FCT
Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR

The Initial State of a Primordial Anisotropic Stage of Inflation

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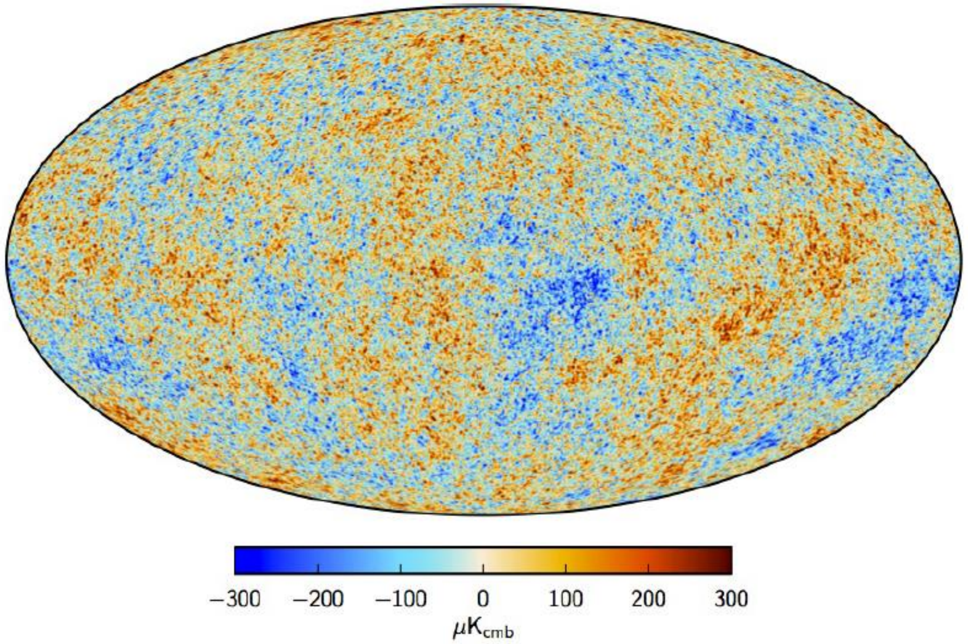
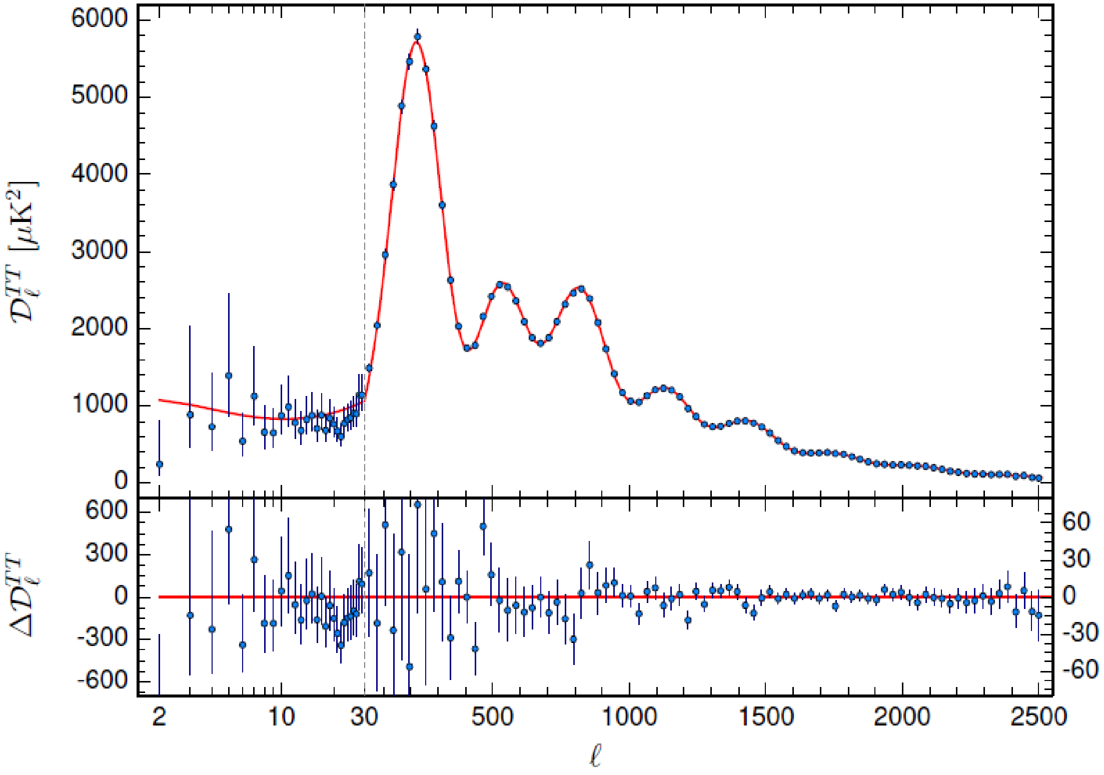
JCAP 1506 (2015) 024 [arXiv: 1501.07427]

Inflation and CMB Anisotropy

- Inflation can nicely explain the observed flatness, homogeneity and isotropy of the present day’s Universe.
- Nearly scale-invariant, Gaussian and isotropic quantum fluctuations can be natural sources of CMB anisotropies and large-scale structures.



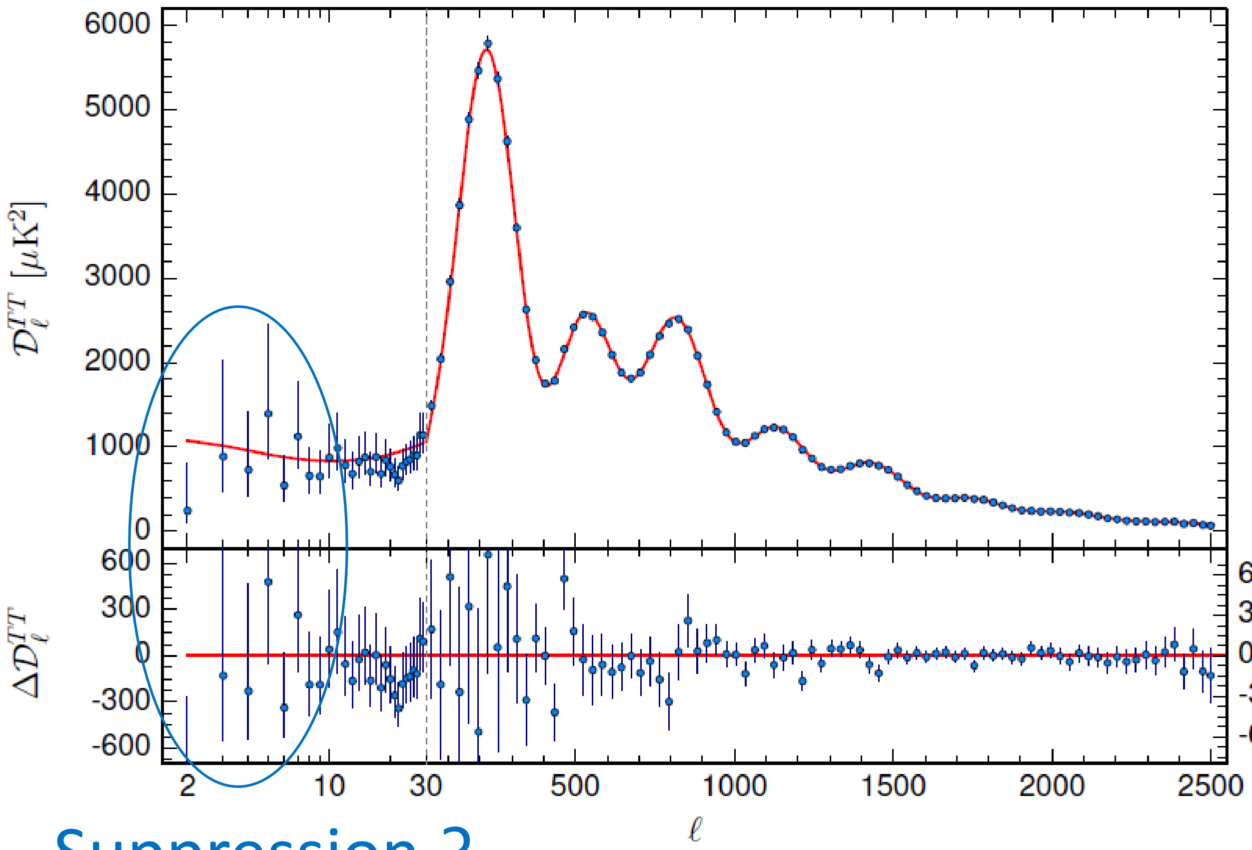
Planck Collaboration, 1502.01582



Large-Scale Anomalies of CMB

Broken scale invariance ?

