Ultra-high energy particle collisions in the vicinity of black holes

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Two kinds of energies as a result of collisions

1) High (unbound) energy in the centre of mass frame E_c.m.

Black holes, naked singularities, quasiblack holes, star-like configurations, wormholes

BHs: rotating or electrically charged

Proximity to horizon
Ergoregion (high angular momentum),
Extremely rapid rotation

Collisions outside and inside BH

In magnetic field

Sclar field

Particle moving towards horizon (BSW effect), Banados-Silk-Wesr PRL 2009
Head-on collisions
Fine-tuned (critical) and typical (usual) particles

2) Possibility to get high (unbound) energies E at infinity (debris after collision) – super-Penrose process (separate talk on 8-th BH Workshop)

Physical explanation and properties of BSW effect

Universal character of BSW effect near BH

Kinematic nature of the BSW effect. Role of critical trajectories

BSW effect and acceleration horizons

Geometric explanation

Kinematic explanation for collisions inside BH

Extremal versus nonextremal BHs

Kinematic censorship
B

BSW effect versus Penrose process: what can be seen at infinity?

Role of self-force due to gravitational radiation

High energy processes near BHs

Key quantity: energy in centre of mass frame

1 particle $$m^2 = \left|P_\mu P^\mu\right|$$

2 particles colliding in some point

$$E^2{}_{cm} = \left|P_\mu P^\mu\right|$$

Total momentum $$P_\mu = p^{(1)}{}_\mu + p^{(2)}{}_\mu$$

$$P_a = (E_{c.m.}, 0, 0, 0) \qquad u^\mu u_\mu = -1$$

Individual E finite, energy in CM frame unbound

Two different kinds of energy

Killing energy $E = -p_{\mu}\xi^{\mu}$ ξ^{μ} Killing vector

E conserved, integral of motion since metric is static or stationary

Energy in the CM frame $E_{c.m.}$

not conserved. Moreover, it is defined in one point only.
point of collision

Head-on collision

1975 - 1977 T. Piran, J. Katz and J. Shanam

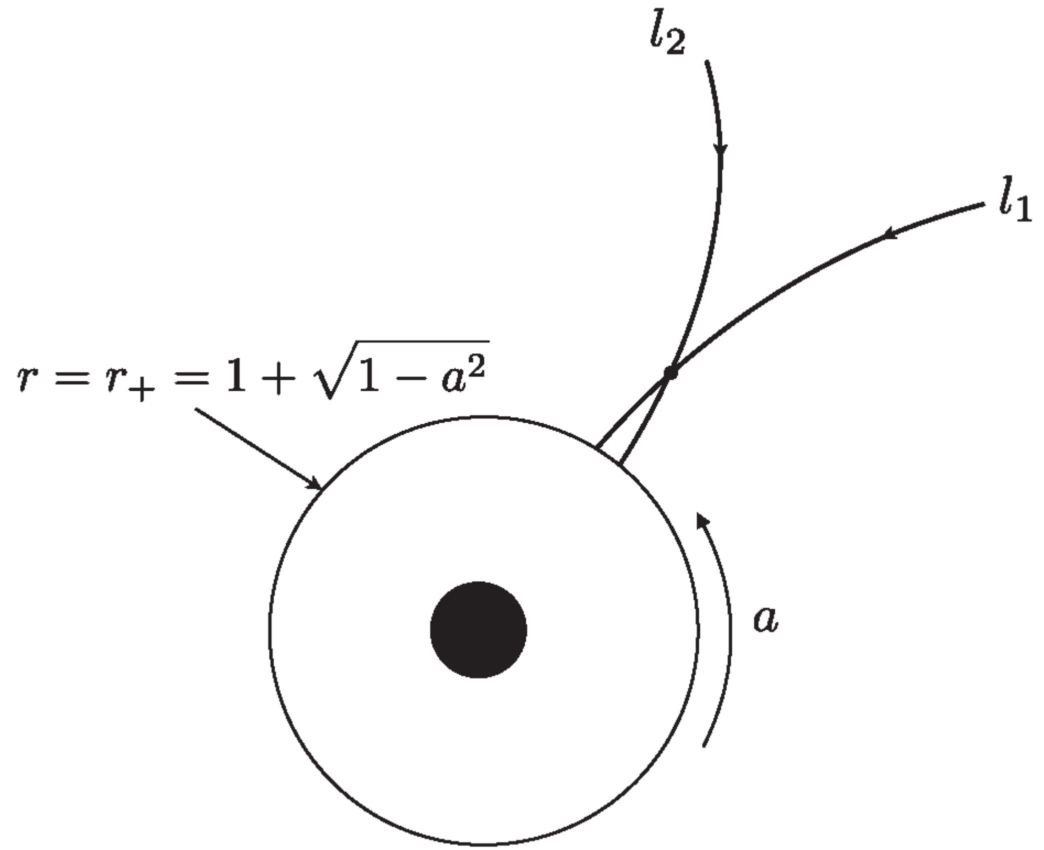
Two particles move in opposite directions near BH

