The cosmological divide: collapse versus expansion

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Refs: MLDM, PRD 81, 123514 (2010) arXiv:0910.5755 [gr-qc]
LDMM, PRD 83: 103528 (2011) arXiv:1103.0976v2[gr-qc]
MLDM, PRD 88 (2013) 043501 Arxiv: 1302.6186,
LDMMFCA, PRD 88 (2013) 027301: 1305.3475, MLDMPRD92(2015)

UID/FIS/04434/2013

- Observations reveal that the large scale universe is expanding, and yet that there is a plethora of bound structures on the smaller scales.

- The idea underlying the viewpoint of structure formation from gravitational instability is that the collapse of the over-densities departs from the general expansion of the universe.

- In this talk I report on investigations of spherically symmetric spacetimes aiming at obtaining the conditions for the existence of shells dividing expanding and collapsing regions.

Motivation

- Models of structure formation assume that small local in-homogeneities grow due to gravitational instability

- so that the over-densities collapse and eventually form the "bound" structures we observe in the present universe.

- An idea underlying this viewpoint is that the collapse of the over-densities eventually departs from the general expansion of the universe.

- Local physics seems to be immune to global Physics

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Are there specific scales for GR being inescapable?

At present, observations seem to indicate that superclusters of galaxies ≥30 Mpc have not yet started to collapse.

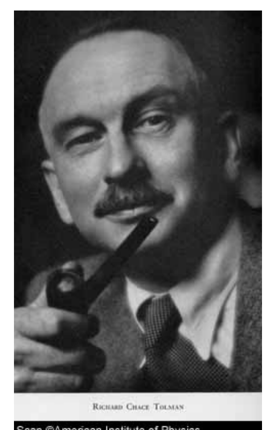
On the other hand, at earlier stages of the expansion of the universe the high mean density makes even modest density fluctuations strongly relativistic at much smaller scales...

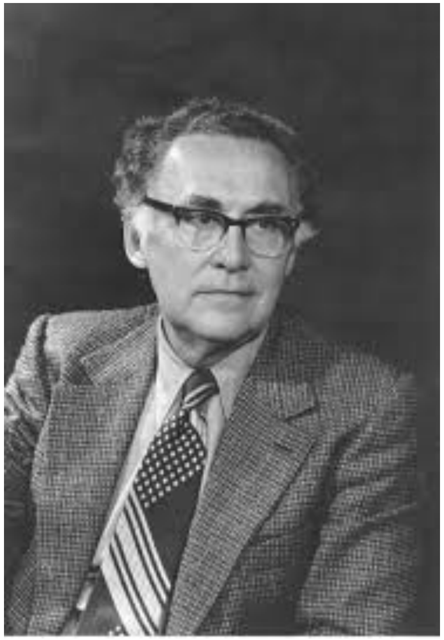
Static (Schwarzschild 1916, de Sitter

Spherically symmetric Models

Expanding (Friedmann 1921, Lemaître 1926-1933 Tolman 1934,...)

Contracting (Lemaître 1933, Tolman 1934, Bondi 1947, Chandrasekhar 1930,...)





(1939) R. Oppenheimer and G. Volkoff derived a condition for the equilibrium of a spherical configuration such as a (neutron) star. It became known as the TOV equation of state

$$\frac{dP}{dr} = -\frac{(\rho + p)(M/r^2 + 4\pi \mathrm{Pr})}{1 - 2M/r}$$

Also in 1939, Oppenheimer and Snyder put forward an influential Model of spherical collapse: a closed FLRW embedded in a Schwarzschild solution.

In 1972 Gunn and Gott extended the latter idea for spherical collapse model with isothermal spheres.

[more recently…Manera & Mota, 2006, Nunes & Mota 2006, Pace et al 2010…]

Similar concerns have been pursued in several works in the literature:

H. Bondi, MNRAS 142 (1969) 142); (bounces of uniform spheres)
W. B. Bonnor, Mon. Not. R. Astron. Soc. 167, 55 (1974) (inhomogeneous)
J. Barrow, G Galloway, and F.Tipler, MNRAS 223 (1986) (recollapse)
B. Carr, A. Coley, Phys.Rev. D62 (2000) 044023 (self-similar collapse)

How local physics departs and becomes immune from the global expansion?.

A.Einstein and E.G. Strauss, Rev.Mod. Phys. 17,120 (1945),
ibid 18,148 (1946)
G. F. R. Ellis, Local and Global Physics, Int.J. Mod. Phys A17, 2667 (2002)
V. Faraoni and A.Jacques, Phys. Rev. D 76, 063510 (2007),

A.Einstein and E.G. Strauss, Rev.Mod. Phys. 17,120 (1945),
ibid 18,148 (1946)

G. F. R. Ellis, Local and Global Physics, Int.J. Mod. Phys A17, 2667 (2002)
V. Faraoni and A.Jacques, Phys. Rev. D 76, 063510 (2007),

A. Chamorro, "A Kerr cavity with a small rotation parameter embedded in Friedmann universes", GRG 20, pp.1309-1323:

V. Marra, E. W.Kolb, S. Matarrese, A. Riotto, PRD 76, 123004, (2007)

Review (see refs therein):
Mars, Mena, Vera, General Relativity and Gravitation, Vol. 45, Issue 11, pp. 2143-2173

-On a different context, L. Herrera and co-workers have studied the "cracking" of compact objects in astrophysics using small anisotropic perturbations around spherically symmetric homogeneous fluids in equilibrium.

L. Herrera, Phys . Lett. A1 6 5 , 206 (1992)
A. Di Prsico, L. Herrera, V. Varela, GRG 29, 1239 (1997)

- The latter references are concerned with the existence of a shell where there is a change in the direction of the radial force acting on the particles of the shells. Whenever this happens one has what they termed as a *cracking* situation, a concept introduced by Herrera in 1992 (PLA 165)

