma_2 & -\sigma_1 \end{bmatrix} + \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} + \begin{bmatrix} \Theta/2 & 0 \\ 0 & \Theta/2 \end{bmatrix}$$

3 independent effects:
shear, rotation,
convergence

$$\begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

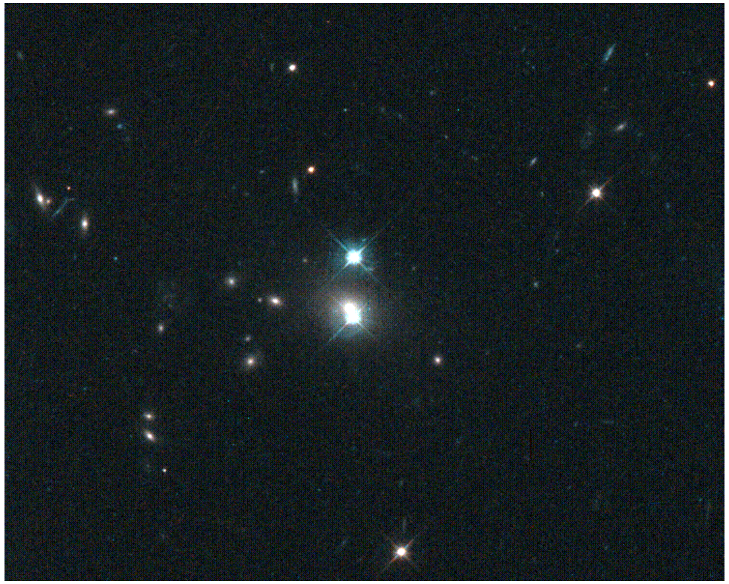
Image magnification

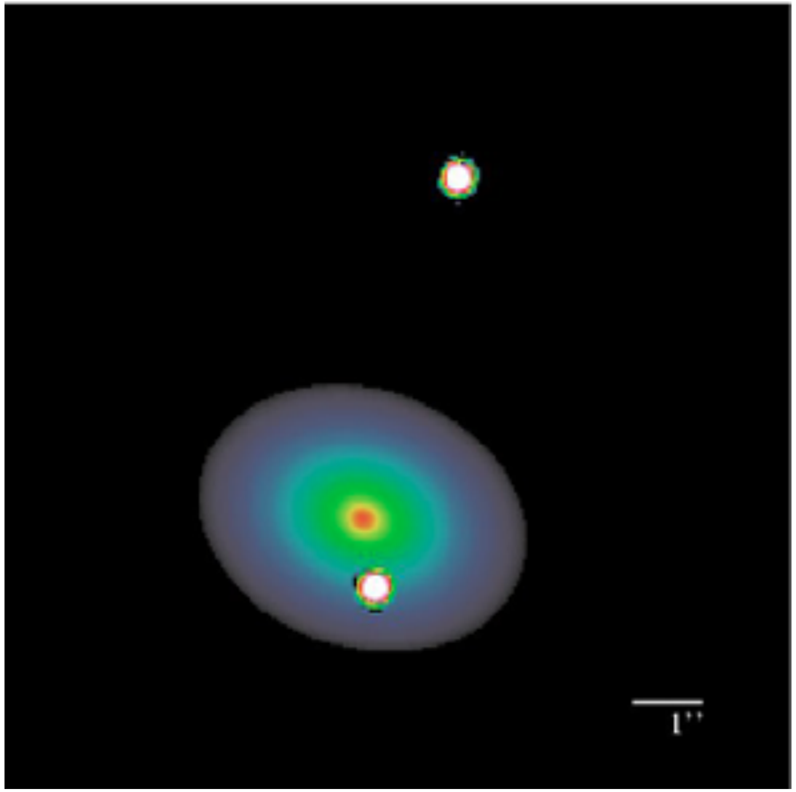
$$\mu = \frac{1}{det\ A} = \frac{1}{(1-\kappa)^2 - \gamma^2}$$

New observations

Discovery of quasars (Schmidt 1963): very distant, luminous and compact objects → good lensing sources

First observation of the lensing effect (Walsh, Carswell and Weymann 1979)





z_lens galaxy = 0.36
z_source quasar = 1.4

separation = 5''.7

Gravitational Lensing:

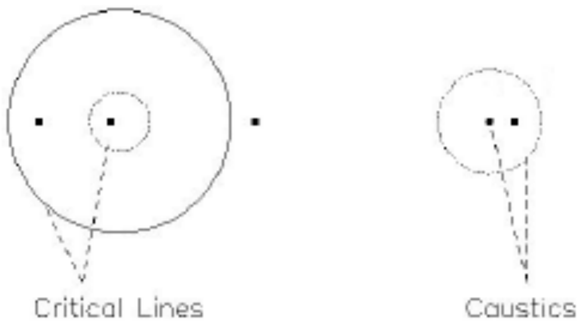
the modern times

A multitude of lensing systems and applications

det A = 0 defines critical lines in the image plane,
mapped to caustic lines in the source plane:

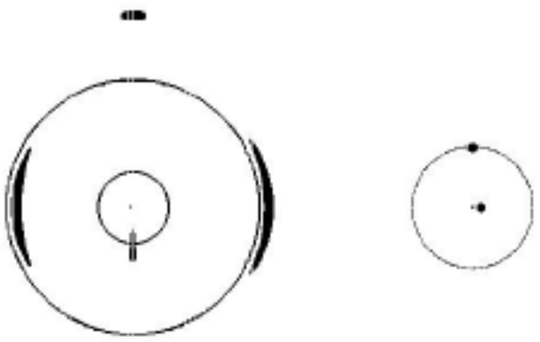
regions of high distortion, multiple images