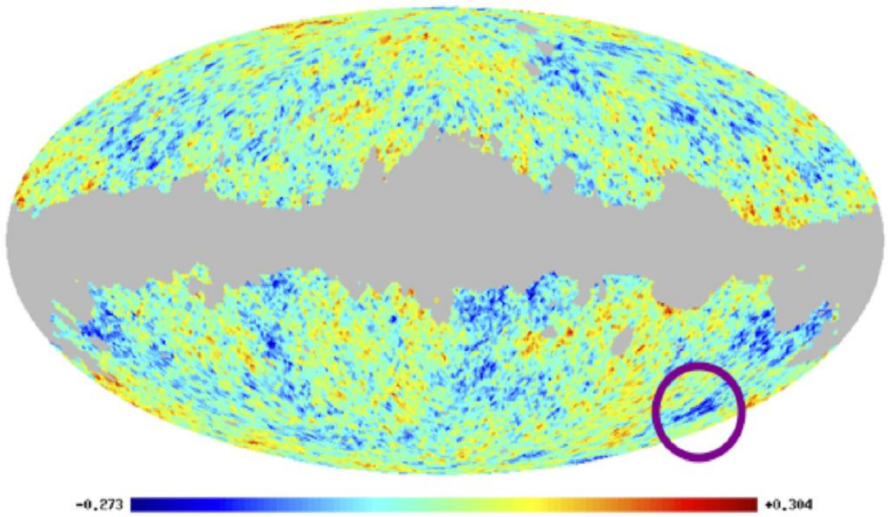
- Suppression of angular power



Suppression ?

non-Gaussianity ?

- Cold spot Vielva, 1008.3051

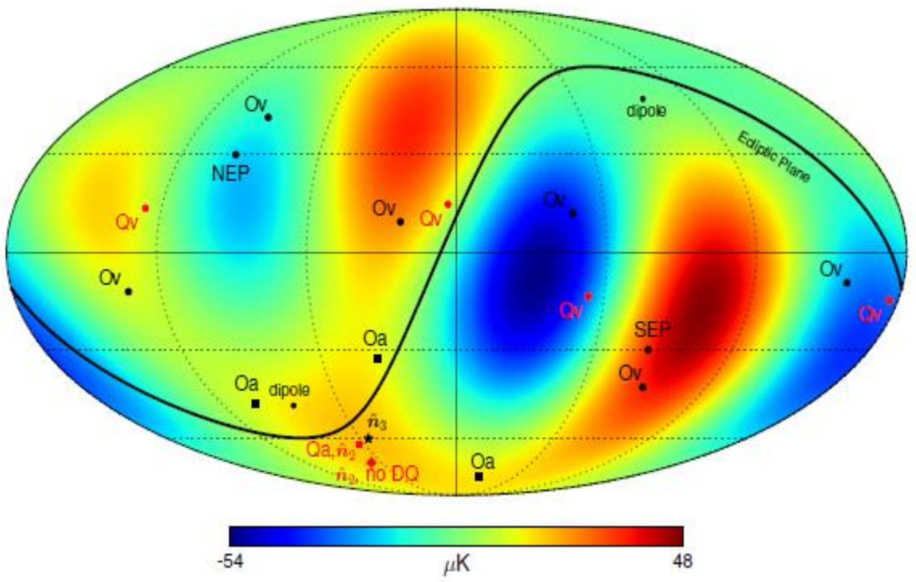


$$(l, b) \sim (209°, -57°)$$

- Non-Gaussian over 10 degrees
- Large voids or textures ?

Statistical Anisotropies ?

- Alignment of lower multipoles

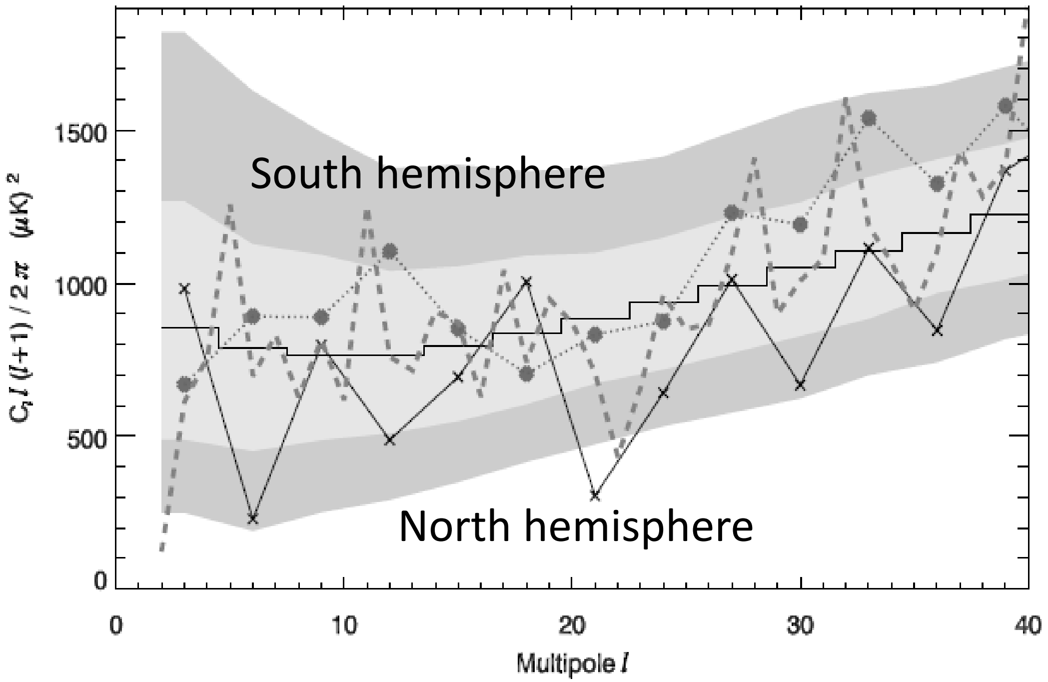


$$(\ell = 2) + (\ell = 3)$$

Copi, Huterer, Schwarz and Starkman (15)

- Octopole is unusally planar and aligned with quadrupole.
- Alignment is perpendicular to the Ecliptic plane.

- Hemispheric power asymmetry



Eriksen, et al, astro-ph/0307507

- Dipolar modulation ?

$$T(\hat{\boldsymbol{e}}) = T_0(\hat{\boldsymbol{e}})[1 + A\,\hat{\boldsymbol{e}} \cdot \hat{\boldsymbol{d}}]$$

$$(l, b) = (230°, -16°) \pm 24°$$

$$A = (0.066 \pm 0.021)$$ *Planck* 2015

- *Anomalies* are more or less statistically independent.
- None reaches 5σ detection individually, but it is hard to realize *all* anomalies within the ΛCDM model by chance.
- Instrumental effects would be unlikely, as they were detected by both WMAP and *Planck* consistently and independently.
- While the foreground effects depend on frequencies, the observed anomalies are almost independent of them.
- A variety of cosmological mechanisms
  - Local voids
  - Vector fields
  - Pre-inflationary physics
  - Non-trivial topology

None have been demonstrated to be detectable at the statistically significant level.