Familiar LTB form of Einstein equations:

$$ds^2 = -dt^2 + \frac{(\partial_R r)^2}{1+E(t,R)} dR^2 + r^2 d\Omega^2$$

$$\dot{r}^2 = \frac{2M}{r} + \frac{\Lambda}{3} r^2 + E$$

$$\ddot{r} = -\frac{M}{r^2} + \frac{\Lambda}{3} r \leq / > 0$$

The LTB dust solutions

$$p = 0 \Rightarrow \quad \begin{aligned} M &= M(R) \\ E &= E(R) \end{aligned}$$

$$d\eta = \sqrt{E}\,\frac{dt}{r}$$

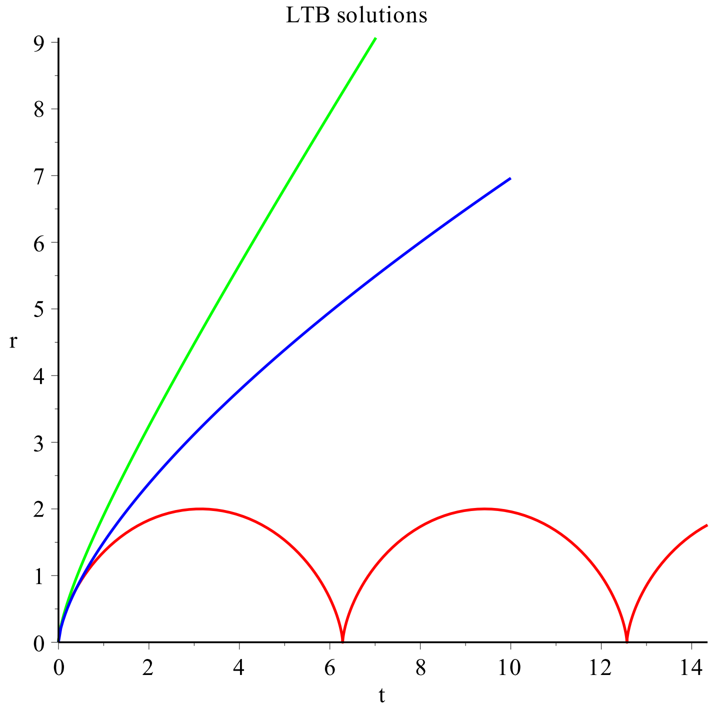
$$r(\eta, R) = \frac{M(R)}{2E(R)}\, h_E'(\eta)$$

$$t - t_{BB}(R) = \frac{M(R)}{2(\sqrt{E}\,)^3}\, h(\eta)$$

$$h(\eta) = \eta - \sin\eta \quad \mathit{if} \quad E < 0$$

$$h(\eta) = \sinh\eta - \eta \quad \mathit{if} \quad E > 0$$

$$r^{3/2}(t, R) = \pm\frac{3\sqrt{2M(R)}}{2}\,(t - t_{BB}(R))$$



LTB solutions
9
8
7
6
5
r
4
3
2
1
0
0
2
4
6
8
10
12
14
t

Turning point

$$\dot{r}^2 = 0 \quad \Leftrightarrow \frac{2M}{r} + \frac{\Lambda}{3}r^2 + E = 0$$

If $\Lambda = 0$ requires $E < 0$

But

$$\ddot{r} = -\frac{M}{r^2} < 0$$

Yet

$$\ddot{r} = -\frac{M}{r^2} + \frac{\Lambda}{3}r \leq/> 0$$

One needs to consider more general models than the canonical dust LTB

3+1 Splitting

$$N^a_b := -n^a n_b, \qquad h^{ab} := g^{ab} + n^a n^b,$$

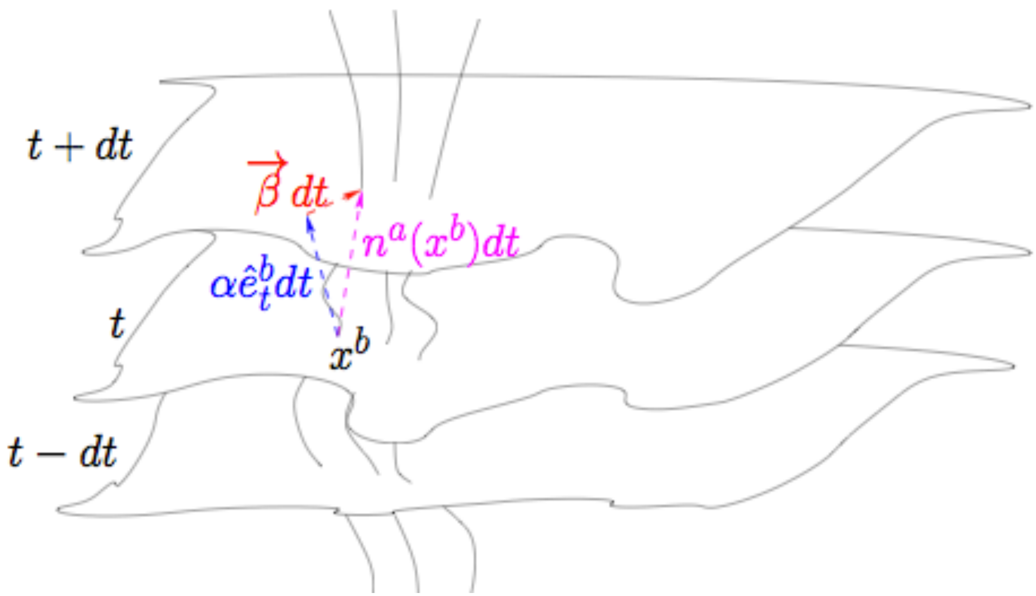
$$n_a := -\alpha \nabla_a t = [-\alpha, 0, 0, 0] \; (n_a n^a = -1)$$

Use Generalised Painlevé-Gullstrand coords (also, Gautreau)

[Lasky and Lun PRD 2006, 2007]

$$ds^2 = -\alpha^2 dt^2 + \frac{\left(dR + \beta(t,R)\, dt\right)^2}{1 + E(t,R)} + r^2(t,R)\left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

α = lapse function, β =shift function, E =curvature-energy



Do 3+1 decomposition

$$n_{a;b} = N^a_b n_{a;c} + n_{\bar{a};\bar{b}} = -n_b \dot{n}_a + \frac{1}{3}\Theta h_{ab} + \sigma_{ab} + \omega_{ab}$$

$$T^{ab} = \rho n^a n^b + p h^{ab} + \Pi^{ab} + 2 j^{(a} n^{b)}$$

$$\sigma_{ab} = \sigma(t,R)\, P_{ab}$$

$$P_i^{\ j} = diag(-2,1,1)$$

$$E_{ab} = \Sigma(t,R)\, P_{ab}$$

$$\Pi_{ab} = \Pi(t,R)\, P_{ab}$$

$$\frac{1}{\alpha}\left(D_a D_b - \frac{1}{3}\, {}^3g_{ab} D^c D_c\right)\alpha = \varepsilon(t,R)\, P_{ab}$$

$$\theta = n^a{}_{;a}$$

$${}^3R_{ab} - \frac{1}{3}\, {}^3R\, {}^3g_{ab} = q(t,R)\, P_{ab}$$

Introducing the Misner-Sharpe (also ADM) mass

$$M = \int \rho r^2 dr$$

And restricting to a perfect fluid

$$\dot{M} = 4\pi \mathrm{P} r^2 \beta = 4\pi \mathrm{P} r^2 \alpha \sqrt{\frac{2M}{r} + \frac{\Lambda}{3} r^2 + E}$$

$$\dot{E} r' = 2\frac{1+E}{\rho + P} \mathrm{P}' \beta = 2\frac{1+E}{\rho + P} \mathrm{P}' \alpha \sqrt{\frac{2M}{r} + \frac{\Lambda}{3} r^2 + E}$$

$$T^a_{b;a} = 0$$

Bianchi contracted identities

$$\Rightarrow n^b T^a_{b;a} = -L_n \rho - (\rho + P)\Theta = 0$$

$$\Rightarrow P' = -(\rho + P)\frac{\alpha'}{\alpha}$$

[Lasky and Lun PRD 2006, 2007]