Almost infinite relative blue shift

E in CM frame almost diverges

Special scenario. Particle near black (not white) hole moving away from horizon and colliding with another particle

M. Banados, J. Silk, and S. M. West PRL 2009



$$-2(1 + \sqrt{1 + a}) < l < 2(1 + \sqrt{1 - a})$$

Both particles experience blue shift, centre of mass frame is in free fall.

Acceleration of particles as universal property of rotating black holes

O. Z., PRD 2010

Role of horizon

Universality of black hole physics

Unified approach to nonextremal versus extremal black holes

Energy in CM frame

$$E_{c.m.}{}^2 = -(m_1 u^\mu{}_1 + m_2 u^\mu{}_2)(m_1 u_{1\mu} + m_2 u_{2\mu})$$

$$E_{c.m.}{}^2 = m^2{}_1 + m_2{}^2 + 2 m_1 m_2 \gamma$$

$$\gamma = -(u_1 u_2)$$

$$ds^2 = -N^2 dt^2 + g_\phi (d\phi - \omega dt)^2 + \frac{dr^2}{A} + g_\theta d\theta^2,$$

equatorial plane $\theta = \dfrac{\pi}{2} \quad (z = 0)$ Is a symmetry one

$u_0 = -E \qquad u_\phi = L$ conserved quantities

Integrals of geodesic equations

$$g_{\mu\nu} u^\mu u^\nu = -1$$

$$\dot{t} = u^0 = \frac{E - \omega L}{N^2} = \frac{X}{N^2}. \qquad X = E - \omega L$$

$$\frac{E_{c.m.}^2}{2m^2} = \frac{X_1 X_2 - \varepsilon_1 \varepsilon_2 Z_1 Z_2}{N^2} + 1 - Y, \qquad Z_i = \sqrt{(E_i - \omega L_i)^2 - N^2 b_i},\ b_i = 1 + \frac{L_i^2}{g_{\phi\phi}},$$

$$Y = \frac{L_1 L_2}{g_{\phi\phi}}.$$

$\varepsilon = -1$ for particle moving towards horizon

$\varepsilon = +1$ away from horizon

$\varepsilon_1 \varepsilon_2 = -1$ head-on collision, Piran et al

$$\frac{E_{c.m.}^2}{2m^2} = \frac{X_1 X_2 + Z_1 Z_2}{N^2} + 1 - Y,$$

$E_{c.m.}^2$ always unbound near horizon

For any relationship between energies and angular momenta

$$\varepsilon_1 = \varepsilon_2 = -1$$

Energy in CM frame

$$\frac{E_{c.m.}^2}{2m^2} = \frac{X_1 X_2 - Z_1 Z_2}{N^2} + 1 - Y,$$

$$X_i \equiv E_i - \omega L_i$$

$$Z_i = \sqrt{(E_i - \omega L_i)^2 - N^2 b_i}, \; b_i = 1 + \frac{L_i^2}{g_{\phi\phi}}, \qquad Y = \frac{L_1 L_2}{g_{\phi\phi}}.$$

Three kinds of mechanism leading to unbound energy in CM frame

1) $N \to 0$ proximity to horizons BSW

2) $L_2 \to -\infty$ inside ergoregion, NOT near horizon Grib and Pavlov, Kerr metric

Generalization OZ

3) $\omega \to \infty$ rapid totation (wormholes)

$$\varepsilon_1 = \varepsilon_2 = -1$$

BSW

$$\frac{E_{c.m.}^2}{2m^2} = \frac{X_1 X_2 - Z_1 Z_2}{N^2} + 1 - Y,$$

$$X_i \equiv E_i - \omega L_i$$

$$Z_i = \sqrt{(E_i - \omega L_i)^2 - N^2 b_i},\ b_i = 1 + \frac{L_i^2}{g_{\phi\phi}}, \qquad Y = \frac{L_1 L_2}{g_{\phi\phi}}.$$

In general case, $E_{c.m.}^2$ remains bound in horizon limit $N \to 0$

Special conditions for unbound $E_{c.m.}^2$

Two kinds of particles (trajectories)