Schwarz, Copi, Huterer and Starkman, 1510.07929.

Pre-inflationary Anisotropy

- An initially anisotropic universe rapidly approaches an inflationary universe in the presence of the positive potential energy.

Wald (84)

- Our Universe could naturally begin with a highly anisotropic state.

Gumrukcuoglu, Contaldi and Peloso (07) Pitrou, Pereira and Uzan (08)

- CMB anomalies could be identified as the consequences of the pre-inflationary anisotropy.

- requests the minimal duration of inflation, namely 50-60 e-folds, in order for the corresponding modes to be observed at the largest possible scales of the CMB today.

- simple enough in the sense that it does not invoke any additional anisotropic energy source.

- The Kasner-de Sitter solution provides a very good approximation as the geometry of the primordial universe, interpolating the initial anisotropic ($t \approx 0$) and the late-time de Sitter ($t \to \infty$) stages.

Ellis and MaCallum (69) , Gumrukcuoglu, Contaldi and Peloso (07), Pitrou, Pereira and Uzan (08)

$$ds^2 = -dt^2 + \sum_{i=1}^{3} \sinh^{\frac{2}{3}}(3Ht) \underbrace{\left\{ \tanh\left(\frac{3Ht}{2}\right) \right\}^{2(p_i - \frac{1}{3})}}_{\text{Anisotropy}} dx_i^2 ,$$

$$\sum_i p_i = \sum_i p_i^2 = 1.$$

- Kasner universe as $t \approx 0$ Kasner (21)

$$ds^2 = -dt^2 + \sum_{i=1}^{3} t^{2p_i} dx_i^2$$

- de Sitter universe as $t \to \infty$

$$ds^2 \approx -dt^2 + e^{2Ht} \sum_{i=1}^{3} dx_i^2$$

- Regularity at $t = 0$ is requested to find a well-defined adiabatic vacuum and selects the branch of $p_1 = 1, \quad p_2 = p_3 = 0$.

$$ds^2 = -dt^2 + \left(\frac{2}{3}H^{-1}\sinh\frac{3Ht}{2}\left(\cosh\frac{3Ht}{2}\right)^{-\frac{1}{3}}\right)^2 dr^2 + \left(\cosh\frac{3Ht}{2}\right)^{\frac{4}{3}} dx_\perp^2$$

R^2

- highly anisotropic initial geometry as $Milne_2 \times R^2$

$$ds^2 \approx -dt^2 + t^2 dr^2 + dx_\perp^2 .$$

$t = 0$ surface represents the *past horizon* of 2d Milne universe.

- in the "conformal" time; $\sinh(-3H\eta) = 1/\sinh(3Ht)$

$$ds^2 = -\frac{d\eta^2}{\sinh^2(-H_{2d}\eta)} + \alpha^4 \frac{e^{4H_{2d}\eta/3}}{\sinh^{2/3}(-H_{2d}\eta)} dr^2 + \alpha^{-2}\frac{e^{-2H_{2d}\eta/3}}{\sinh^{2/3}(-H_{2d}\eta)} dx_\perp^2$$

$$\alpha = 2^{1/3} \qquad H_{2d} = 3H$$

Kasner $\eta \to -\infty (t = 0) \Rightarrow$ de Sitter $\eta \to 0 (t = +\infty)$

Power Spectrum from the KdS Universe

- Quantization massless scalar field

$$\phi(\eta, r, x_\perp) = \int dk \sum_{k_\perp} \left[\frac{1}{(2\pi)^{3/2}} \tilde{a}_{k_\perp,k,i} \; f_{k_\perp,k}(\eta) e^{ik_\perp x_\perp} e^{-ikr} + \mathrm{h.c} \right] .$$

$$\left[\frac{d^2}{d\eta^2} + \Omega^2(k_\perp, k, \eta) \right] f(\eta_R) = 0$$

$$\Omega^2(k_\perp, k, \eta) = \alpha^{-4} \sinh^{-4/3}(-H\eta) \;\; e^{2H\eta/3} \left(\alpha^6 k_\perp^2 + e^{-2H\eta} k^2 \right)$$

- In the initial adiabatic vacuum of the KdS universe

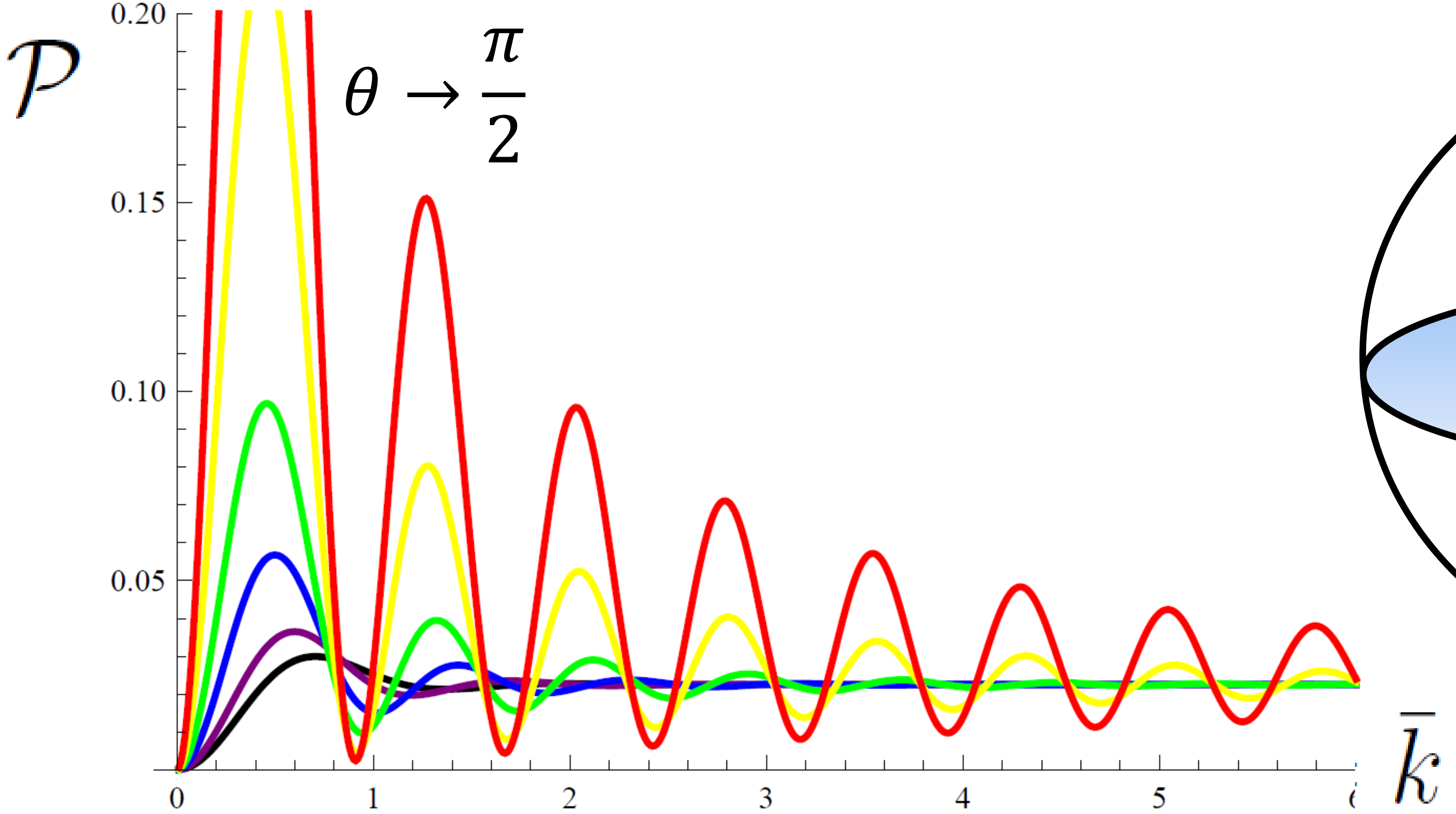
$$f^{(c)}_{k_\perp,k}(\eta) = \sqrt{\frac{\pi}{2H_{2d} \sinh(\pi \tilde{k})}} J_{-i\tilde{k}} \left(2\tilde{k}_\perp e^{H_{2d}\eta} \right)$$