Spherical lens



image

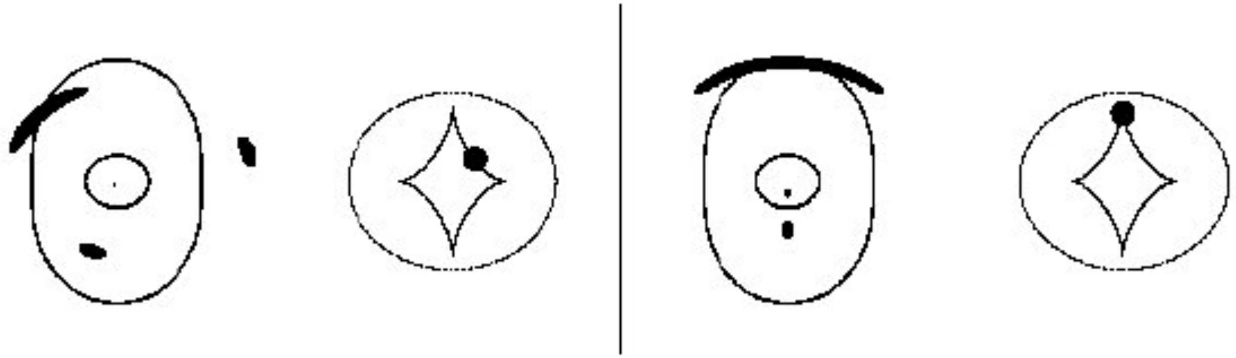
point source



image

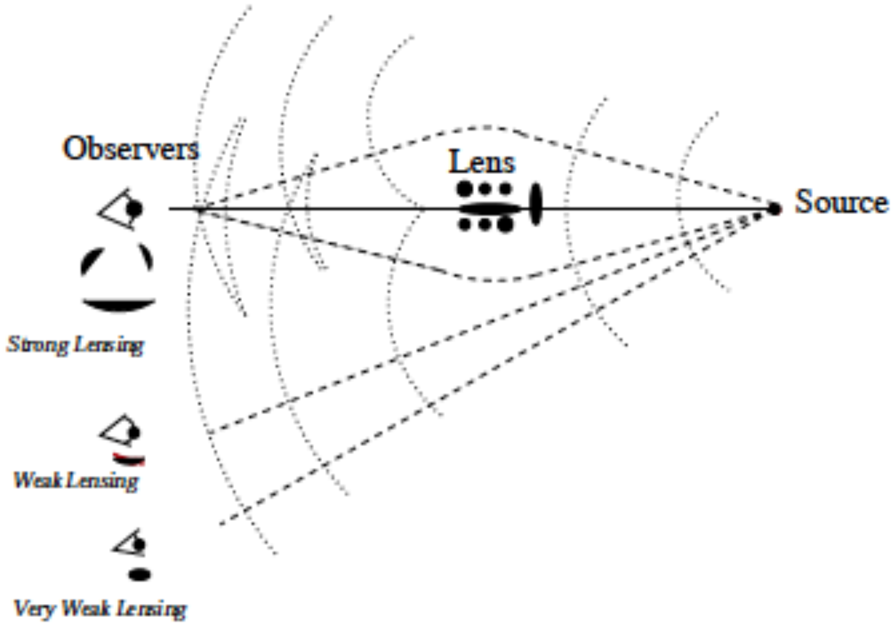
extended source

Elliptical lens



Far from that area, regions of small K, no arcs or multiple images:

Strong Lensing / Weak Lensing



Measurements of
positions, fluxes,
distortions

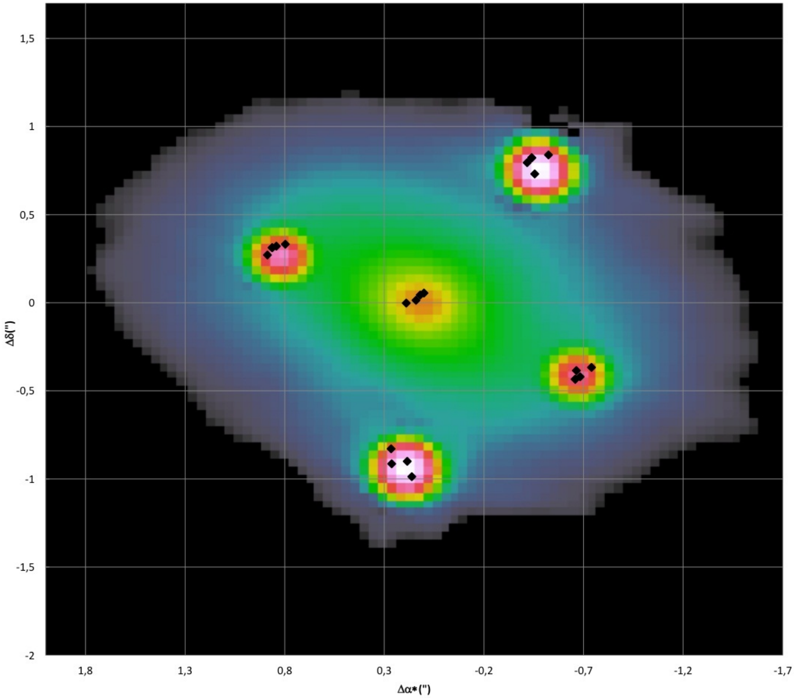
used to constrain
the lens models
(assuming gravity theory)

or to test gravity
(assuming lens model)

#	**Source**	**Lens**	**Effects**	**Applications**
1	Quasar	Galaxy	**SL:** multiple images, Einstein rings, time delays, **ML:** variable lightcurves	mass of galaxies, satellites, substructure, H_0, Ω_Λ, size of quasars
2	Star (extra-galactic)	Compact object (galactic)	**ML:** peak in lightcurve	dark matter in Milky Way halo
3	Star (galactic)	Star (galactic)	**ML:** perturbed peak in lightcurve	extrasolar planets, MW inner structure, M31 structure, limb darkening
4	Galaxy	Cluster	**SL:** multiple images, giant arcs **WL:** arclets, magnification bias	total mass of cluster, Ω_Λ, redshifts, cluster mass profile, clusters morphology
5	Galaxy	Galaxy	**StWL:** ellipticity bias, shear-galaxy correlation	galaxies parameters, haloes properties
6	Galaxy	Large scale structure	**StWL:** cosmic shear, shear-shear correlation	cosmological parameters, matter power spectrum
7	Quasar	Large scale structure	**StWL:** cosmic magnification, quasar-galaxy correlation	cosmological parameters, matter power spectrum, bias
8	Last Scattering surface	Large scale structure	**StWL:** smoothing of CMB T-T correlation	cosmological parameters from lensed C_ℓ, matter power spectrum

Galaxy strong lensing:
RCS2 032727-132623
(HST)

Quasar strong lensing:
Einstein Cross
(GAIA)