Usual $X_H \equiv E - \omega_H L \neq 0$

Critical $X_H \equiv E - \omega_H L = 0$

Different limiting transitions

1) point of collision approaches the horizon, $N \to 0$ and $L_1 \to L_{1(H)}$ afterwards

2) $L_1 \to L_{1(H)}$ and $N \to 0$ afterwards

In both cases

$$\lim_{L_1 \to L_{1(H)}} \lim_{N \to 0} E_{cm} = \lim_{N \to 0} \lim_{L_1 \to L_{1(H)}} E_{cm} = \infty$$

particle 1 is critical, particle 2 is usual

Extremal versus nonextremal

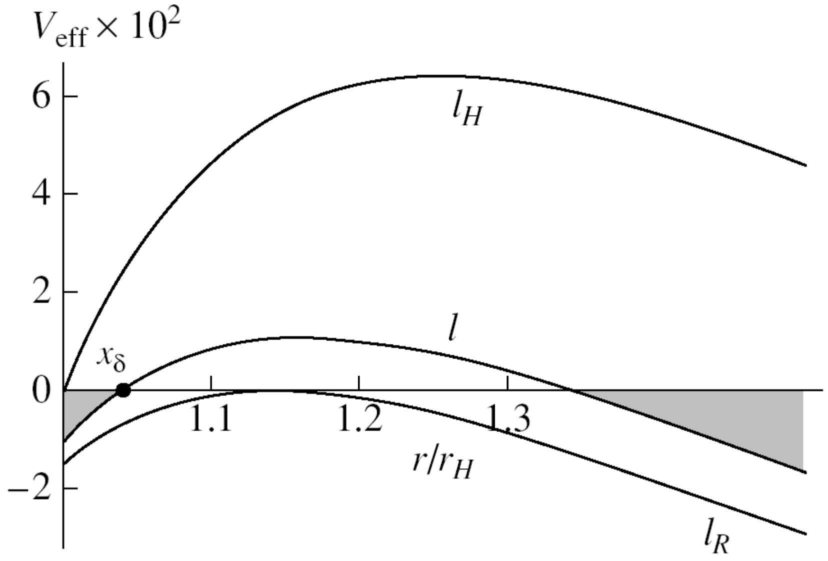
Problems with attaining extremality, a=0,998 (Thorne)

Jacobson et al, Berti at al: difficulties in realization

Grib and Pavlov: nonextremal Kerr

Extremal case: collision near horizon

$$E_{c.m.} \approx \frac{m}{\sqrt{\delta}} \sqrt{\frac{2(L_H - L_2)}{1 - \sqrt{1 - a^2}}} \qquad L_1 = L_{(H)} - \delta$$



The effective potential for $A = 0.95$ and $l_R \approx 2.45$, $l = 2.5$, $l_H \approx 2.76$. Allowed zones for $l = 2.5$ are shown by the gray color.

Multiple scattering (Grib and Pavlov)

$$-2\left(1+\sqrt{1+a}\right)=L_L \le L \le L_R = 2\left(1+\sqrt{1-a}\right). \qquad a = 1-\varepsilon$$

$$L_H - L_R = 2\frac{\sqrt{1-a}}{a}(\sqrt{1+a}+\sqrt{1-a}-a) \approx 2(\sqrt{2}-1)\sqrt{\varepsilon}$$

$$\varepsilon \to 0$$

Geometric explanation

$g_{\alpha\beta} = -l_\alpha N_\beta - l_\beta N_\alpha + \sigma_{\alpha\beta}$ $\sigma_{\alpha\beta}=a_\alpha a_\beta+b_\alpha b_\beta$ lightlike vectors l^μ and N^μ