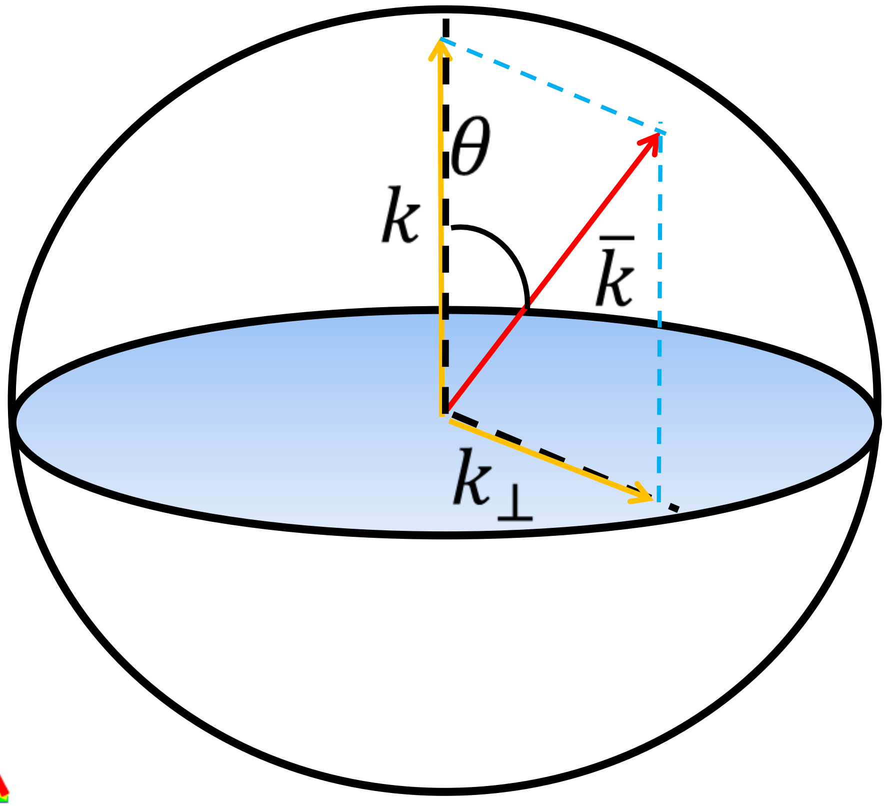
Kim and Minamitsuji (10)

$$\tilde{k} = \frac{k}{H} \quad \tilde{k}_\perp = \frac{k_\perp}{H} \qquad H_{2d} = 3H$$

- Scalar power spectrum suffers *divergence* on the plane, and the significant backreaction make perturbative approach invalid.

$$P = \frac{1}{2\pi^2}(\alpha^{-4}k^2 + \alpha^2 k_\perp^2)^{3/2} \times \left| f_{k_\perp,k}^{(C)}(\eta \to 0) \right|^2$$





Quantum State in the Vicinity of Singularity

- The KdS universe = The *planar* Schwarzchild-de Sitter spacetime

$$ds^2 = -dt^2 + (HL)^2 \left(\sinh \frac{3Ht}{2} \left(\cosh \frac{3Ht}{2} \right)^{-\frac{1}{3}} \right)^2 d\tilde{r}^2 + L^2 \left(\cosh \frac{3Ht}{2} \right)^{\frac{4}{3}} d\tilde{x}_\perp^2 .$$

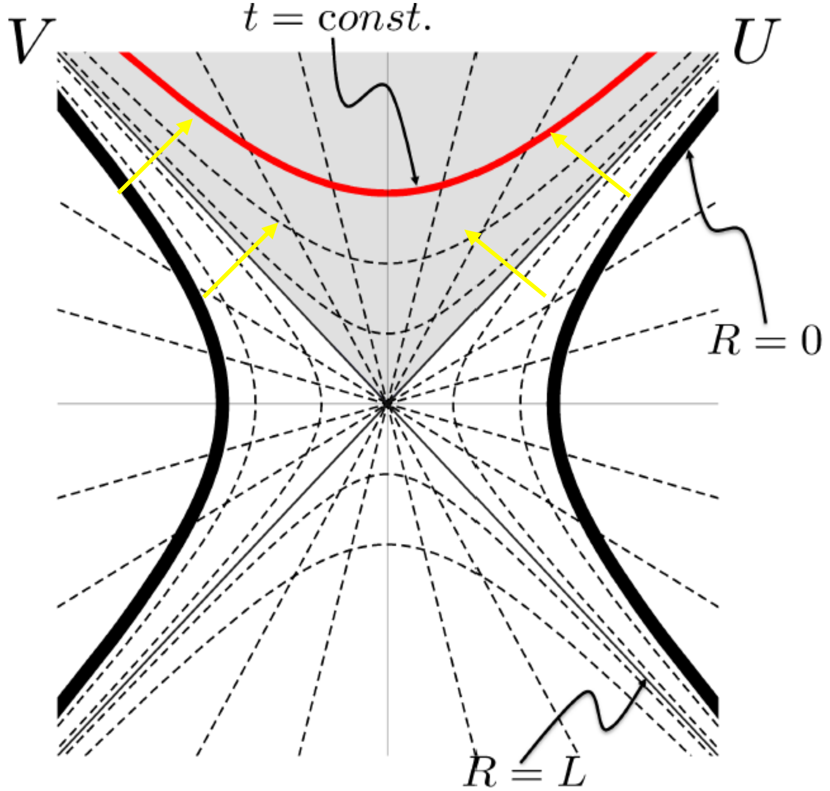
$$\Updownarrow \qquad t = \frac{2}{3} H^{-1} \log \left[\left(\frac{T}{L} \right)^{3/2} + \sqrt{\left(\frac{T}{L} \right)^3 - 1} \right] \qquad L < T < \infty$$

$$ds^2 = -\frac{dT^2}{f(T)} + f(T) d\tilde{r}^2 + T^2 d\tilde{x}_\perp^2$$

$$f(T) = H^2 \left(T^2 - \frac{L^3}{T} \right)$$

The initial surface $t = 0 \Leftrightarrow$ The cosmological horizon $T = L$

- Maximally-extended Kasner-de Sitter spacetime



- KdS universe (the shaded region) is influenced by the timelike singularity.

- The existence of singularity makes the quantum state unpredictable.

Horowitz and Marolf (95)

$\Rightarrow$ We adopt the new interpretation of KdS universe as the consequence of the quantum tunneling from a *regular* lower-dimensional vacuum.