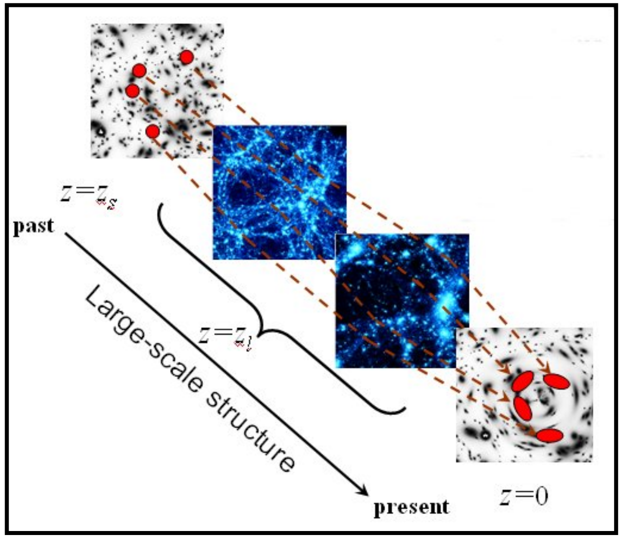


Galaxy weak lensing:
Bullet Cluster
(HST, Chandra, ESO)

Cosmic shear:

a powerful cosmological application of gravitational lensing

Constraining the lens model (evolution and distribution of large scale structure, dark matter, dark energy)
or testing general relativity on cosmological scales



Statistical properties of the convergence and shear fields - related to statistical properties of the lensing potential field.

Ex: power spectrum

$$P_\gamma^{ij}(\ell) = \frac{9}{4}\Omega_{\rm m}^2 H_0^4 \int_0^{w_{\rm h}} dw \frac{g_i(w)g_j(w)}{a^2(w)} \Sigma^2(a) P_\delta\left(\frac{\ell}{w}, w\right)$$

$$\Sigma(a) = Q(a)(1+\eta(a)/2)$$

$$\Psi = (1+\eta)\Phi$$

G_eff = G Q

First detection of shear correlations in field galaxies - evidence of Dark matter

4 papers submitted do arXiv on march 2000 by 4 independent teams

Detection of correlated galaxy ellipticities from CFHT data: first evidence for gravitational lensing by large-scale structures ⋆

L. Van Waerbeke[1], Y. Mellier[2,3], T. Erben[4], J.C. Cuillandre[5], F. Bernardeau[6], R. Maoli[2,3], E. Bertin[2,3], H.J. Mc Cracken[7], O. Le Fèvre[7], B. Fort[2], M. Dantel-Fort[3], B. Jain[8], P. Schneider[4]

Detection of Weak Gravitational Lensing by Large-scale Structure

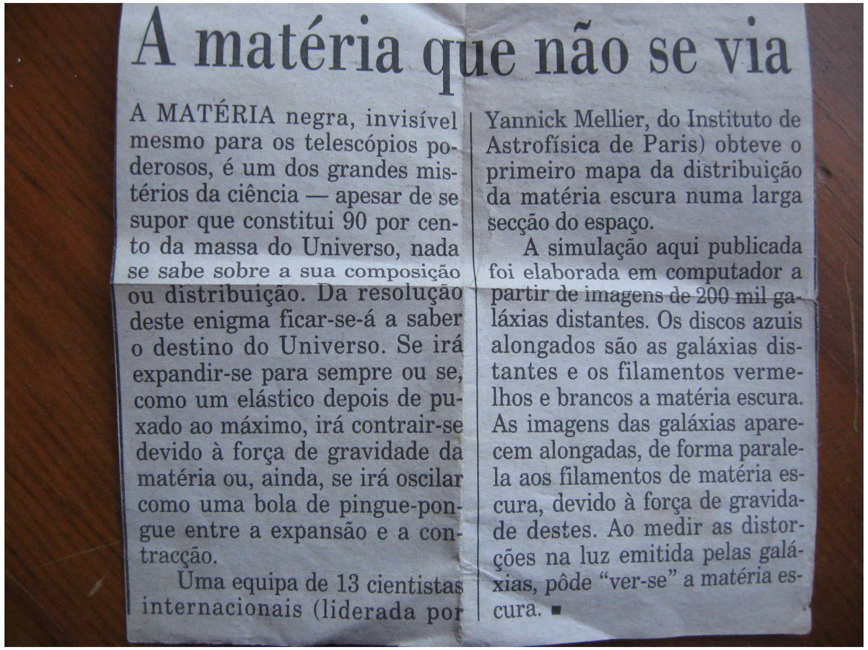
David J. Bacon,[1⋆] Alexandre R. Refregier[1] & Richard S. Ellis[1,2]

Detection of weak gravitational lensing distortions of distant galaxies by cosmic dark matter at large scales

David M. Wittman*, J. Anthony Tyson, David Kirkman,
Ian Dell'Antonio†, and Gary Bernstein‡

LARGE-SCALE COSMIC SHEAR MEASUREMENTS

NICK KAISER, GILLIAN WILSON AND GERARD A. LUPPINO



A matéria que não se via

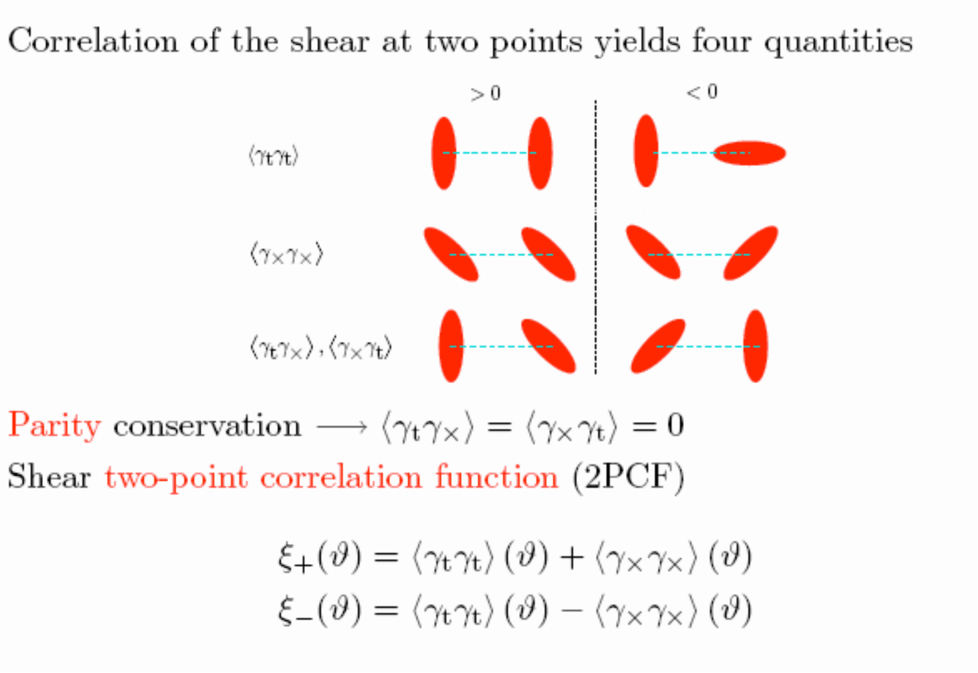
A MATÉRIA negra, invisível mesmo para os telescópios poderosos, é um dos grandes mistérios da ciência — apesar de se supor que constitui 90 por cento da massa do Universo, nada se sabe sobre a sua composição ou distribuição. Da resolução deste enigma ficar-se-á a saber o destino do Universo. Se irá expandir-se para sempre ou se, como um elástico depois de puxado ao máximo, irá contrair-se devido à força de gravidade da matéria ou, ainda, se irá oscilar como uma bola de pingue-pongue entre a expansão e a contracção.

