The Initial State of a Primordial Anisotropic Stage of Inflation

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Inflation and CMB Anisotropy

- Inflation can nicely explain the observed flatness, homogeneity and isotropy of the present day’s Universe.
- Nearly scale-invariant, Gaussian and isotropic quantum fluctuations can be natural sources of CMB anisotropies and large-scale structures.
Large-Scale Anomalies of CMB

Broken scale invariance?

- Suppression of angular power

non-Gaussianity?

- Cold spot

Vielva, 1008.3051

- Non-Gaussian over 10 degrees
- Large voids or textures?
Statistical Anisotropies?

- Alignment of lower multipoles

\[ (\ell = 2) + (\ell = 3) \]

Copi, Huterer, Schwarz and Starkman (15)

- Octopole is unusually planar and aligned with quadrupole.
- Alignment is perpendicular to the Ecliptic plane.

- Hemispheric power asymmetry

\[ T(\hat{e}) = T_0(\hat{e})[1 + A \hat{e} \cdot \hat{d}] \]

\[ (l, b) = (230^\circ, -16^\circ) \pm 24^\circ \]

\[ A = (0.066 \pm 0.021) \]

Planck 2015
• **Anomalies** are more or less statistically independent.

• None reaches $5\sigma$ detection individually, but it is hard to realize *all* anomalies within the $\Lambda$CDM model by chance.

• **Instrumental** effects would be unlikely, as they were detected by both WMAP and *Planck* consistently and independently.

• While the **foreground** effects depend on frequencies, the observed anomalies are almost independent of them.

• A variety of **cosmological** mechanisms
  - Local voids
  - Pre-inflationary physics
  - Vector fields
  - Non-trivial topology

  None have been demonstrated to be detectable at the statistically significant level.

Schwarz, Copi, Huterer and Starkman, 1510.07929.
Pre-inflationary Anisotropy

- An initially anisotropic universe rapidly approaches an inflationary universe in the presence of the positive potential energy.  
  Wald (84)

- Our Universe could naturally begin with a highly anisotropic state.  
  Gumrukcuoglu, Contaldi and Peloso (07)  Pitrou, Pereira and Uzan (08)

  - CMB anomalies could be identified as the consequences of the pre-inflationary anisotropy.

  - requests the minimal duration of inflation, namely 50-60 e-folds, in order for the corresponding modes to be observed at the largest possible scales of the CMB today.

  - simple enough in the sense that it does not invoke any additional anisotropic energy source.
The Kasner-de Sitter solution provides a very good approximation as the geometry of the primordial universe, interpolating the initial anisotropic \((t \approx 0)\) and the late-time de Sitter \((t \to \infty)\) stages.

\[ ds^2 = -dt^2 + \sum_{i=1}^{3} \sinh^2 (3Ht) \left\{ \tanh \left( \frac{3Ht}{2} \right) \right\}^{2(p_i - \frac{1}{3})} dx_i^2 , \]

\[ \sum_i p_i = \sum_i p_i^2 = 1. \]

- Kasner universe as \(t \approx 0\) \(\text{Kasner (21)}\)

\[ ds^2 = -dt^2 + \sum_{i=1}^{3} t^{2p_i} dx_i^2 \]

- de Sitter universe as \(t \to \infty\)

\[ ds^2 \approx -dt^2 + e^{2Ht} \sum_{i=1}^{3} dx_i^2 \]
• Regularity at $t = 0$ is requested to find a well-defined adiabatic vacuum and selects the branch of $p_1 = 1$, $p_2 = p_3 = 0$.

$$ds^2 = -dt^2 + \left( \frac{2}{3} H^{-1} \sinh \frac{3Ht}{2} \left( \cosh \frac{3Ht}{2} \right)^{-\frac{1}{3}} \right)^2 dr^2 + \left( \cosh \frac{3Ht}{2} \right)^\frac{4}{3} \frac{dx^2}{R^2}$$

- highly anisotropic initial geometry as $\text{Milne}_2 \times R^2$

$$ds^2 \approx -dt^2 + t^2 dr^2 + dx^2_\perp.$$ 

t = 0$ surface represents the past horizon of 2d Milne universe.