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Local conditions for a dividing shell:

$$L_n r = r\left(\frac{\Theta}{3}+a\right) = -\frac{\beta}{\alpha} = 0$$ → $$E_* = -\frac{2M}{r_*}$$

$$-L_n^2 r = \quad r\left[L_n\left(\frac{\Theta}{3}+a\right)-\left(\frac{\Theta}{3}+a\right)^2\right] = 0$$

→ $$gTOV = \left[\frac{1+E}{\rho+P}P' + 4\pi P r + \frac{M}{r^2}\right] = 0$$

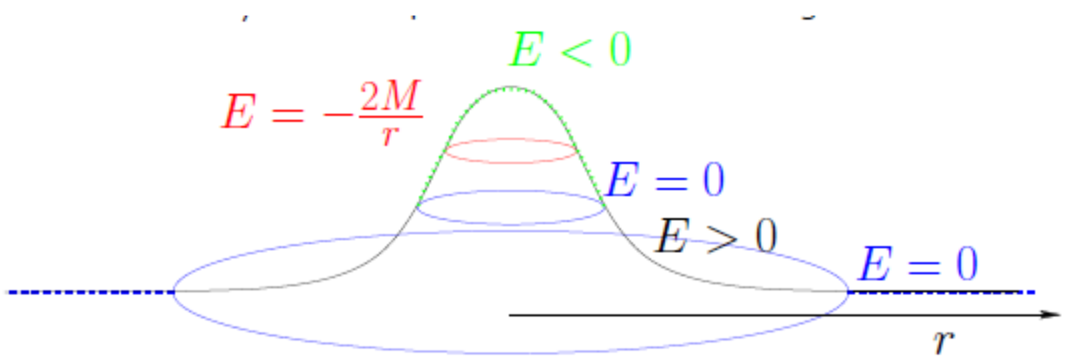
$\Theta_* = 0$ → $\left(\frac{\beta}{\alpha}\right)' = 0$ → a (shear) $= 0$

NB: E is not 3-curvature!

$$-\frac{^3R}{2} = \frac{1}{r^2}\left[(1+E)r'^2 + E'rr' + 2(1+E)rr'' - 1\right]$$

But the turning point requires positive curvature

$$\frac{\Theta^2}{3} + \frac{^3R}{2} = 8\pi\rho + 3a^2 + \Lambda$$



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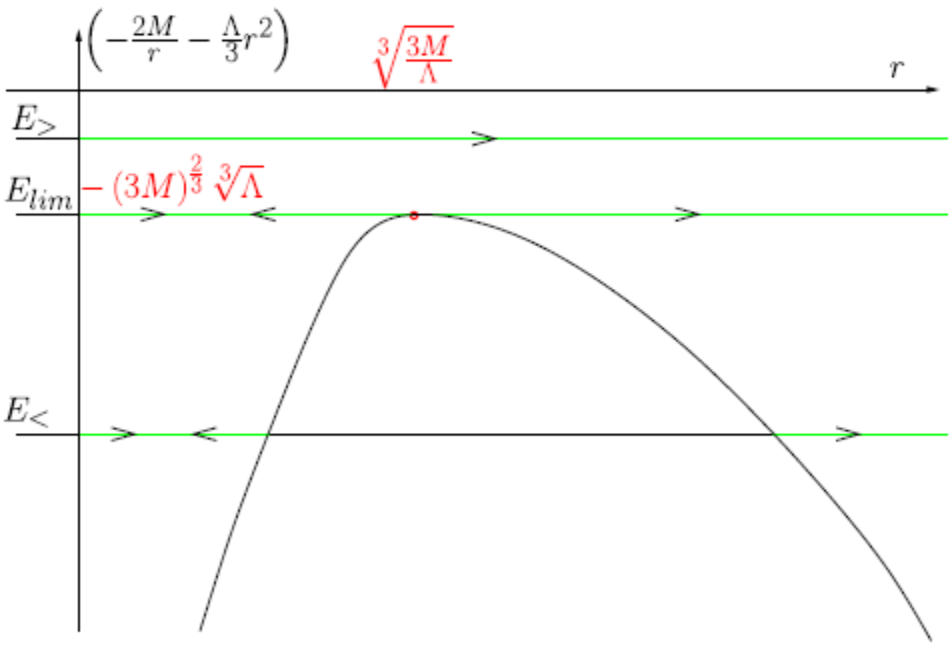
"Illustration" :: Λ-CDM



$$r_{lim} = \sqrt[3]{\frac{3M}{\Lambda}},$$

$$E_{lim} = -\,(3M)^{\frac{2}{3}}\,\Lambda^{\frac{1}{3}} = -\Lambda r_{lim}^2,$$

FIG. 1 (color online). Kinematic analysis for a given shell of constant M and E. Depending on E relative to E_{lim}, the fate of the shell is either to remain bound ($E_< < E_{\mathrm{lim}}$) or to escape and cosmologically expand ($E_> > E_{\mathrm{lim}}$). There exists a critical behavior where the shell will forever expand, but within a finite, bound radius ($E = E_{\mathrm{lim}}$, $r \leq r_{\mathrm{lim}}$). The maximum occurs at $r_{\mathrm{lim}} = \sqrt[3]{3M/\Lambda}$.