Uma equipa de 13 cientistas internacionais (liderada por Yannick Mellier, do Instituto de Astrofísica de Paris) obteve o primeiro mapa da distribuição da matéria escura numa larga secção do espaço.

A simulação aqui publicada foi elaborada em computador a partir de imagens de 200 mil galáxias distantes. Os discos azuis alongados são as galáxias distantes e os filamentos vermelhos e brancos a matéria escura. As imagens das galáxias aparecem alongadas, de forma paralela aos filamentos de matéria escura, devido à força de gravidade destes. Ao medir as distorções na luz emitida pelas galáxias, pôde "ver-se" a matéria escura. ■

The Euclid Mission (2020):

a long way since the first cosmic shear detections



Correlation of the shear at two points yields four quantities

Parity conservation $\longrightarrow \langle\gamma_t\gamma_\times\rangle = \langle\gamma_\times\gamma_t\rangle = 0$

Shear two-point correlation function (2PCF)

$$\xi_+(\vartheta) = \langle\gamma_t\gamma_t\rangle(\vartheta) + \langle\gamma_\times\gamma_\times\rangle(\vartheta)$$
$$\xi_-(\vartheta) = \langle\gamma_t\gamma_t\rangle(\vartheta) - \langle\gamma_\times\gamma_\times\rangle(\vartheta)$$

ground based
2 sqdeg
10^5 galaxies

space
15 000 sqdeg
10^9 galaxies

This will allow to measure the cosmic shear power spectrum with a precision better than 1% if:

shape measurement bias < 0.001
mean redshift precision > 0.002

This precision is required to
understand the nature of dark energy and dark matter by:

- reaching a DE FoM > 400

- measuring the growth factor parameter γ with precision < 0.02,
and constrain Ψ and Φ separately, using also galaxy clustering

(Mass distribution of non-relativistic matter depends only on Ψ
while lensing depends on light propagation $\Psi + \Phi$)

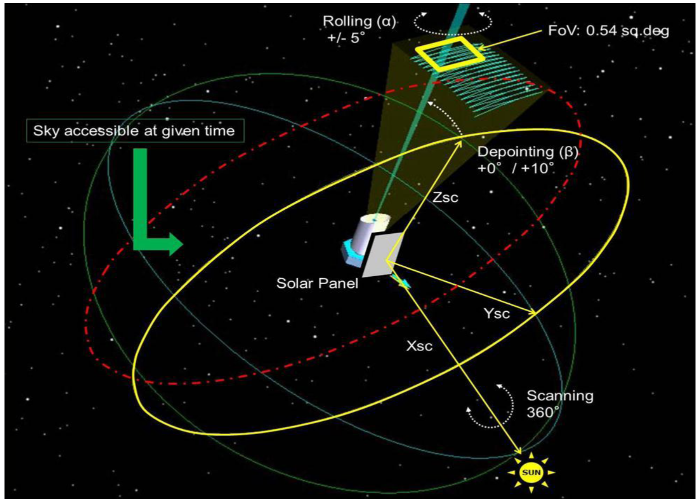
euclid

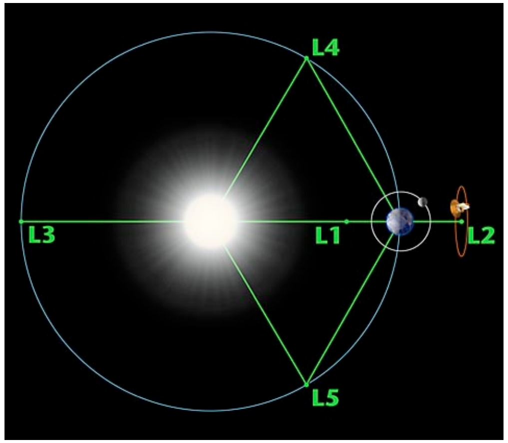
euclid
consortium

A modern cosmological test of GR with lensing - but it is not the heir of the eclipse experiments, it is not an astrometry mission → Gaia, precision 1’’ → 10^-5 ’’

The Euclid Mission at IA / FCUL:

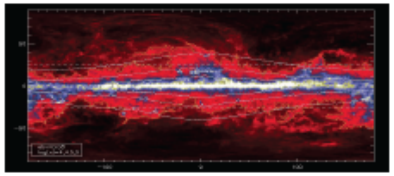
Survey Implementation



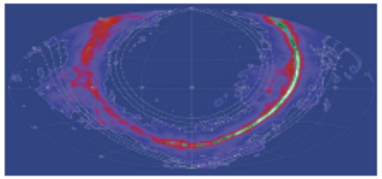


Find a 6-year sequence of fields (FoVs) in a step-and-stare procedure with constraints:

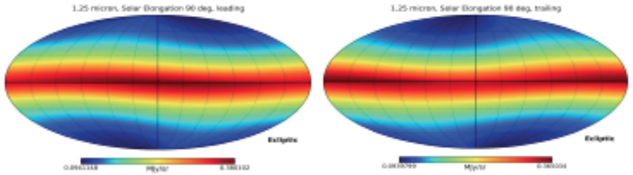
- operational (SAA range, propellant, maximum number of slews, exposure time)
- scientific (coverage > 15000 deg2, holes, exposure time, sky quality: <ngal> > 30 /sqamin
- calibration plan
- deep fields (40 deg2, 2 mag deeper)



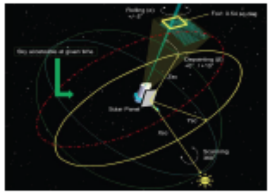
DUST : Extinction in the galactic plane (E(B-V) contours)