Anisotropic Inflation after Quantum Tunneling

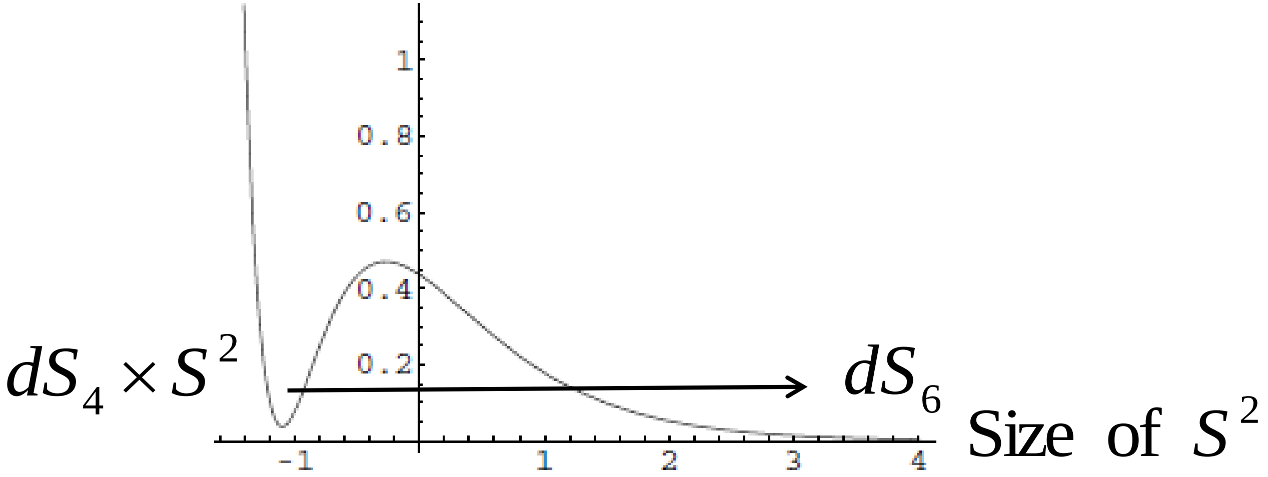
- Initially , the $x_\perp$ directions are static and smoothly matched to a lower-dimensional parent vacuum compactified on T^2

$$ds^2 \approx -dt^2 + t^2 dr^2 + \underbrace{dx_\perp^2}_{T^2}.$$

- A transdimensional transition was argued in the context of the landscape of multiple vacua in a higher-dimensional theory.

Blanco-Pillado, Schwartz-Perlov and Vilenkin (09), Carroll, Johnson and Randall (09)

In 6d Einstein-Maxwell theory



- Decompactification *within* our 4D universe.

$$dS_3 \times S^1 \to dS_4 \qquad dS_2 \times S^2 \to dS_4$$

Blanco-Pillado and Salem (10) Adamek, Campo, and Niemeyer (10)

- Anisotropic spatial curvature leads to late-time anisotropic expansion, which dominates over the primordial effects.

Demianski and Doroshkevich (07)
Graham, Harnik and Rajendran (10)

$$\Rightarrow \Omega_K < 10^{-4}$$

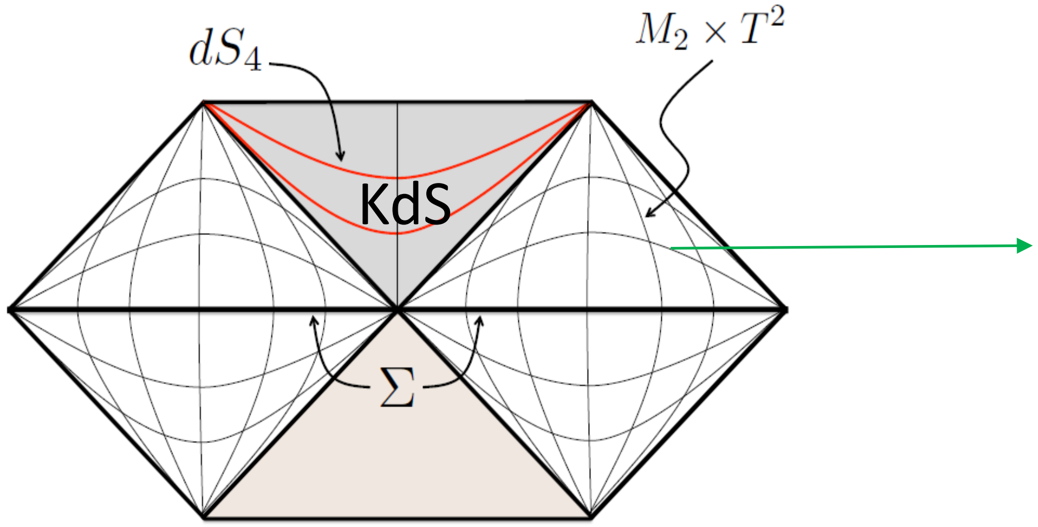
- Our T^2 compactification model is free from the late-time anisotropy.

- Instanton transitions between 4D vacuum and 2D vacuum on T^2 are mediated by Casimir interactions.

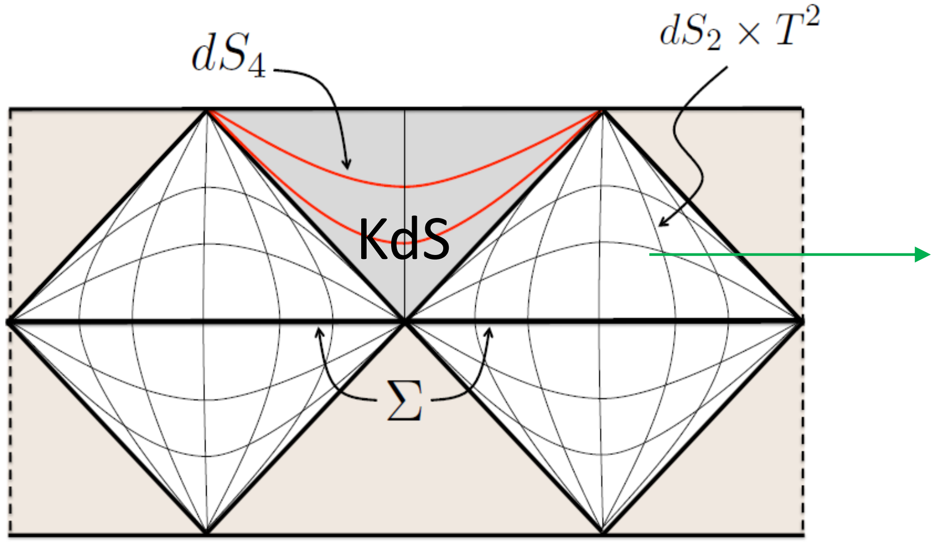
Arnold, Fornal & Ishiwata (11)

- Observational signatures from toy models describing the decompactification from a 2D vacuum on T^2 to 4D will be argued.