- in the “conformal” time; $\sinh(-3H\eta) = 1/\sinh(3Ht)$

$$ds^2 = -\frac{d\eta^2}{\sinh^2(-H_{2d}\eta)} + \alpha^4 \frac{e^{4H_{2d}\eta/3}}{\sinh^{2/3}(-H_{2d}\eta)} dr^2 + \alpha^{-2} \frac{e^{-2H_{2d}\eta/3}}{\sinh^{2/3}(-H_{2d}\eta)} dx^2_\perp$$

$$\alpha = 2^{1/3} \quad H_{2d} = 3H$$

Kasner $\eta \to -\infty (t = 0) \Rightarrow$ de Sitter $\eta \to 0 (t = +\infty)$
Power Spectrum from the KdS Universe

- Quantization massless scalar field

\[
\phi(\eta, r, x_{\perp}) = \int dk \sum_{k_{\perp}} \left[ \frac{1}{(2\pi)^{3/2}} \tilde{a}_{k_{\perp}, k, i} f_{k_{\perp}, k}(\eta) e^{ik_{\perp}x_{\perp}} e^{-ikr} + \text{h.c} \right].
\]

\[
\left[ \frac{d^2}{d\eta^2} + \Omega^2(k_{\perp}, k, \eta) \right] f(\eta_R) = 0
\]

\[
\Omega^2(k_{\perp}, k, \eta) = \alpha^{-4} \sinh^{-4/3}(-H\eta) e^{2H\eta/3} \left( \alpha^6 k_{\perp}^2 + e^{-2H\eta} k^2 \right)
\]

- In the initial adiabatic vacuum of the KdS universe

\[
f^{(c)}_{k_{\perp}, k}(\eta) = \sqrt{\frac{\pi}{2H_{2d} \sinh(\pi \tilde{k})}} J_{-i\tilde{k}} \left( 2\tilde{k}_{\perp} e^{H_{2d}\eta} \right)
\]

Kim and Minamitsuji (10)

\[
\tilde{k} = \frac{k}{H} \quad \tilde{k}_{\perp} = \frac{k_{\perp}}{H} \quad H_{2d} = 3H
\]
• Scalar power spectrum suffers *divergence* on the plane, and the significant backreaction make perturbative approach invalid.

\[ P = \frac{1}{2\pi^2} \left( \alpha^{-4} k^2 + \alpha^2 k_\perp^2 \right)^{3/2} \times \left| f_{k_\perp,k}^{(C)} (\eta \to 0) \right|^2 \]
Quantum State in the Vicinity of Singularity

- The KdS universe = The \textit{planar} Schwarzschild-de Sitter spacetime

\[ ds^2 = -dt^2 + (HL)^2 \left( \sinh\frac{3Ht}{2} \left( \cosh\frac{3Ht}{2} \right)^{-\frac{1}{3}} \right)^2 d\tilde{r}^2 + L^2 \left( \cosh\frac{3Ht}{2} \right)^{\frac{4}{3}} d\tilde{x}_\perp^2. \]

\[ t = \frac{2}{3} H^{-1} \log \left[ (\frac{T}{L})^{3/2} + \sqrt{(\frac{T}{L})^3 - 1} \right] \quad L < T < \infty \]

\[ ds^2 = -\frac{dT^2}{f(T)} + f(T)d\tilde{r}^2 + T^2d\tilde{x}_\perp^2 \]

\[ f(T) = H^2 \left( T^2 - \frac{L^3}{T} \right) \]

The initial surface \( t = 0 \Leftrightarrow \) The cosmological horizon \( T = L \)
• Maximally-extended Kasner-de Sitter spacetime

- KdS universe (the shaded region) is influenced by the timelike singularity.

- The existence of singularity makes the quantum state **unpredictable**.