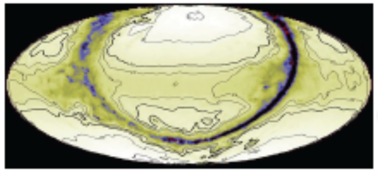
STAR DENSITY: Contamination in galactic plane



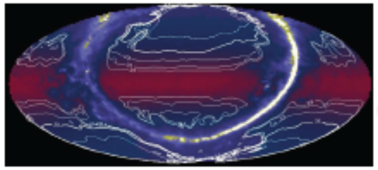
ZODIACAL LIGHT emission maps: Contamination in the ecliptic plane (with leading/trailing asymmetry)



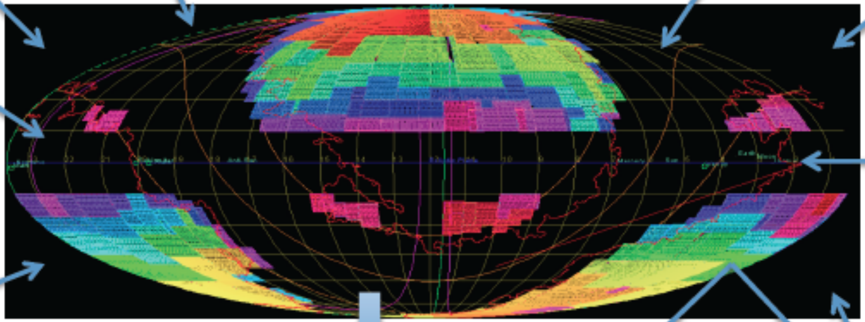
SPACECRAFT: limited range of rotations (in pitch ~ SAA and roll ~ α) ; limited propeller



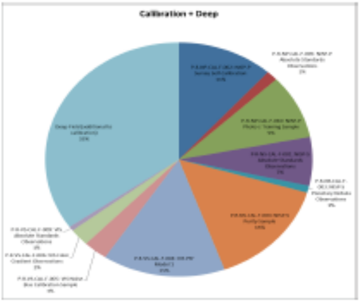
WEAK LENSING SAMPLING: galaxy density contours (arcmin^{-2})



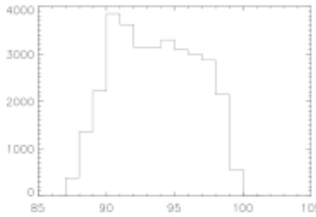
GALAXY CLUSTERING SAMPLING: galaxy density contours (arcmin^{-2})



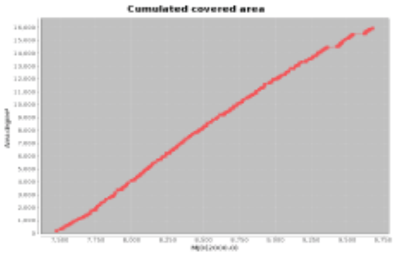
REFERENCE SURVEY: in ecliptic coordinates; color code (red to blue) shows the scheduled observation sequence. Released in the Mission Operations Concept Document, Dec 2013)



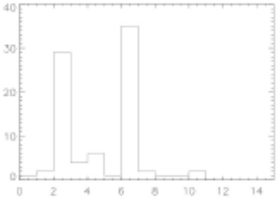
CALIBRATION PLAN: distribution of science and instrument calibrations, including targets and cadences



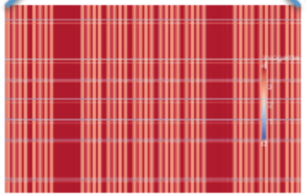
Histogram of Solar Aspect Angle over the total 40 000 pointings



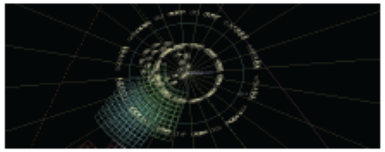
Rate of survey coverage with time. 15000 deg² are reached in 5.5 years



Histogram of duration of calibration sequences (in days) over the total time

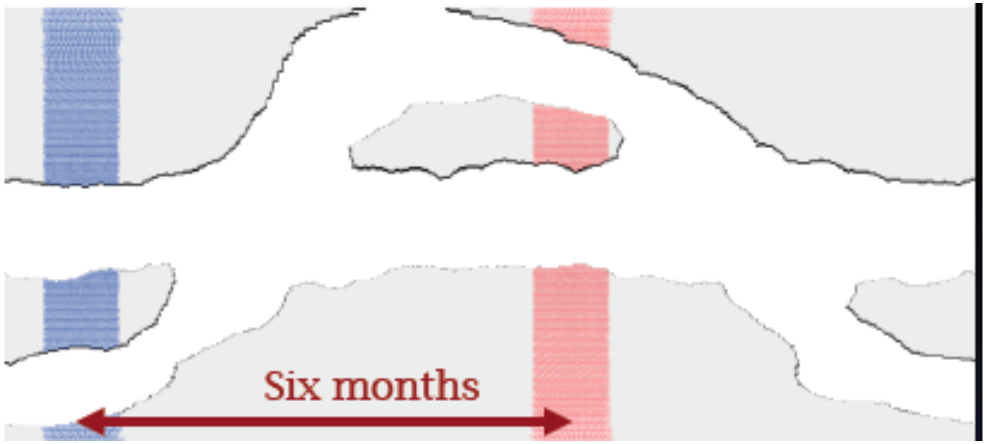


INTEGRATION TIME maps: number counts per pixel over the dither sequence for 1 FoV