Saddle

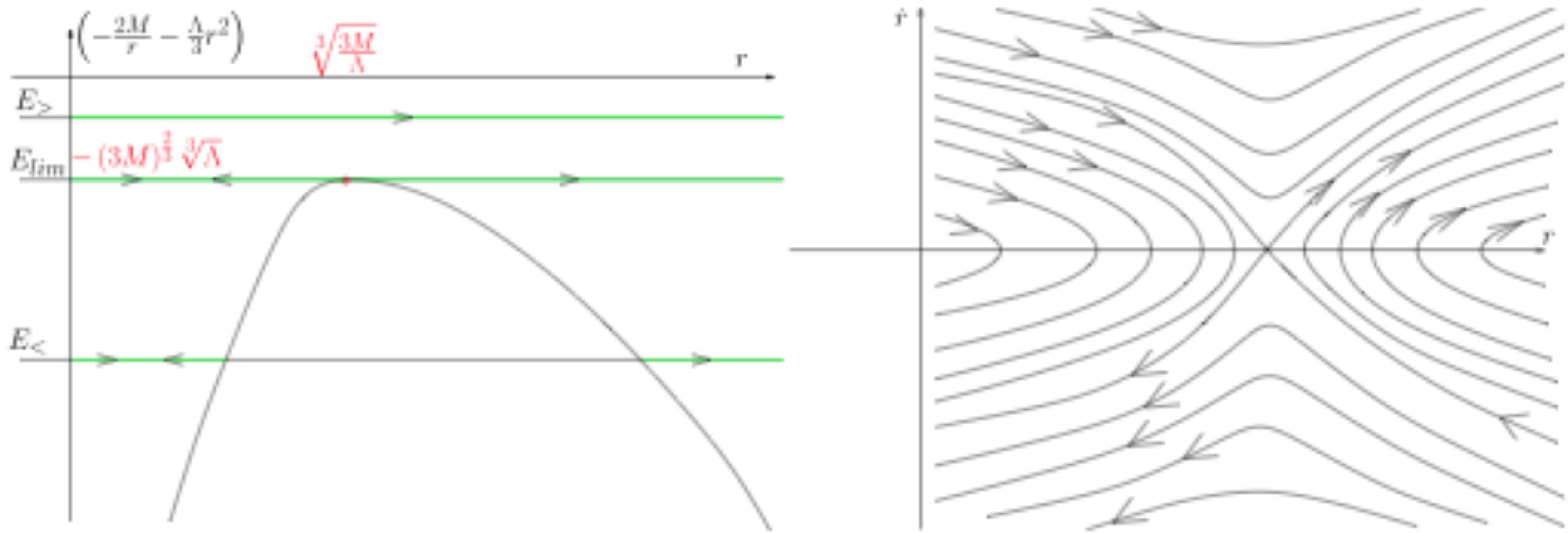


FIG. 1 (color online). Effective potential kinematic analysis (left) and phase space analysis (right) from [5]. The kinematic analysis for a given shell of constant M and E depict the fate of the shell, depending on E relative to E_{lim}. It either remains bound ($E_< < E_{lim}$) or escapes and cosmologically expands ($E_> > E_{lim}$). There exists a critical behavior where the shell will forever expand, but within a finite, bound radius ($E = E_{lim}$, $r \leq r_{lim}$). The maximum occurs at $r_{lim} = \sqrt{3M/\Lambda}$. The corresponding phase space behavior follows, the scales are set by the value of $r_{lim} = \sqrt{3M/\Lambda}$ while the actual kinematic of the shell is given by E.

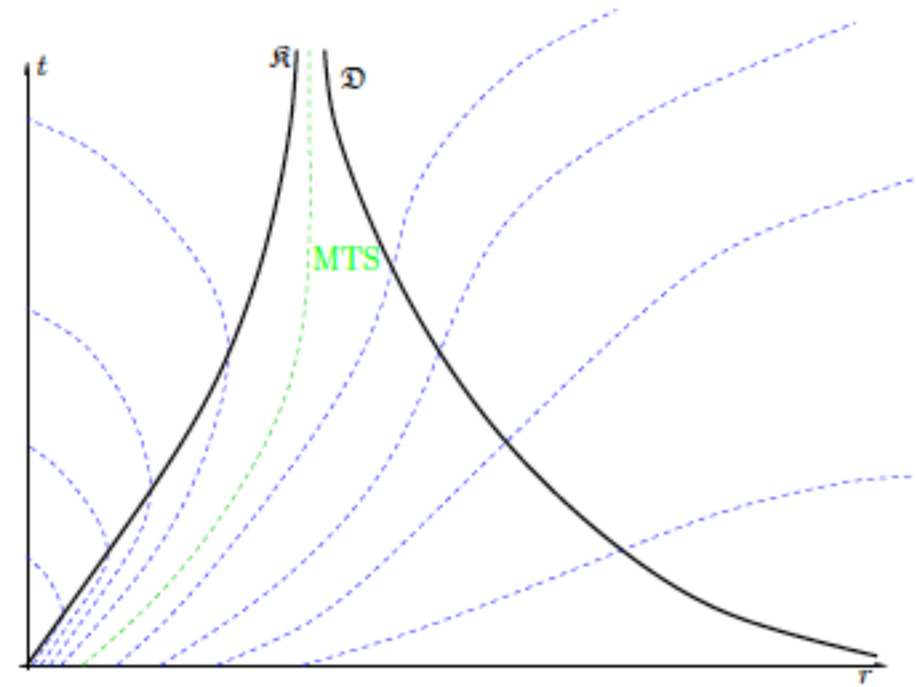


Figure 1. A sketch of the worldlines for the second dust+Λ example of Ref. [20]. The initial condition are typical of cosmology. In this case, the points of 𝔎 are the areal radius turnaround events while those of 𝔇 are its zero acceleration events. Both asymptote to the MTS.

A more realistic approach requires the consideration of a more general content with anisotropic stresses and, most likely, energy transfer (heat flow) as well.

Models with anisotropic pressures (but without heat fluxes)

$$T_{\mu\nu} = (\rho + P)n_\mu n_\nu + P g_{\mu\nu} + 2 j_{(\mu} n_{\nu)} + \Pi_{\mu\nu}.$$

$$\Pi_{ij} := \Pi(t, r) P_{ij}.$$

$$K_{ij} - \tfrac{1}{3} \perp_{ij} K := a(t, r) P_{ij}.$$

$$^3R_{ij} - \tfrac{1}{3} \perp_{ij} {}^3R := q(t, r) P_{ij}.$$

Eij = (t; r) Pij

$$R = r \Rightarrow \partial R / \partial r = 1$$

$$(\mathcal{L}_n R)^2 = \frac{2M}{R} + (1 + E)\left(\frac{\partial R}{\partial r}\right)^2 - 1.$$

$$\frac{M}{R^2} + 4\pi(P - 2\Pi)R = \frac{1 + E}{\alpha}\frac{\partial \alpha}{\partial r}\frac{\partial R}{\partial r} - \mathcal{L}_n{}^2 R,$$

$$\mathcal{L}_n M = -4\pi R^2\left[(P - 2\Pi)\mathcal{L}_n R + j\frac{\partial R}{\partial r}(1 + E)\right]$$

$$
\begin{aligned}
{}^3R + \frac{2}{3}\theta^2 - 6a^2 =& 16\pi\rho + 2\Lambda \\
2\mathcal{L}_n\theta - \frac{1}{2}{}^3R - \theta^2 - 9a^2 + \frac{2}{\alpha}D^a D_a\alpha =& 24\pi P - 3\Lambda, \\
-\mathcal{L}_n a + a\theta + \epsilon - q =& 8\pi\,\Pi.
\end{aligned}
$$