  Horowitz and Marolf (95)

  We adopt the new interpretation of KdS universe as the consequence of the quantum tunneling from a *regular* lower-dimensional vacuum.
Anisotropic Inflation after Quantum Tunneling

- Initially, the $x_\perp$ directions are static and smoothly matched to a lower-dimensional parent vacuum compactified on $T^2$

$$d s^2 \approx -d t^2 + t^2 d r^2 + d x_\perp^2.$$  

- A transdimensional transition was argued in the context of the landscape of multiple vacua in a higher-dimensional theory.

Blanco-Pillado, Schwartz-Perlov and Vilenkin (09), Carroll, Johnson and Randall (09)

In 6d Einstein-Maxwell theory
• Decompactification within our 4D universe.

$$dS_3 \times S^1 \rightarrow dS_4$$

Blanco-Pillado and Salem (10)

$$dS_2 \times S^2 \rightarrow dS_4$$

Adamek, Campo, and Niemeyer (10)

- Anisotropic spatial curvature leads to late-time anisotropic expansion, which dominates over the primordial effects.

- Instanton transitions between 4D vacuum and 2D vacuum on $T^2$ are mediated by Casimir interactions.

Arnold, Fornal & Ishiwata (11)

- Our $T^2$ compactification model is free from the late-time anisotropy.

• Observational signatures from toy models describing the decompactification from a 2D vacuum on $T^2$ to 4D will be argued.

$$\Rightarrow \Omega_K < 10^{-4}$$

Demianski and Doroshkevich (07)
Graham, Harnik and Rajendran (10)
Models of Kasner-de Sitter Bubble Nucleation

\[ ds^2 = -r^2 d\tau^2 + dr^2 + dx_{\perp}^2 \]

\[ ds^2 = \left[ -\frac{1}{H_{2d}^2} \sin(H_{2d} r)^2 d\tau^2 + dr^2 \right] + dx_{\perp}^2 \]
• KdS bubble nucleation in the vacuum of $M_2 \times T^2$

- As the lightcone limit of KdS universe is identical to 2d Milne, the Milne wedge (M) can be *smoothly* replaced by the KdS universe. 
- The modes of a massless scalar field are quantized on the global Cauchy surface $\Sigma$ stretched over the parent vacuum of $M_2 \times T^2$. 
• KdS bubble nucleation in the vacuum of $dS_2 \times T^2$

- Both the lightcone limits of the M region of $dS_2$ and KdS universe are identical to 2d Milne, (M) can be smoothly replaced by the KdS universe.

- The modes of a massless scalar field are quantized on the global Cauchy surface $\Sigma$ stretched over the parent vacuum of $dS_2 \times T^2$. 
Power Spectrum of a Massless Scalar Field

- The initial state from the parent vacuum of $M_2 \times T^2$

$$f^{(M)}_{k_{\perp}, k}(\eta) = \frac{1}{2} \sqrt{\frac{\pi}{H_{2d}}} e^{\pi \tilde{k}/2} H^{(2)}_{i\tilde{k}} \left(2\tilde{k}_{\perp} e^{H_{2d}\eta}\right)$$

- An excited state with respect to the vacuum inside the anisotropic bubble

$$f^{(M)}_{k_{\perp}, k} = \alpha_{\tilde{k}} f^{(c)}_{k_{\perp}, k} + \beta_{\tilde{k}} \left(f^{(c)}_{k_{\perp}, k}\right)^*$$

$$\alpha_{\tilde{k}} = \frac{e^{\pi \tilde{k}/2}}{\sqrt{e^{\pi \tilde{k}} - e^{-\pi \tilde{k}}}} \quad \beta_{\tilde{k}} = -\frac{e^{-\pi \tilde{k}/2}}{\sqrt{e^{\pi \tilde{k}} - e^{-\pi \tilde{k}}}}$$

$$f^{(c)}_{k_{\perp}, k}(\eta) = \sqrt{\frac{\pi}{2H_{2d} \sinh(\pi \tilde{k})}} J_{-i\tilde{k}} \left(2\tilde{k}_{\perp} e^{H_{2d}\eta}\right)$$