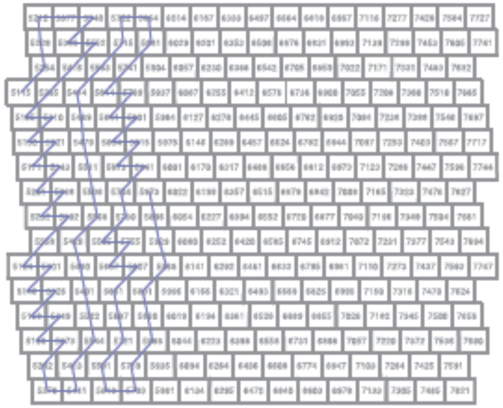


CALIBRATION TARGETS: high ecliptic latitudes; Deep fields; HST fields

Small-scale tiling

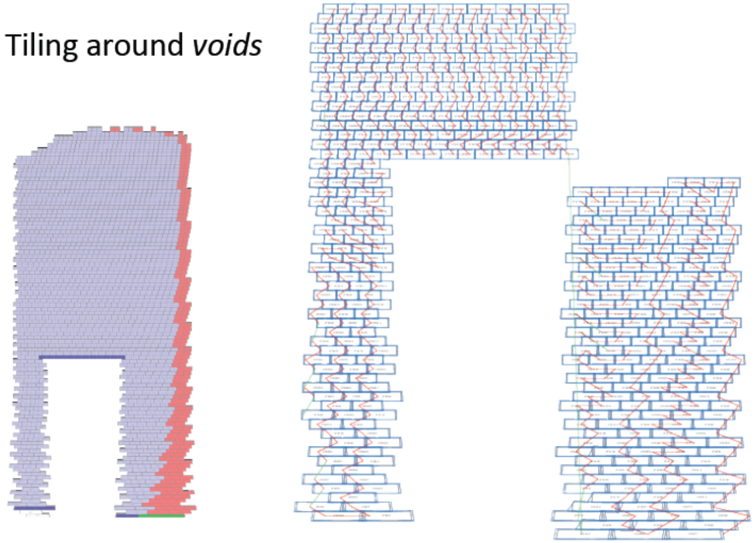


Patch tiling algorithm



A tiling sequence is computed by finding a valid observation sequence, traversing the fields with small-slews. This is computed iteratively.

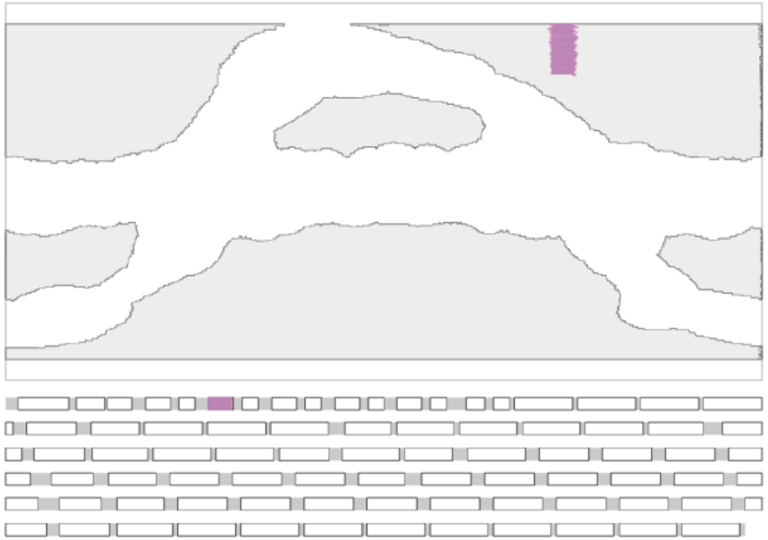
Tiling around *voids*



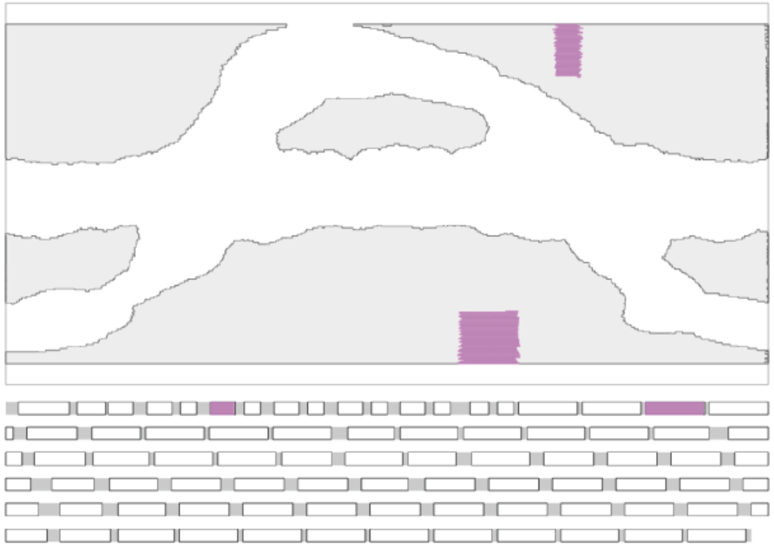
One final trick is to **split survey-windows** in vertical stripes (chosen at natural edges) plus allow for slews slightly **larger than 1.2 deg**. This produces multi-part patches, joined by one large-slew.

Large-scale tiling

Solving the puzzle



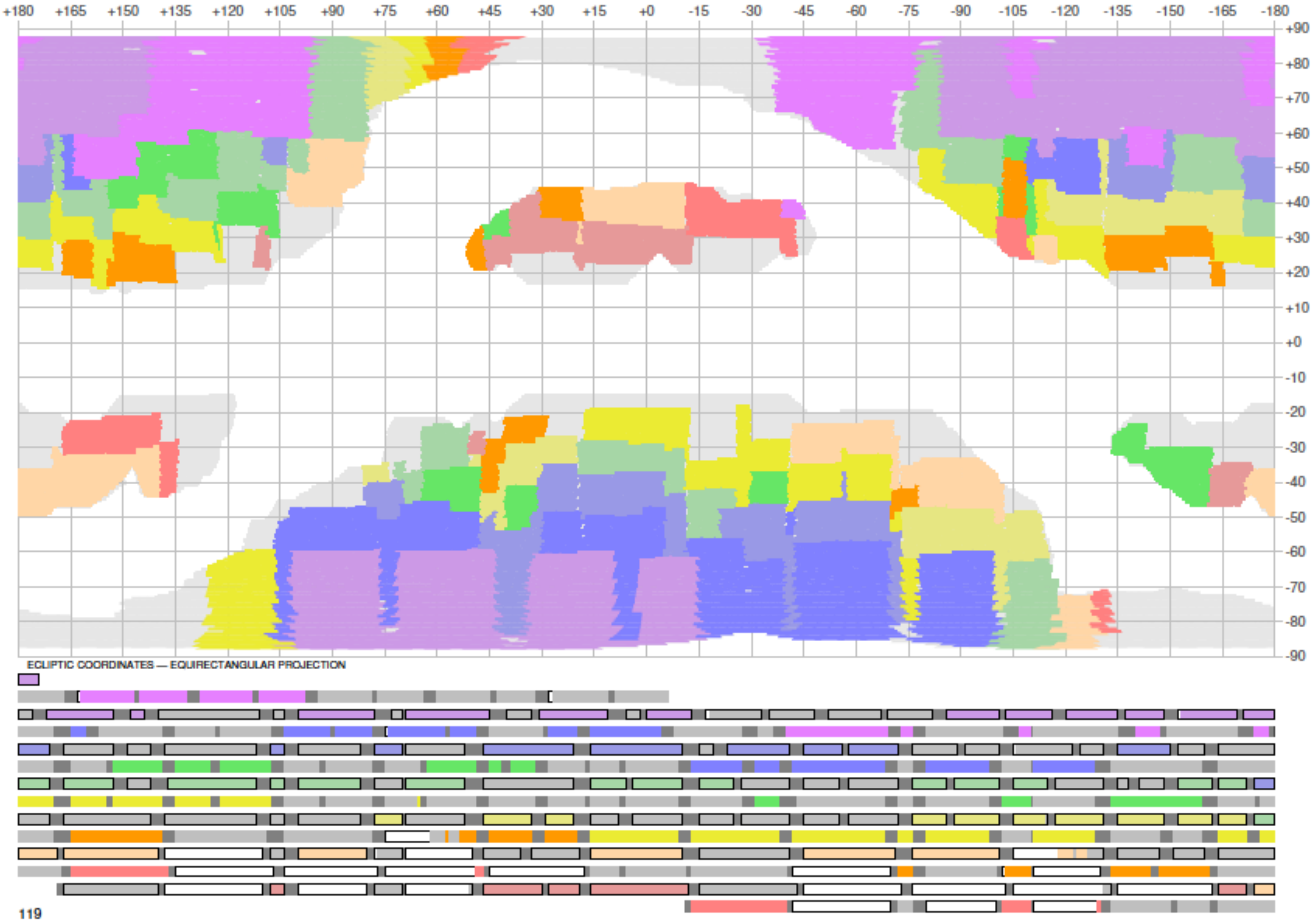
Generating a survey is like solving a jigsaw puzzle: placing a piece at a time striving to match them. Begin by selecting and placing the first patch on the board.