$$
(\mathcal{L}_n r)^2 = \frac{2M}{r} + E = 0
$$

$$
-\mathcal{L}_n^2\, r = \frac{M}{r^2} + 4\pi(P - 2\Pi)r - \frac{1+E}{\alpha}\frac{\partial\alpha}{\partial r} = 0\ .
$$

$$
-\frac{1}{\alpha}\frac{\partial\alpha}{\partial r} = \frac{1}{(\rho + P - 2\Pi)}\left[\frac{\partial}{\partial r}(P - 2\Pi) - \frac{6\Pi}{r}\right]
$$

$$-2\mathcal{L}_n\Theta = \frac{{}^3R}{2} + \Theta^2 + 9a^2 - \frac{2}{\alpha}D^\mu D_\mu\alpha + 24\pi P - 3\Lambda,$$

$$\mathcal{L}_n a = -a\Theta + \epsilon - q + 8\pi\Pi$$

$$\mathcal{L}_n\Sigma = -4\pi\mathcal{L}_n\Pi - 4\pi\left(\rho + P - 2\Pi\right)a - \left(3\Sigma + 4\pi\Pi\right)\left(\frac{\Theta}{3} + a\right),$$

$$\left(\frac{\Theta}{3} + a\right)' = -3a\frac{r'}{r},$$

$$\frac{4\pi}{3}\left(\rho + 3\Pi\right)' = -\Sigma' - 3\left(\Sigma + 4\pi\Pi\right)\frac{r'}{r},$$

$${}^3R + \frac{2}{3}\Theta^2 - 6a^2 = 16\pi\rho + 2\Lambda,$$

$$\mathcal{L}_n\rho = -\Theta(\rho + P) - 6\Pi a$$

$$0 = \left(D^k + \dot{n}^k\right)\left(\Pi_{ik} + h_{ik}P\right) + \left[\rho - \left(P - 2\Pi\right)\right]\dot{n}_i - n_i\left[\Theta P + 6\Pi a\right].$$

Dictionary

$$\left(\frac{\Theta}{3}+a\right)=\frac{\mathcal{L}_n r}{r}$$

$$\frac{2M}{r^3}+(1+E)\left(\frac{r'}{r}\right)^2-\frac{1}{r^2}=\frac{8\pi\rho}{3}-\frac{{}^3R}{2}+2a\left(\frac{\Theta}{3}+a\right)$$

$$\begin{aligned}\frac{{}^3R}{3}+\frac{8\pi\rho}{3}+2a\left(\frac{\Theta}{3}+a\right)\\ =\frac{2M}{r^3}+2\frac{\sqrt{1+E}}{r}\left(\sqrt{1+E}r'\right)\end{aligned}$$

$$\begin{aligned}q&=\frac{1}{6}\left\{r\left(\frac{Er'}{r^2}\right)'+E\frac{r''}{r}+\frac{2}{r^2}\left[1+r^2\left(\frac{r'}{r}\right)'\right]\right\}\\ &=\frac{1}{6}\left[r\left(\frac{Er'}{r^2}\right)'+(2+E)\frac{r''}{r}+\frac{2}{r^2}\left(1-r'^2\right)\right],\end{aligned}$$

$$\begin{aligned}\frac{1}{\alpha}D^\mu D_\mu\alpha&=\frac{\sqrt{1+E}}{\alpha r^2}\left(r^2\sqrt{1+E}\alpha'\right)',\\ \epsilon&=-\frac{r\sqrt{1+E}}{3\alpha}\left(\frac{\sqrt{1+E}}{r}\alpha'\right)'\end{aligned}$$

$$L_n r = 0 = \frac{2M}{r} + E$$

$$gTOV = L_n{}^2 r = 0 = \frac{M}{r^2} + 4\pi\left(P - 2\Pi\right) r - \frac{(1+E)}{\alpha}\frac{\partial\alpha}{\partial r}$$

Illustrations are given by Herrera's cracking phenomenon, and by an extrapolation from the solution by R.Sussman and D. Pavón, PRD 60, 104023 (1999) generalising it to the cases where E is not vanishing

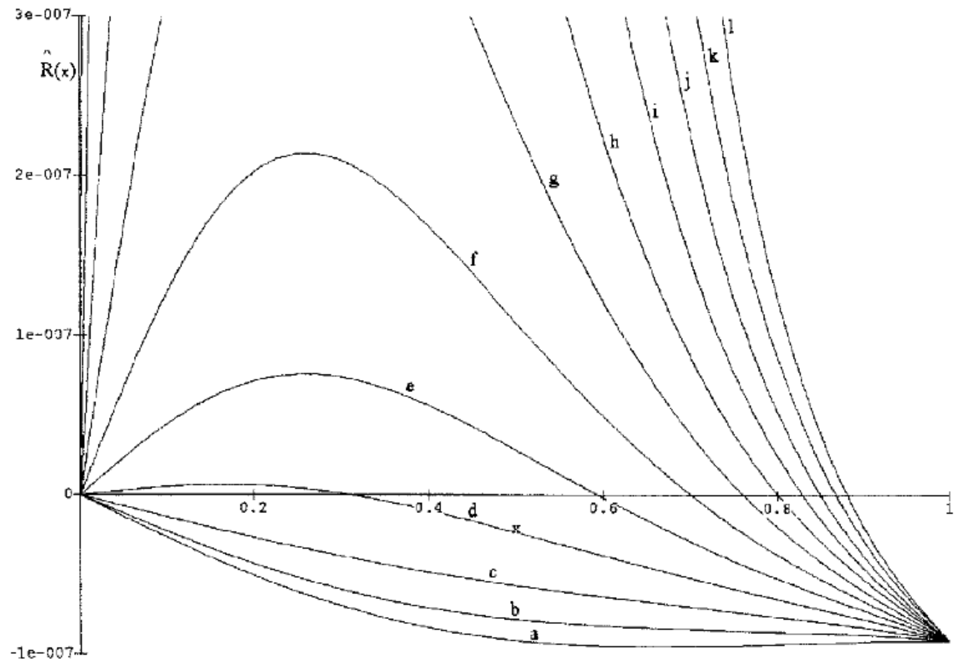
$$L_n a - a\Theta + \varepsilon - q = 8\pi\,\Pi$$

$$\varepsilon - q = 8\pi\,\Pi = 2\Sigma$$

[Mimoso & Crawford, CQG 1993, Coley & Mac Manus, CQG 1994]

Cracking

$$R \equiv \frac{dp_r}{dr} + \frac{(\rho + p_r)(m + 4\pi r^3 p_r)}{r^2(1 - 2m/r)} + 2\frac{(p_r - p_\perp)}{r}$$