Models of Kasner-de Sitter Bubble Nucleation

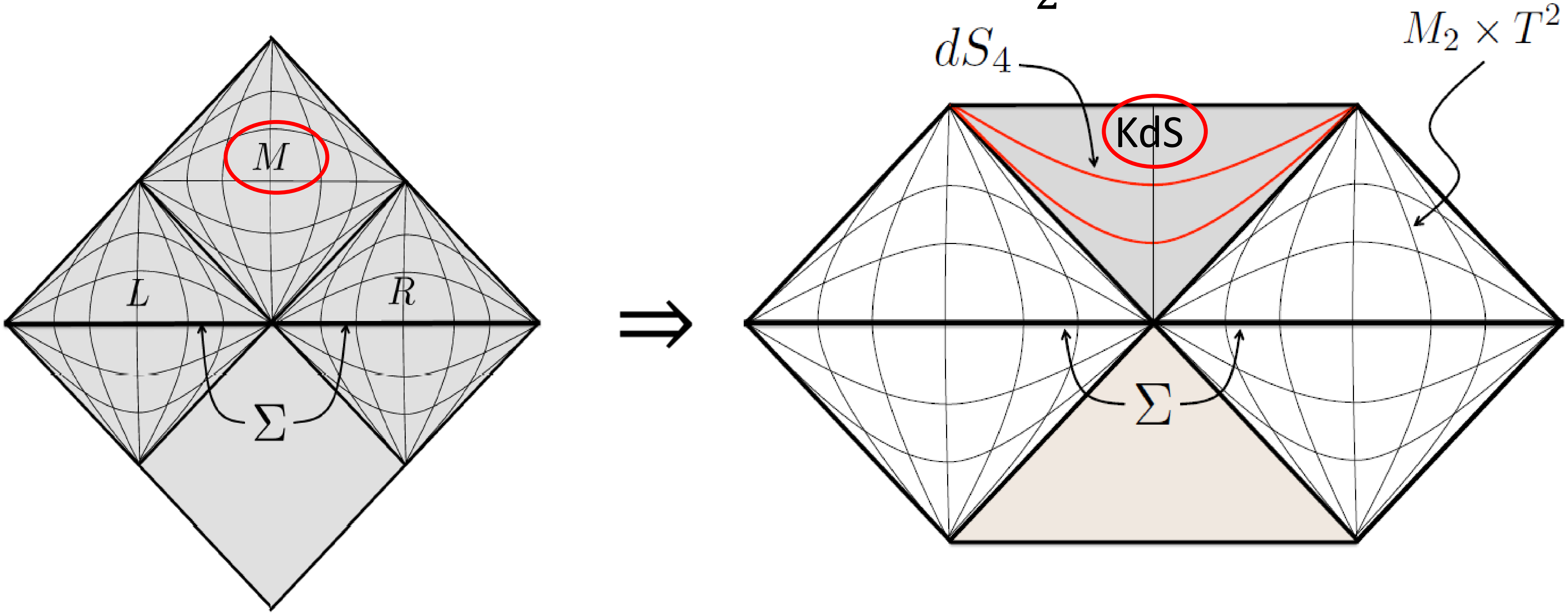


$$ds^2 = -r^2 d\tau^2 + dr^2 + dx_\perp^2$$



$$ds^2 = \left[-\frac{1}{H_{2d}^2}\sin(H_{2d}\, r)^2 d\tau^2 + dr^2\right] + dx_\perp^2$$

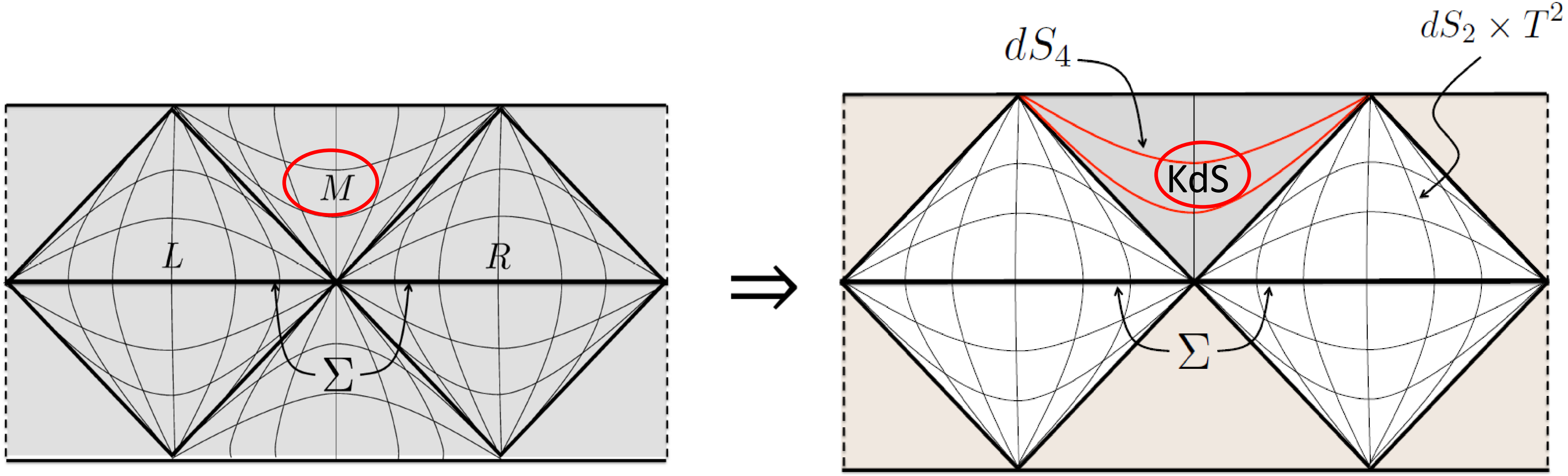
- KdS bubble nucleation in the vacuum of $M_2 \times T^2$



- As the lightcone limit of KdS universe is identical to 2d Milne, the Milne wedge (M) can be *smoothly* replaced by the KdS universe.

-The modes of a massless scalar field are quantized on the global Cauchy surface Σ stretched over the parent vacuum of $M_2 \times T^2$.

- KdS bubble nucleation in the vacuum of $dS_2 \times T^2$



- Both the lightcone limits of the M region of dS_2 and KdS universe are identical to 2d Milne, (M) can be *smoothly* replaced by the KdS universe.

-The modes of a massless scalar field are quantized on the global Cauchy surface Σ stretched over the parent vacuum of $dS_2 \times T^2$.

Power Spectrum of a Massless Scalar Field

- The initial state from the parent vacuum of $M_2 \times T^2$

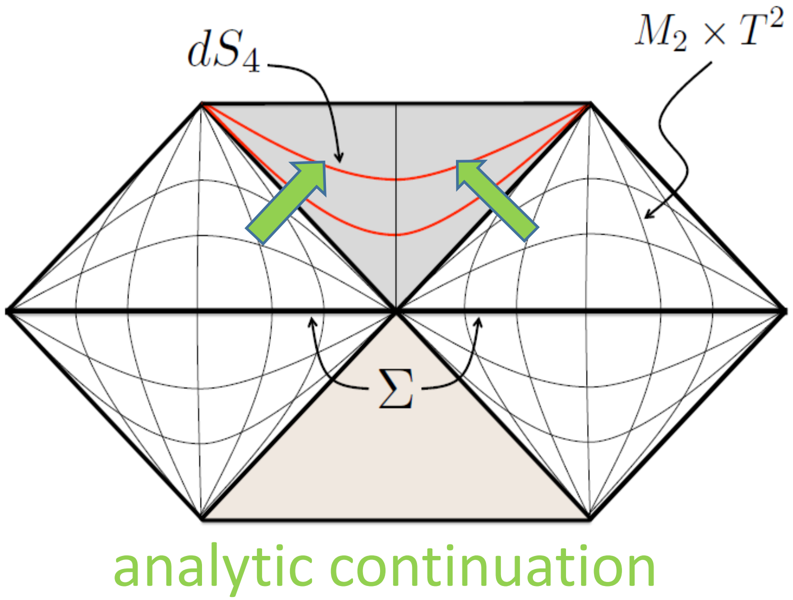
$$f^{(M)}_{k_\perp,k}(\eta) = \frac{1}{2}\sqrt{\frac{\pi}{H_{2d}}} e^{\pi\tilde{k}/2}\, H^{(2)}_{i\tilde{k}}\left(2\tilde{k}_\perp e^{H_{2d}\eta}\right)$$