spacelike vectors a^μ, b^μ orthogonal to them

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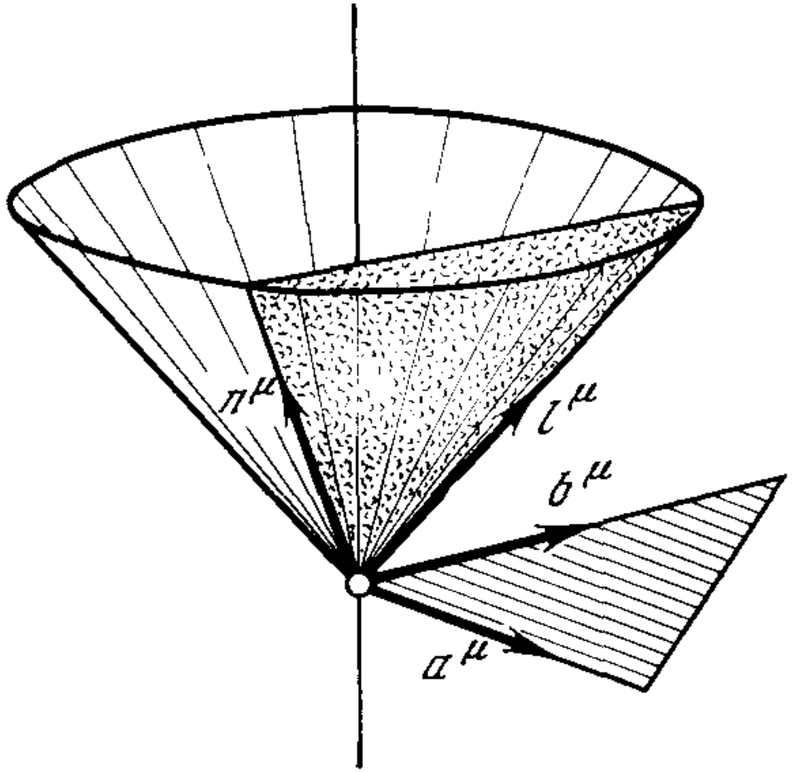


Four-velocity

$$u_i^\mu = \frac{l^\mu}{2\alpha_i} + \beta_i N^\mu + s_i^\mu,\ s_i^\mu = A_i a^\mu + B_i b^\mu$$

Case 1
always

$$-(u_1 u_2) = \frac{1}{2}\left(\frac{\beta_1}{\alpha_2} + \frac{\beta_2}{\alpha_1}\right) - (s_1 s_2).$$

$\alpha=0$

For head-on collision
means that particle
cannot cross horizon

$$E_{c.m.}^2 = m_1^2 + m_2^2 - 2m_1 m_2 (u_1 u_2)$$

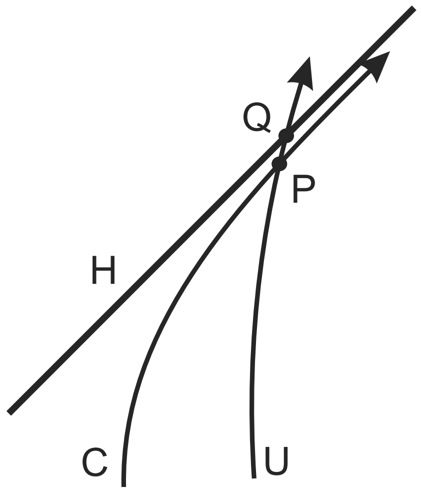
Case 2
Special condition

$$E_{c.m.}^2 = m_1^2 + m_2^2 + m_1 m_2 \left[\frac{\beta_1}{\alpha_2} + \frac{\beta_2}{\alpha_1} - 2(s_1 s_2)\right].$$

Case 2

$\alpha_1 \to 0.$

Now, special condition



$$E^2_{c.m.} \to \infty$$

Kruskal-like coordinates

$$ds^2 = -CdUdV + \gamma_{ab}dx^a dx^b$$

$$Cu^X u^Y = 1$$

$$u^X \sim \alpha \to 0 \qquad \tau \sim -\ln X \to \infty$$

Proper time grows unbound (T. Jacobson, Grib and Pavlov, O. Z.)

Kinematic explanation

$$E_{c.m.}^2 = -(p_1^\mu + p_2^\mu)(p_{1\mu} + p_{2\mu}) = m_1^2 + m_2^2 - 2m_1 m_2 u_1^\mu u_{2\mu}.$$