Then, discard overlapped patches, re-generate patches from modified stripes, re-compute scores, and randomly select the second patch.

Can make several independent runs

The Euclid Reference Survey



Current Status : Preliminary Design Review

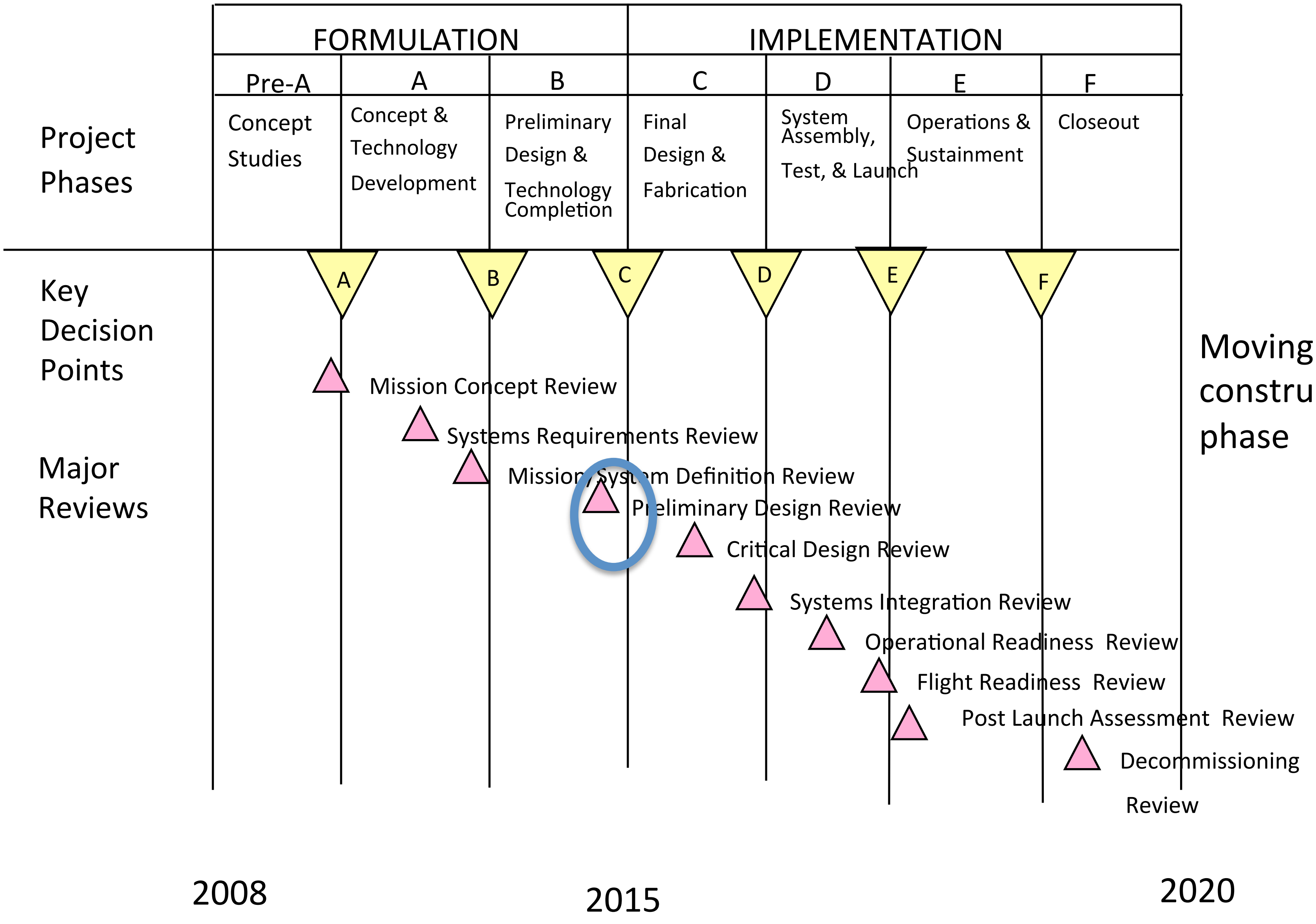
Euclid Reference Survey delivered to ESA as part of the technical review Mission-PDR

Mission Concept Review - Does the proposed concept meet the mission need and objectives?

System Requirements Review and/or Mission Definition Review - Do the functional and performance requirements and the selected concept satisfy the mission?