- An excited state with respect to the vacuum inside the anisotropic bubble

$$f^{(M)}_{k_\perp,k} = \alpha_k f^{(c)}_{k_\perp,k} + \beta_k \left(f^{(c)}_{k_\perp,k}\right)^*$$

$$\alpha_{\tilde{k}} = \frac{e^{\pi\tilde{k}/2}}{\sqrt{e^{\pi\tilde{k}} - e^{-\pi\tilde{k}}}} \qquad \beta_{\tilde{k}} = -\frac{e^{-\pi\tilde{k}/2}}{\sqrt{e^{\pi\tilde{k}} - e^{-\pi\tilde{k}}}}$$

$$f^{(c)}_{k_\perp,k}(\eta) = \sqrt{\frac{\pi}{2H_{2d}\sinh(\pi\tilde{k})}} J_{-i\tilde{k}}\left(2\tilde{k}_\perp e^{H_{2d}\eta}\right)$$



- Regular in any direction

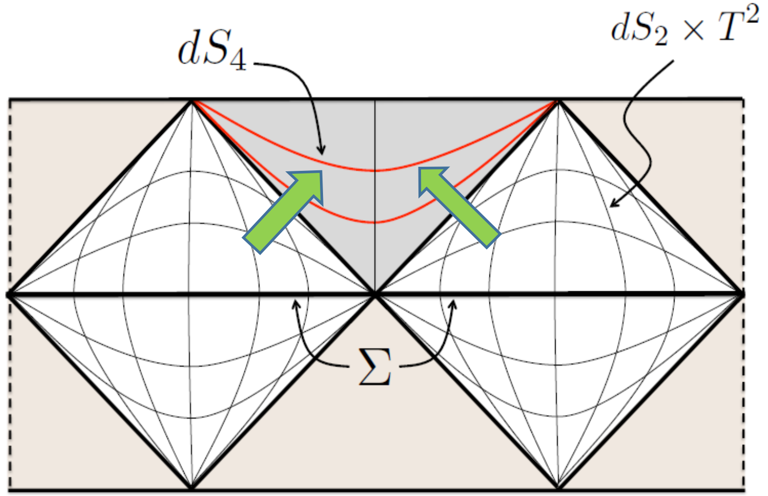
- The initial state from the parent vacuum of $dS_2 \times T^2$

$$f^{(1)}_{k_\perp,k}(\eta) = \frac{1}{\sqrt{2k}} \frac{e^{\pi k/2H_{2d}}}{\sqrt{2\sinh(\pi k/H_{2d})}} N(k,k_\perp)\ \tilde{f}^{(1)}_{k_\perp,k}(\eta)$$

$$f^{(2)}_{k_\perp,k}(\eta) = \frac{1}{\sqrt{2k}} \frac{e^{\pi k/2H_{2d}}}{\sqrt{2\sinh(\pi k/H_{2d})}} \left(L(k,k_\perp)\ \tilde{f}^{(1)}_{k_\perp,k}(\eta) + e^{-\pi k/H_{2d}} \tilde{f}^{(2)}_{k_\perp,k}(\eta)\right)$$

$$\tilde{f}^{(1)}_{k_\perp,k}(\eta) = e^{-ik\eta} F\left[-\nu, \nu+1, 1-\mu, \frac{1+\xi_i}{2}\right]$$

$$\tilde{f}^{(2)}_{k_\perp,k}(\eta) = e^{ik\eta} F\left[-\nu, \nu+1, 1+\mu, \frac{1+\xi_i}{2}\right]$$



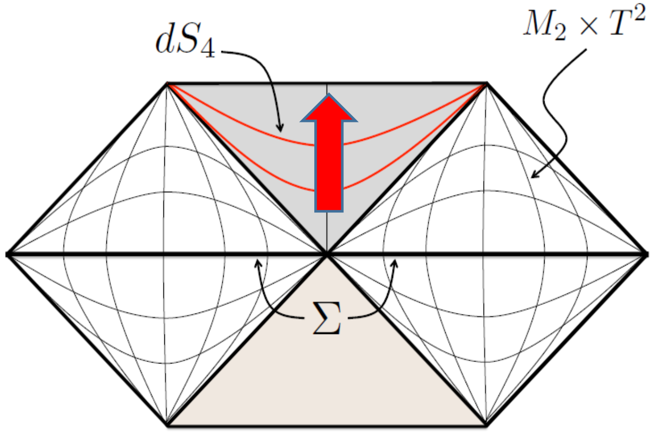
$$N(k,k_\perp) = \frac{\Gamma(1+\nu-\mu)\Gamma(-\mu-\nu)}{\Gamma(1-\mu)\Gamma(-\mu)} \qquad L(k,k_\perp) = -\frac{\Gamma(1+\mu)\Gamma(1+\nu-\mu)\Gamma(-\mu-\nu)}{\Gamma(1-\mu)\Gamma(-\nu)\Gamma(1+\nu)}.$$

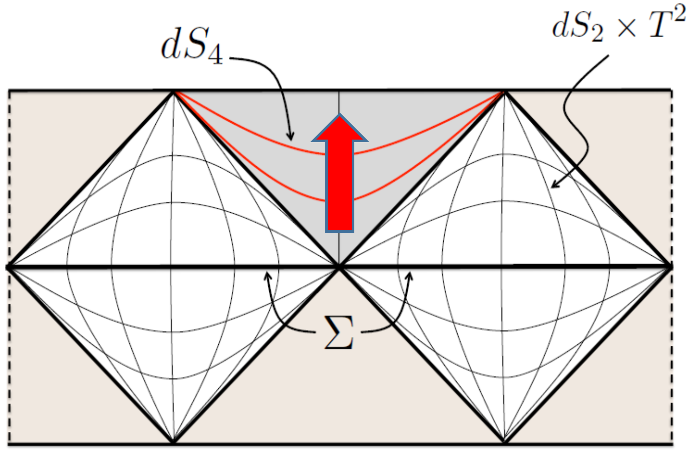
- Also regular in any direction.

- Time evolution inside the KdS bubble

$$\left[\frac{d^2}{d\eta^2} + \Omega^2(k_\perp, k, \eta)\right] f_{k_\perp,k}(\eta) = 0$$

$$\Omega^2(k_\perp, k, \eta) = \alpha^{-4}\sinh^{-4/3}(-H_{2d}\eta)\ \ e^{2H_{2d}\eta/3}\left(\alpha^6 k_\perp^2 + e^{-2H_{2d}\eta}k^2\right)$$