L. Herrera, Phys. Lett. A1 6 5, 206 (1992)

A. Di Prsico, L. Herrera, V. Varela, GRG 29, 1239 (1997)

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Cracking

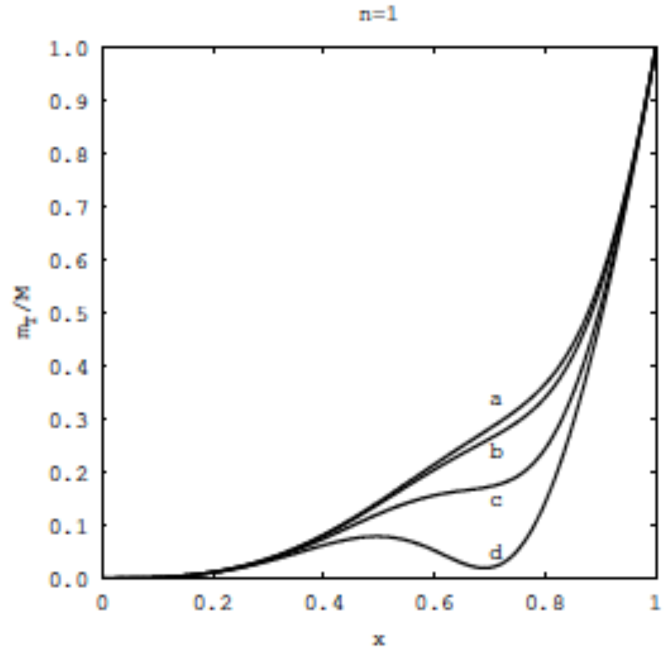


FIG. 5: Case I: m_T/M as a function of x for $n = 1$ and $\alpha(y)$: $8 \times 10^{-11}(0.3991)$ (curve a), $10^{-10}(0.4019)$ (curve b), $2 \times 10^{-10}(0.3998)$ (curve c), $4 \times 10^{-10}(0.3858)$ (curve d).

Herrera, Di Prisco, Barreto, Ospino, preprint 2014

More stable configurations should support larger surface gravitational potentials. The Tolman mass, the measure of active gravitational mass, is not a monotically increasing function for the cases c and d. This means that there is a source of instability leading to cracking/splitting under perturbations.

Sussman and Pavon PRD 1999

$$M_{dust} = M(R)$$

$$M_{rad} = \frac{W(R) r_i(R)}{2r(T,R)}$$

$$(P - 2\Pi)' - 6\Pi \frac{r'}{r} = 0$$

$$\dot{r}^2 = c^2 \left(2\frac{M}{r} + \frac{W r_i}{r^2} + E \right)$$

$$\frac{\ddot{r}}{c^2} = -\frac{M}{r^2} - \frac{W r_i}{r^3},$$

$$\frac{W r_i}{2r^3} = 4\pi \left(P - 2\Pi\right) r.$$

$$W_\star = - M_\star \quad \Rightarrow W(R_\star) < 0 \quad \Leftrightarrow P_\star < 2\Pi_\star.$$

$$r_\star = \sqrt[3]{\frac{M_\star}{8\pi\left(2\Pi_\star - P_\star\right)}}$$

$$E_\star = -\frac{M_\star}{r_\star} = -\sqrt[3]{8\pi\left(2\Pi_\star - P_\star\right)} M_\star^{\frac{2}{3}} = \frac{M_\star^{\frac{2}{3}} W_\star^{\frac{1}{3}}}{r_\star} < 0.$$

From local to global?

Shell crossing in initially expanding Λ-CDM

Assumptions:

(1) Regular density distribution (no vacuum at the center, finiteness of mass)

(2) Initial Hubble-type flow >> E<Elim at center

$$r_{lim} = \sqrt[3]{\frac{3M}{\Lambda}},$$
$$E_{lim} = -\left(3M\right)^{\frac{2}{3}} \Lambda^{\frac{1}{3}} = -\Lambda r_{lim}^2,$$

(3) Asymptotic spatial cosmological behaviour (--> FLRW)

Non-locality:

$$\left(\frac{\beta}{\alpha}\right) = -r\left(a + \frac{\Theta}{3}\right) = \left(\frac{\beta}{\alpha}\right)_{r_0}\left(\frac{r_0}{r}\right)^2 - \frac{1}{r^2}\int_{r_0}^{r}\Theta r^2 dr,$$

$$r = r_\star,$$

$$\left(\frac{\beta}{\alpha}\right)_{r_0} r_0^2 = \int_{r_0}^{r_\star}\Theta r^2 dr,$$

$$\left(a + \frac{\Theta}{3}\right) = \left(\frac{r_0}{r}\right)^3\left(a_{r_0} + \frac{\Theta_{r_0}}{3}\right) + \frac{1}{r^3}\int_{r_0}^{r}\Theta r^2 dr,$$

$$\mathcal{L}_n\left(\frac{\beta}{\alpha}\right) = \left(\frac{\beta}{\alpha}\right)\left[\frac{2}{r^3}\left(r_0^2\left(\frac{\beta}{\alpha}\right)_{r_0} - \int_{r_0}^{r}\Theta r^2 dr\right) + \Theta\right] + \frac{1}{\alpha r^2}\left[r_0^2\partial_t\left(\frac{\beta}{\alpha}\right)_{r_0} - \int_{r_0}^{r}\dot{\Theta} r^2 dr\right] = \mathrm{gTOV}.$$

$$\mathrm{gTOV} = 0,$$

$$\left(\frac{\beta}{\alpha}\right)\left[\frac{2}{r^3}\left(r_0^2\left(\frac{\beta}{\alpha}\right)_{r_0} - \int_{r_0}^{r}\Theta r^2 dr\right) + \Theta\right] = \frac{1}{\alpha r^2}\left[r_0^2\partial_t\left(\frac{\beta}{\alpha}\right)_{r_0} - \int_{r_0}^{r}\dot{\Theta} r^2 dr\right].$$

$$r_0^2\partial_t\left(\frac{\beta}{\alpha}\right)_{r_0} = \int_{r_0}^{r_\star}\dot{\Theta} r^2 dr,$$