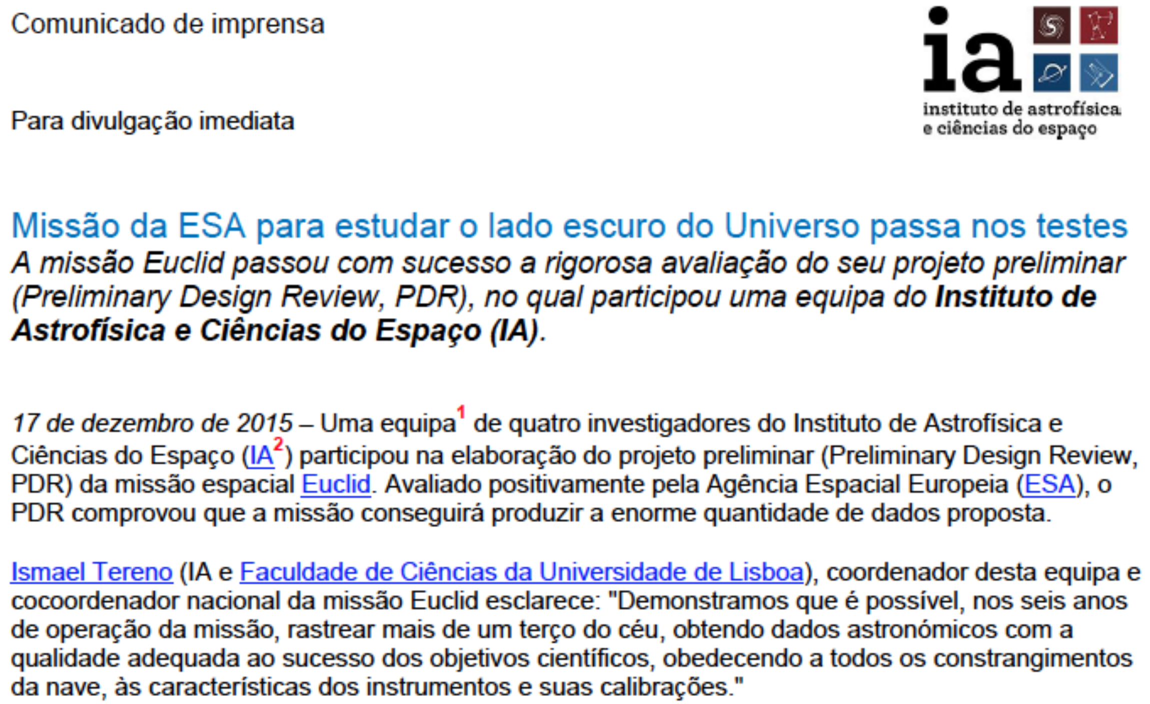
Preliminary Design Review - **Does the preliminary design meet all the system requirements within acceptable cost, schedule, and risk?**

Critical Design Review - Is the system design mature enough to proceed with full-scale fabrication, assembly, integration and test?



Moving to the construction phase

Press Releases - 17 December 2015 - (yesterday)



Comunicado de imprensa

Para divulgação imediata

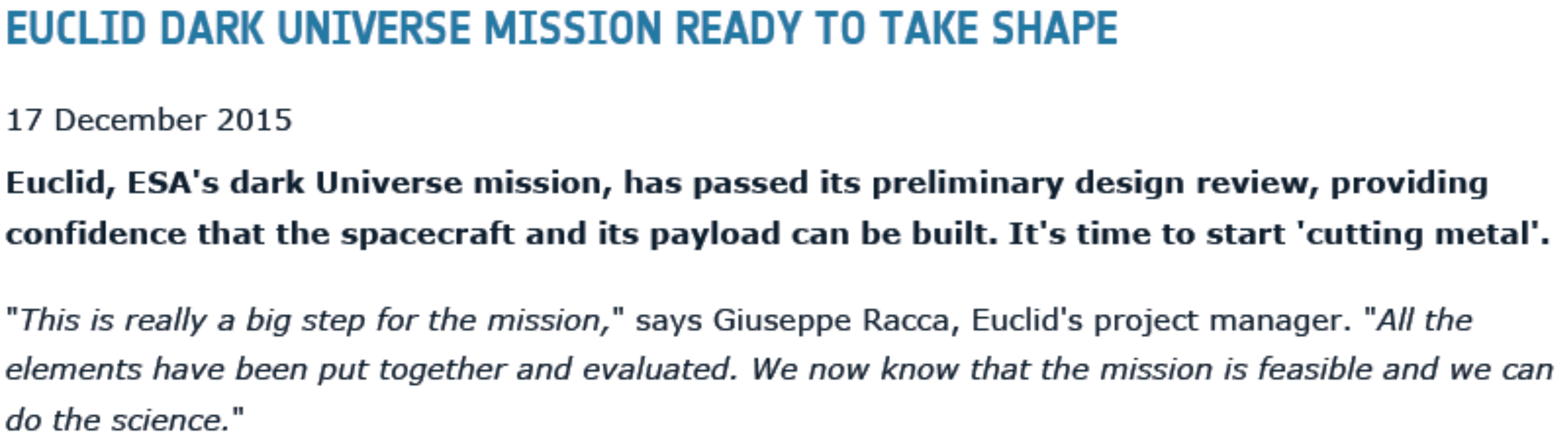
instituto de astrofísica
e ciências do espaço

Missão da ESA para estudar o lado escuro do Universo passa nos testes

*A missão Euclid passou com sucesso a rigorosa avaliação do seu projeto preliminar (Preliminary Design Review, PDR), no qual participou uma equipa do **Instituto de Astrofísica e Ciências do Espaço (IA)**.*

17 de dezembro de 2015 – Uma equipa[1] de quatro investigadores do Instituto de Astrofísica e Ciências do Espaço (IA[2]) participou na elaboração do projeto preliminar (Preliminary Design Review, PDR) da missão espacial Euclid. Avaliado positivamente pela Agência Espacial Europeia (ESA), o PDR comprovou que a missão conseguirá produzir a enorme quantidade de dados proposta.

Ismael Tereno (IA e Faculdade de Ciências da Universidade de Lisboa), coordenador desta equipa e cocoordenador nacional da missão Euclid esclarece: "Demonstramos que é possível, nos seis anos de operação da missão, rastrear mais de um terço do céu, obtendo dados astronómicos com a qualidade adequada ao sucesso dos objetivos científicos, obedecendo a todos os constrangimentos da nave, às características dos instrumentos e suas calibrações."



EUCLID DARK UNIVERSE MISSION READY TO TAKE SHAPE

17 December 2015

Euclid, ESA's dark Universe mission, has passed its preliminary design review, providing confidence that the spacecraft and its payload can be built. It's time to start 'cutting metal'.

"*This is really a big step for the mission,*" says Giuseppe Racca, Euclid's project manager. "*All the elements have been put together and evaluated. We now know that the mission is feasible and we can do the science.*"

100 Years of Gravitational Lensing (in Lisbon)