- Regular in any direction
The initial state from the parent vacuum of \( dS_2 \times T^2 \)

\[
\begin{align*}
    f^{(1)}_{k_{\perp},k}(\eta) &= \frac{1}{\sqrt{2k} \sqrt{2 \sinh(\pi k / H_{2d})}} e^{\pi k / 2 H_{2d}} N(k, k_{\perp}) \tilde{f}^{(1)}_{k_{\perp},k}(\eta) \\
    f^{(2)}_{k_{\perp},k}(\eta) &= \frac{1}{\sqrt{2k} \sqrt{2 \sinh(\pi k / H_{2d})}} e^{\pi k / 2 H_{2d}} \left( L(k, k_{\perp}) \tilde{f}^{(1)}_{k_{\perp},k}(\eta) + e^{-\pi k / H_{2d}} \tilde{f}^{(2)}_{k_{\perp},k}(\eta) \right)
\end{align*}
\]

\[
\begin{align*}
    \tilde{f}^{(1)}_{k_{\perp},k}(\eta) &= e^{-ik\eta} F \left[ -\nu, \nu + 1, 1 - \mu, \frac{1 + \xi_i}{2} \right] \\
    \tilde{f}^{(2)}_{k_{\perp},k}(\eta) &= e^{ik\eta} F \left[ -\nu, \nu + 1, 1 + \mu, \frac{1 + \xi_i}{2} \right]
\end{align*}
\]

\[
\begin{align*}
    N(k, k_{\perp}) &= \frac{\Gamma(1 + \nu - \mu) \Gamma(-\mu - \nu)}{\Gamma(1 - \mu) \Gamma(-\mu)} \\
    L(k, k_{\perp}) &= -\frac{\Gamma(1 + \mu) \Gamma(1 + \nu - \mu) \Gamma(-\mu - \nu)}{\Gamma(1 - \mu) \Gamma(-\nu) \Gamma(1 + \nu)}
\end{align*}
\]

- Also regular in any direction.
• Time evolution inside the KdS bubble

\[
\left[ \frac{d^2}{d\eta^2} + \Omega^2(k_\perp, k, \eta) \right] f_{k_\perp, k}(\eta) = 0
\]

\[
\Omega^2(k_\perp, k, \eta) = \alpha^{-4} \sinh^{-4/3}(-H_2 d\eta) \ e^{2H_2 d\eta/3} \left( \alpha^6 k_\perp^2 + e^{-2H_2 d\eta} k^2 \right)
\]

\[
\mathcal{P} = \frac{1}{2\pi^2} \left( \alpha^{-4} k^2 + \alpha^2 k_\perp^2 \right)^3 \times \left\{ \begin{array}{l}
|f_{k_\perp, k}^{(M)}(\eta \to 0)|^2 \quad (M_2 \times T_2) \\
\sum_{i=1}^2 |f_{k_\perp, k}^{(i)}(\eta \to 0)|^2 \quad (dS_2 \times T_2)
\end{array} \right. 
\]

• Power spectrum
\[ M_2 \times T^2 \]

\[ \theta = \frac{\pi}{2} \]

\[ \theta = \frac{3\pi}{8} \]

\[ \theta = \frac{\pi}{4} \]

Suppression as well as milder angular variation of the power are compatible with the CMB data.

\[ k = \bar{k} \cos \theta \text{ and } k_\perp = \bar{k} \sin \theta \]
A large enhancement of the power on large scales would conflict with the CMB data.
Summary

• Large-scale anomalies in the CMB may be caused by the nontrivial modifications in the initial quantum states before onset of inflation.

• A naïve quantization in the vacuum of 2D Milne universe leads to several divergences, making the choice of the initial state highly questionable.

• We took the approach that the KdS universe is an outcome of quantum tunneling from the regular universe with stabilized dimensions

  - The vacuum of $M_2 \times T^2$ leads to favorable features to explain to the observed CMB anomalies.

  - The vacuum of $dS_2 \times T^2$ leads to a large enhancement of the power, which easily conflicts with the current CMB data.
Thank you.