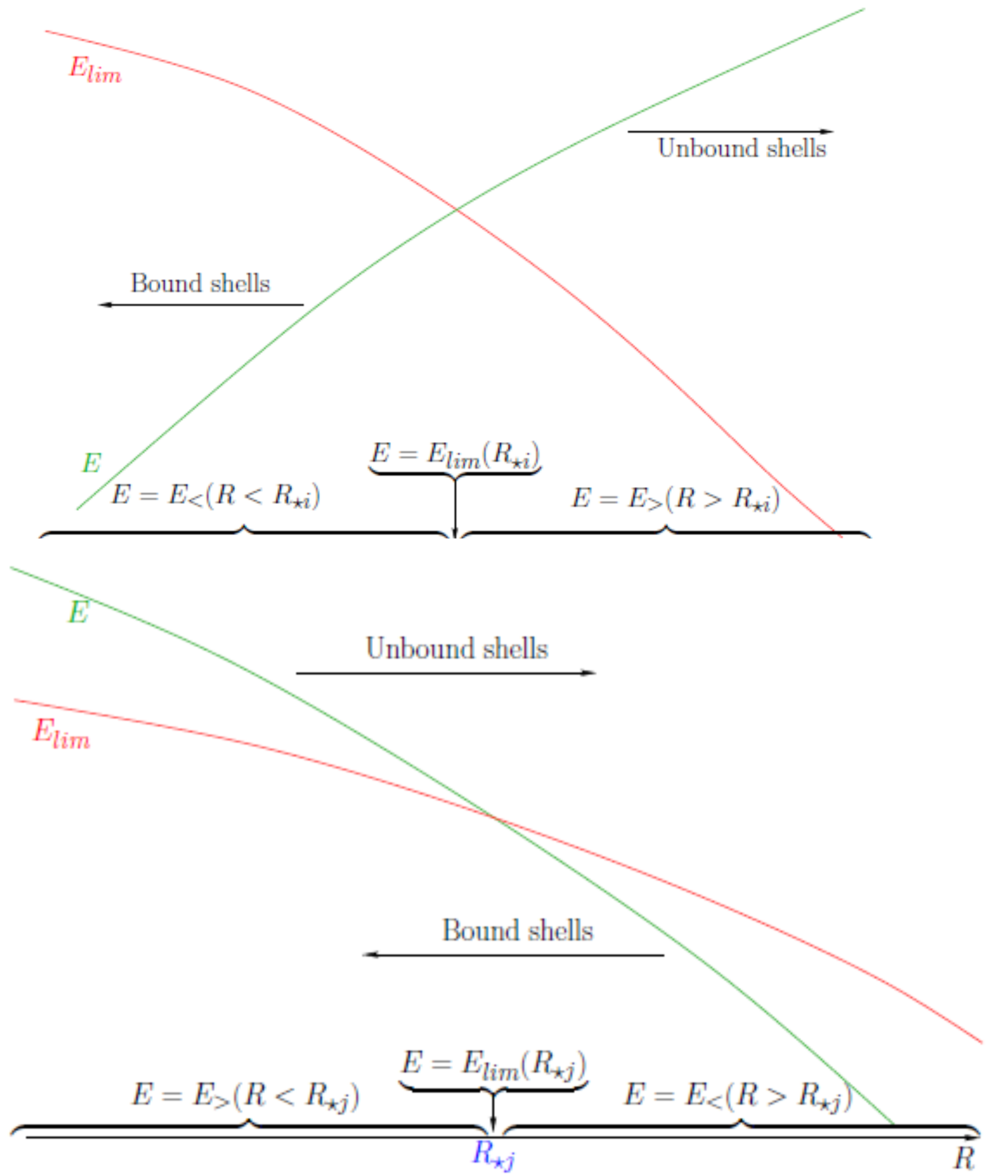
"Cracking" (Herrera 1992)

L. Herrera, Phys. Lett. A **165**, 206 (1992).

A. Di Prisco, L. Herrera, E. Fuenmayor, and V. Varela, Phys. Lett. A **195**, 23 (1994).

(5) Local mass of crossing shell is conserved

$$r_{lim} = \sqrt[3]{\frac{3M}{\Lambda}},$$
$$E_{lim} = -\left(3M\right)^{\frac{2}{3}}\Lambda^{\frac{1}{3}} = -\Lambda r_{lim}^{2},$$



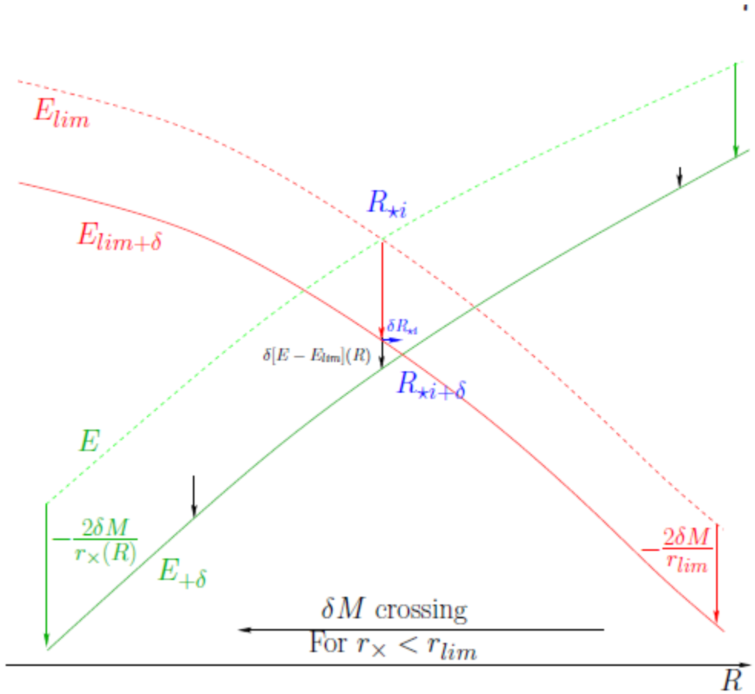


Figure 4. Effect of an ingoing, infinitesimal test shell-crossing on the energy and critical energy profiles, around the *local* initial configuration for the overcoming of E_{lim} by E. The initial intersection shell becomes bound on such perturbations and the local intersection shell shifts outwards in radius.

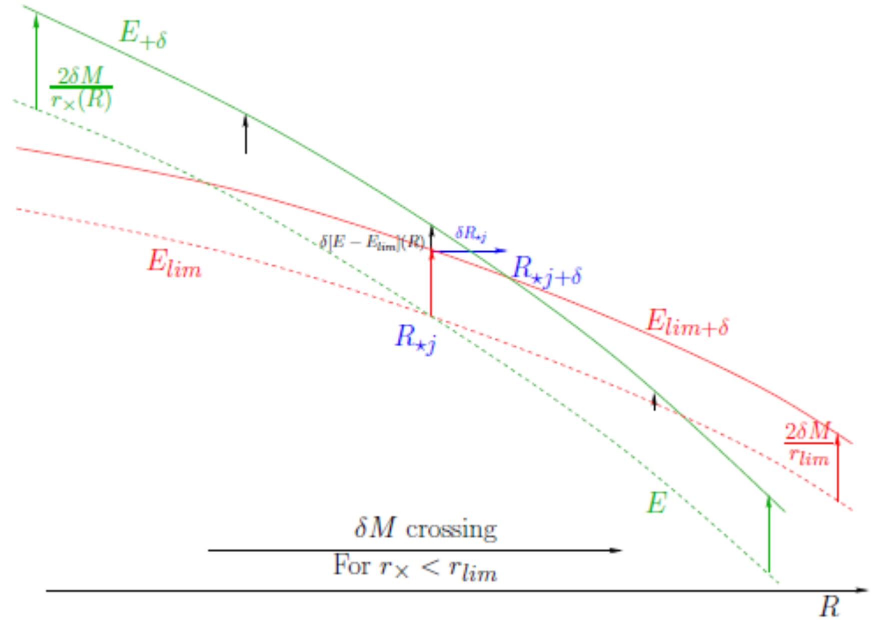


Figure 5. Effect of an outgoing, infinitesimal shell-crossing on the energy and critical energy profiles, around the *local* initial configuration for the undercoming of E_{lim} by E. The initial intersection shell becomes unbound on such perturbations and the local intersection shell shifts outwards in radius.