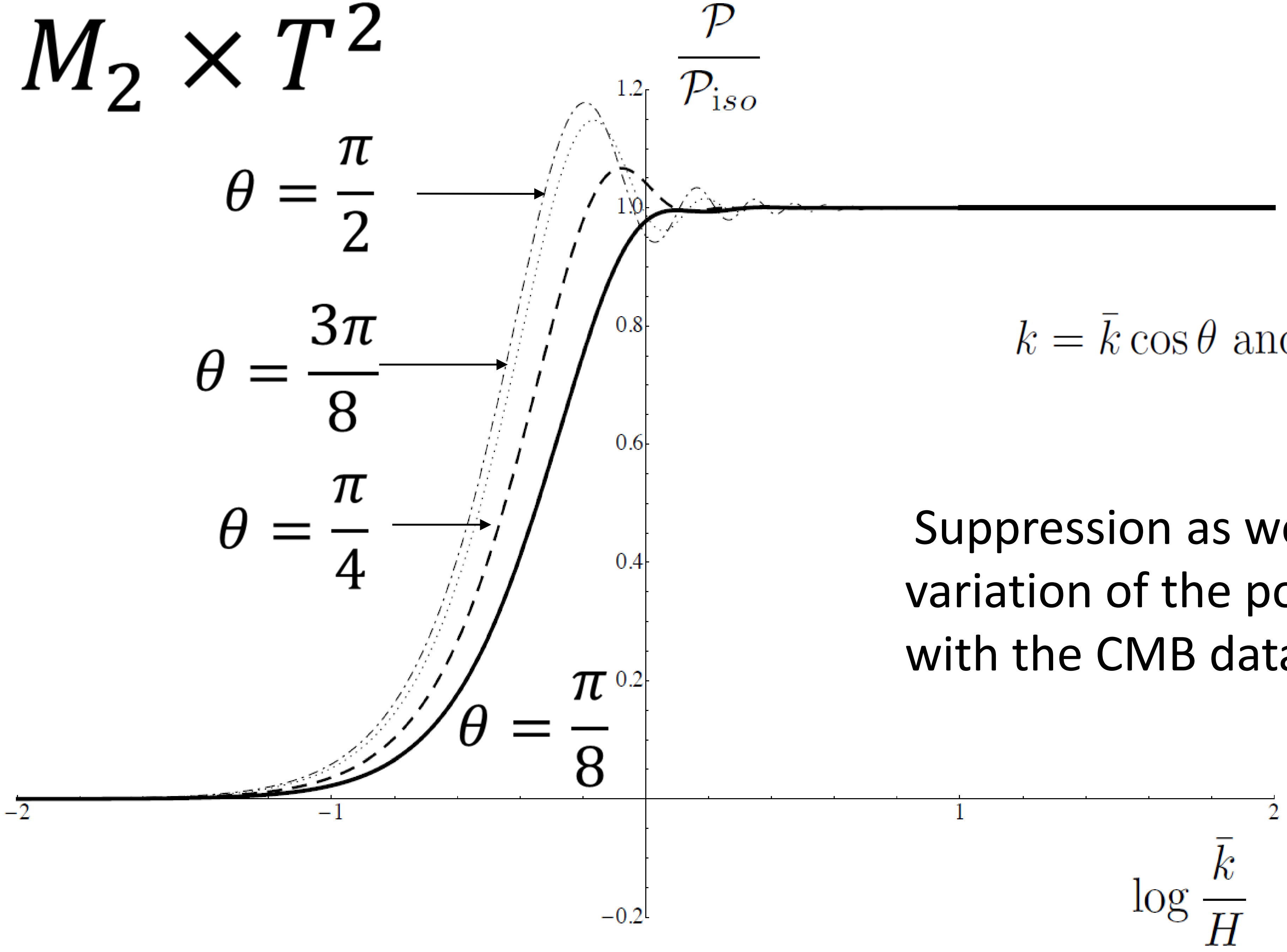




- Power spectrum

$$\mathcal{P} = \frac{1}{2\pi^2}\left(\alpha^{-4}k^2 + \alpha^2 k_\perp^2\right)^{\frac{3}{2}} \times \begin{cases} \left|f^{(M)}_{k_\perp,k}(\eta\to 0)\right|^2 & (M_2\times T_2) \\ \sum_{i=1}^2 \left|f^{(i)}_{k_\perp,k}(\eta\to 0)\right|^2 & (dS_2\times T_2) \end{cases}$$

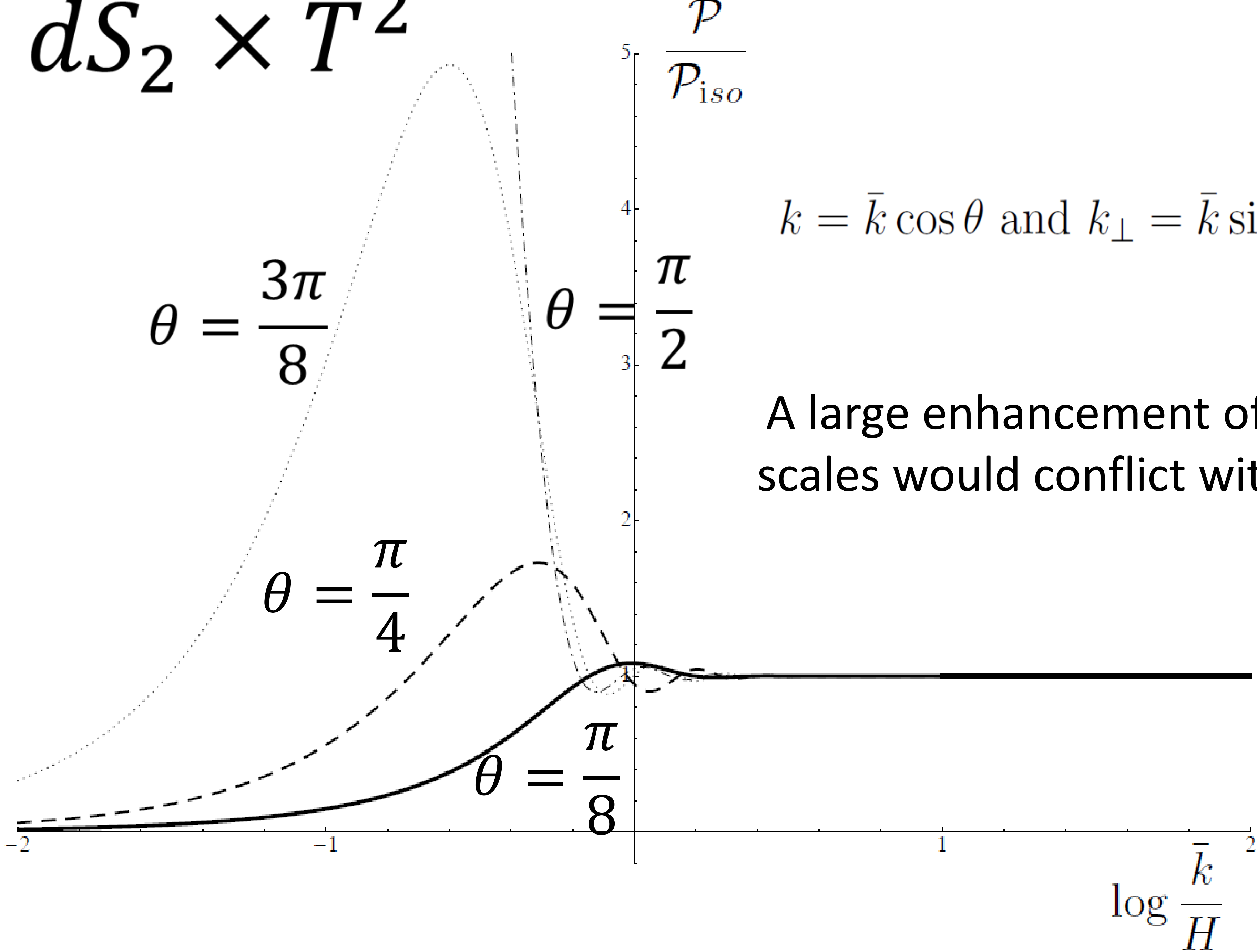
$M_2 \times T^2$



$k = \bar{k}\cos\theta$ and $k_\perp = \bar{k}\sin\theta$

Suppression as well as milder angular variation of the power are compatible with the CMB data.

$dS_2 \times T^2$



$k = \bar{k} \cos\theta$ and $k_\perp = \bar{k} \sin\theta$

A large enhancement of the power on large scales would conflict with the CMB data.

Summary

- Large-scale anomalies in the CMB may be caused by the nontrivial modifications in the initial quantum states *before* onset of inflation.

- A naïve quantization in the vacuum of 2D Milne universe leads to several divergences, making the choice of the initial state highly questionable.

$$\Downarrow$$

- We took the approach that the KdS universe is an outcome of quantum tunneling from the regular universe with stabilized dimensions

  - The vacuum of $M_2 \times T^2$ leads to favorable features to explain to the observed CMB anomalies.

  - The vacuum of $dS_2 \times T^2$ leads to a large enhancement of the power, which easily conflicts with the current CMB data.

Thank you.