$$\gamma = -u_1^\mu u_{2\mu} = \frac{1}{\sqrt{1-w^2}}$$

BSW effect occurs if $w \to 1$ w is relative velocity

$$w^2 = 1 - \frac{(1-v_1^2)(1-v_2^2)}{[1 - v_1 v_2(\vec{n}_1 \vec{n}_2)]^2}$$

$$\vec{v}_1 = v_1 \vec{n}_1$$

The most interesting case: $v_1 < 1, \quad v_2 \to 1$

Collision of rapid particle with target

Relative velocity close to c

$$ds^2 = -N^2 dt^2 + g_{\phi\phi}(d\phi - \omega dt)^2 + dl^2 + g_{zz} dz^2$$

Attached to observer

$$h_{(0)\mu} = -N(1,0,0,0),$$

$$h_{(1)\mu} = (0,1,0,0),$$

$$h_{(2)\mu} = \sqrt{g_{zz}}\,(0,0,0,1),$$

$$h_{(3)\mu} = \sqrt{g_{\phi\phi}}\,(-\omega,0,0,1)$$

$$-u_\mu h^\mu_{(0)} = \frac{E-\omega L}{N},$$

$$u_\mu h^\mu_{(3)} = \frac{L}{\sqrt{g_{\phi\phi}}}.$$

If

$$V_\mu = h_{\mu(0)}$$

then

$$V_\mu \xi^{(3)\mu} = 0$$

ZAMO

$$E - \omega L = \frac{mN}{\sqrt{1-v^2}},$$

Horizon limit $\quad N \to 0$

1) Usual particle, $\quad E \neq \omega_+ L \quad v \to 1$

2) Critical particle $\quad E = \omega_+ L \quad v \to v_0 < 1$

Acceleration versus decceleration

Naïve expectation: to achieve large $E_{c.m.}$

we must have large velocities and individual energies.

No! The condition of criticality selects slow particle among all possible ones

$$E - \omega L = \frac{mN}{\sqrt{1-v^2}},$$

"Almost" any particle is rapid (usual one)

Special subset of slow particles is responsible for large energy in CM frame

Strong gravity ensures BSW effect since it almost "halts" this kind of particles.

Super-Penrose process (collisional)

Schittmann 2014

Debris from head-on collision. Significant enhancement

Critical particle moves away from black hole (outgoing)

Usual particle moves towards black hole

Outgoing usual particle
O. Z. (2014) analytically
V. Cardoso et al (2014) numeric findings

Acceleration of particles by nonrotating charged black holes

O. Z. JETP Letters 2010

Role of rotation $L_1 = \frac{E_1}{\omega_H}$ If $\omega_H \to 0$. $L_1 \to \infty$

Angular momentum versus charge

Reissner-Nordstrom Pure radial motion

$\omega_H = 0$ and $L_1 = L_2 = 0$

particles charged, nongeodesic motion

$$ds^2 = -dt^2 f + \frac{dr^2}{f} + r^2 d\omega^2. \qquad f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

$$m^2 \dot{r}^2 = (E - \frac{qQ}{r})^2 - m^2 f. \qquad mu^0 = m\dot{t} = \frac{1}{f}(E - \frac{qQ}{r}),$$

$$X_i = E_i - \frac{q_i Q}{r}, Z_i = \sqrt{X_i^2 - m^2 f}.$$

$$\frac{E_{cm}^2}{2m^2} = 1 + \frac{X_1 X_2 - Z_1 Z_2}{f m^2}$$

Rotating BH	1 critical + 1 usual	Static charged BH
ω	$E^2_{c.m.} \sim f^{-1}$	Q
L		q

Previous kinematic explanation done in ZAMO or static frame

How does the picture look like from the viewpoint of an observer who falls into black hole?

Paradox of two frames

2 observers

1) ZAMO. He sees horizon, there is BSW effect

2) Falling observer. Does not feel horizon. But the horizon Is essential ingredient of BSW effect

But $E_{c.m.}$ and $\gamma = -u_{1\mu}u^{2\mu}$

They are scalars. Impossible to have them unbound in one frame and Bound in another one.

Explanation of the paradox

Frame 1 (static) horizon, individual Killing energies

E_1 E_2 are finite

Frame 2 (attached to falling observer)

sees no horizon, one of energies diverges!

In flat spacetime case 2 is trivial. But now this is underlying reason for BSW effect in frame 1 (nontrivial at all).

Two different kinematic explanations of BSW effect, complimentary to each other

Collisions near ISCO (innermost circular orbit)

Kerr metric (Harada et al), general case)o. Z.)

Natural way how to realize almost critical trajectories

$$mB(r)\left(\frac{dr}{d\tau}\right)^2 = (E-\omega L)^2 - N^2(1+\frac{L^2}{g_\phi}).$$

Near horizon N is small $\qquad E \approx \omega_H L$

$$X = E - \omega L \sim r - r_H \sim \kappa^{2/3} \qquad \kappa \quad \text{surface gravity}$$

Universal dependence

Near-horizon ISCO provided κ is small

Two variants of BSW effect

O-variant

particle orbiting the ISCO collides with some other particle

$$E_{c.m.} \sim \kappa^{-1/3}.$$

H - variant

one of colliding particles plunges towards the horizon from a circular orbit having the same values of energy and angular momentum which it had there

$$E_{c.m.} \sim \kappa^{-1/2}.$$

Role of gravitational radiation

Naively: it bounds the growth of E in CM, restricts BSW effect

More careful inspection: under rather general assumptions (radial acceleration is finite in OZAMO frame, asimuthal force tends to zero not too slowly) the critical trajectories do exist. As a consequence, the BSW effect persists.

Details: I. V. Tanatarov and O. Z., **PRD 2013**

BSW effect survives!

ISCO in magnetic field and ultra-high energy collisions
(Frolov 2012)

$$b = \frac{q}{m} BM$$ G = c = 1

$BM \ll 1$ Metric unperturbed (aslmost Schwarzschild)

However, $\frac{q}{m} \gg 1$ so we may have $b \gg 1$

Magnetic field affects motion of particle, not metric.

If $b \gg 1$ ISCO close to horizon

$$r_0 - r_H \sim b^{-1}$$

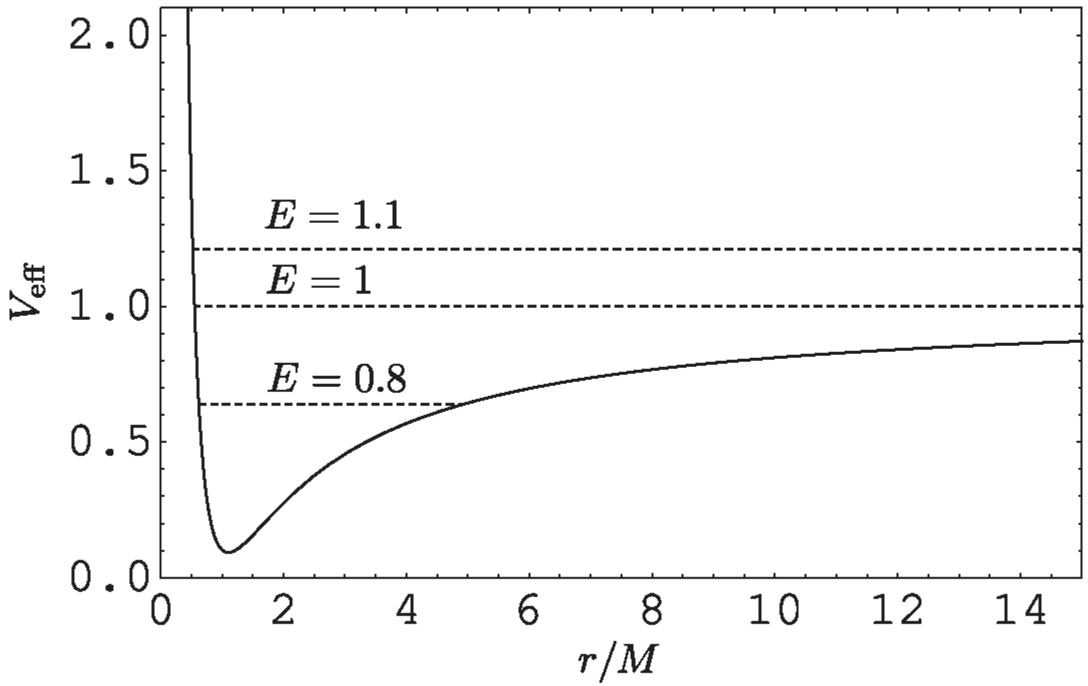
$$E_{c.m.} \sim b^{1/4}$$

Generalization to rotational case (Harada et al)

Alternative mechanisms of getting unbound energies in CM frame

Patil, Joshi, Kimura, Nakao

RN metric, naked singularity



$$E^2_{c.m.} = \frac{4m^2}{1 - \frac{M^2}{Q^2}}.$$

$Q \approx M$

$Q > M$

Black hole

$Q < M$

Naked singularity

Small N

Small f in point of collision

No horizons, no singularities (Patil, Joshi)

Extension to rotating case O. Z.

$$m_1 m_2 \gamma = \frac{X_1 X_2 + \delta Z_1 Z_2}{N^2} - \frac{L_1 L_2}{g_\phi} - g_\theta p_1^\theta p_2^\theta.$$

$\delta = -1$ motion in same direction

$\delta = +1$ motion in opposite directions, head-on

Small N, large energy in CM frame

$$ds^2 = -N^2 dt^2 + g_\phi (d\phi - \omega dt)^2 + \frac{dr^2}{A} + g_\theta d\theta^2,$$

Both particles usual, proper time bounded

Collisions inside ergosphere, not near horizon

Finite Killing energy E, large negative angular momentum L

Grib and Pavlov 2013 (Kerr metric)

O. Z. 2013   (generalization)

Collisions inside ergosphere

$$E_{c.m.}^2 \approx \frac{2|L_2|g_{00}}{N^2 g}[\varepsilon_1\sqrt{(L_1)_+ - L_1} - \varepsilon_2\sqrt{(L_1)_- - L_1}]^2.$$

$g_{00} > 0$

Arbitrarily large

If $\left|L\right|$ so is

Ultra-high rotation

$$m_1 m_2 \gamma = \frac{X_1 X_2 - \varepsilon_1 \varepsilon_2 Z_1 Z_2}{N^2} - \frac{L_1 L_2}{g}.$$

$$X = E - \omega L$$

$$Z = \sqrt{X^2 - N^2 (m^2 + \frac{L^2}{g})}.$$

Small N,
large L,
large ω

N. Tsukamoto and C. Bambi, 2015 for particular wormhole model (Teo wormhole)

General approach, role of rotaiton revealed – O. Z. 2015

Head-on collision, $\sigma_1\sigma_2 = -1$

$$E^2_{c.m.} \approx \frac{4\omega^2 \left|L_1 L_2\right|}{N^2}$$

Motion in the same direction $\sigma_1\sigma_2 = +1$

$$E^2_{c.m.} \approx \frac{2\omega_0\varepsilon|L_2|(X_1 - Z_1)}{N^2}$$

Curvature unbound

BSW effect versus Penrose process

Particles 1 and 2 move towards BH, collide and produce particles 3 and 4

What energy can be observed at infinity?

Large $E_{c.m.}$ but strong redshift In static case $\omega\sqrt{-g_{00}} = \omega_0$

M. Bejger et al, Harada et al (Kerr spacetime), O. Z. (dirty BH)

Rotaing extremal black holes

Conservation laws (energy and radial momentum)

Extraction $$\eta = \frac{E_3}{E_1 + E_2}.$$

Is it possible to achieve this inequality?

In the Kerr case

Scenario IN+

$$\eta_m = \frac{2(2+\sqrt{3})}{q+2} \approx 1.466$$

In two other scenarios no energy extraction

Dirty BHs also restricted

Collisions near inner horizon

Two particles collide inside black hole

$$ds^2 = -dt^2 f + \frac{dr^2}{f} + r^2 d\omega^2.$$

RN $\quad f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = (1 - \frac{r_+}{r})(1 - \frac{r_-}{r})$

$$r_- \le r < r_+ \qquad f = -g \le 0 \qquad r \equiv -T \qquad t \equiv y$$

$$ds^2 = -\frac{dT^2}{g(T)} + g(T) dy^2 + T^2 d\omega^2.$$

Initial moment $r_- < r \le r_1 < r_+$ $r \equiv -T$

Later, r decreases

Collisions near $r = r_-$

Formally, one can achieve

$$\lim E_{c.m.}(r) = \infty$$ when $r \to r_-$

However, by itself this does NOT mean that the effect occurs

There is also kinematic condition that
collision does occur

Collision

$$T_1 = T_2 \qquad\qquad y_1 = y_2$$

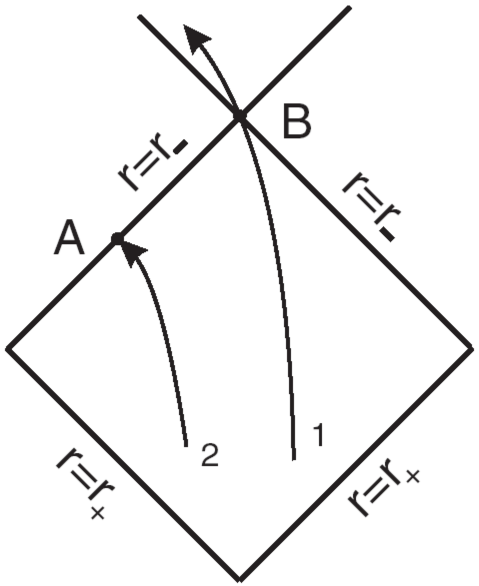
Carter-Penrose diagram, for fixed r different points

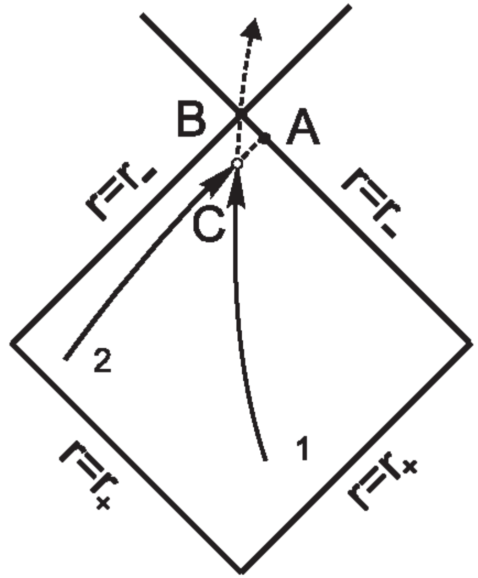
$$(U_1, V_1) \qquad\qquad (U_2, V_2)$$

Kruskal-like coordinates, analytic extension

Collisions near inner horizon

Again, one of two particle should be critical. Then, the following cases are possible.

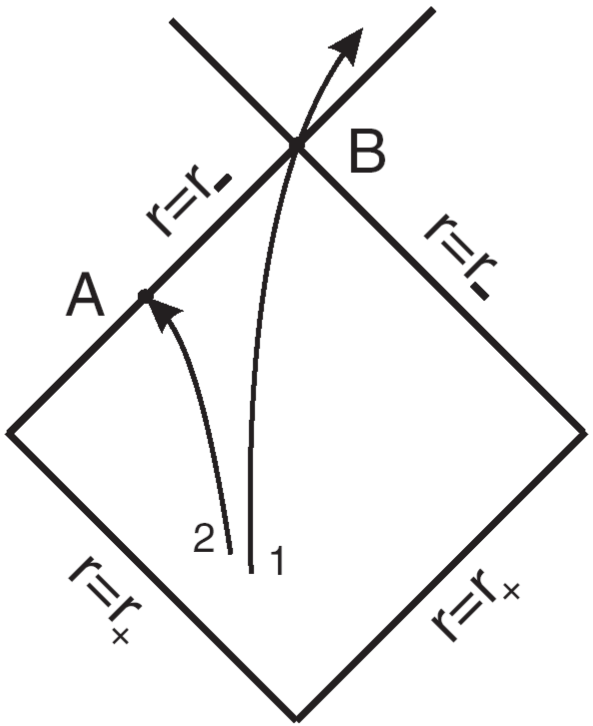




Kinematic censorship preserved

Fig. 1. Impossibility of strong version of BSW effect. Critical particle 1 passes through bifurcation point whereas usual one 2 hits left horizon

Fig. 2. The weak version of BSW effect. Near-horizon collision between Critical particle 1 and usual one 2.



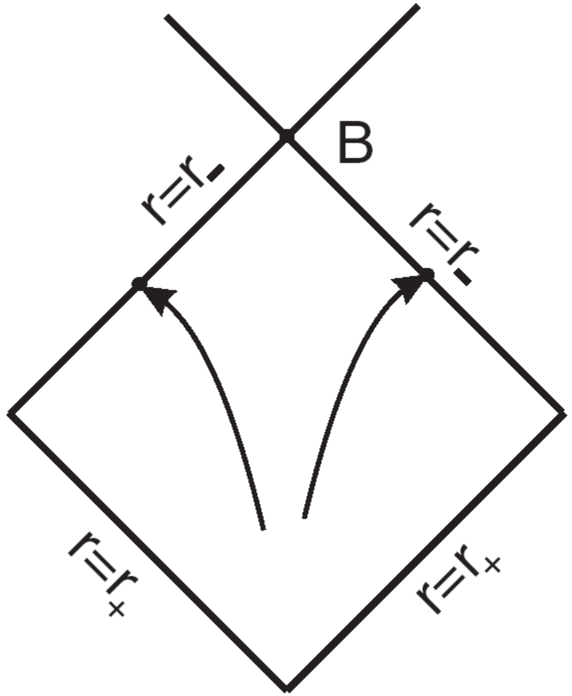


Fig. 4. Impossibility of strong version of PS effect. Two usual particles hit different branches of horizon.

Fig. 3. Impossibility of strong version. Critical particle 1 passes through bifurcation point, whereas a usual one 2 hits left horizon.

Kinematic censorship

CONCLUSION

High energy collisions due to horizon

Outside black hole

Inside black hole

Role of critical trajectories

ISCO

Force does NOT spoil BSW effect, critical trajectories survive

RN metric: example of significant effect at infinity

Relevant physical factors: BH rotation, electric charge, magnetic field

Universality typical of BH physics

Alternative mechanisms

No horizon but system in some sense “close” to its appearance

Ergoregion

A need for further studies of Penrose process in combination with BSW effect

Thank you!