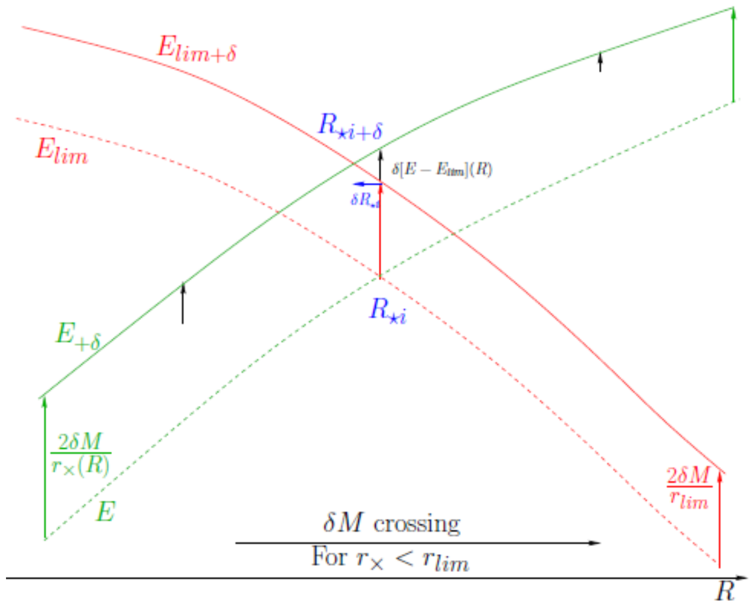


Figure 6. Effect of an outgoing, infinitesimal shell-crossing on the energy and critical energy profiles, around the *local* initial configuration for the overcoming of E_{lim} by E. The initial intersection shell becomes unbound on such perturbations and the local intersection shell shifts inwards in radius.

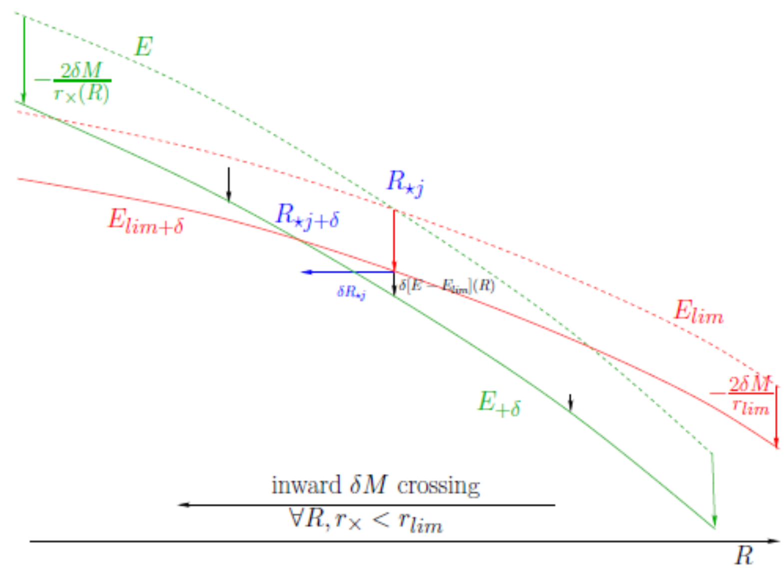


Figure 7. Effect of an ingoing, infinitesimal test shell-crossing on the energy and critical energy profiles, around the *local* initial configuration for the undercoming of E_{lim} by E. The initial intersection shell becomes bound on such perturbations and the intersection shell shifts inwards in radius.

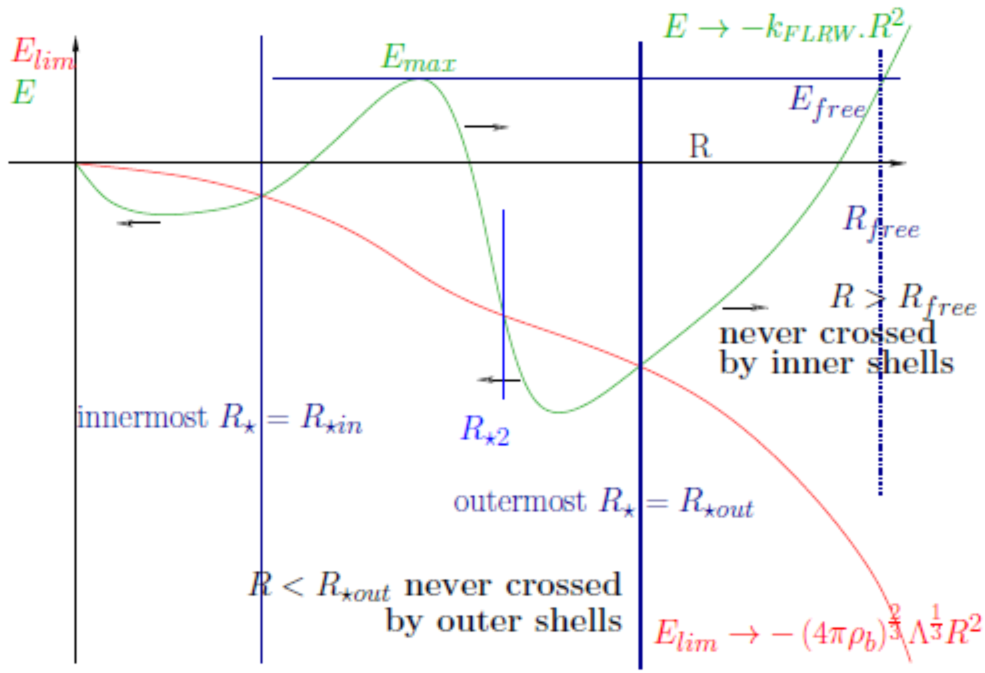


Figure 8. Open background with arbitrary central mass distribution and a single local undercoming intersection. It always gives protected inner shells as well as unmodified cosmological expansion, when keeping integrability despite shell-crossing. Shell crossing entails no fundamental modification.

Dual null formalism

Maciel Le Delliou, JPM PRD92(2015)

Conclusions

2 local conditions for the existence shells separating an inner collapsing region from an outer expansion.

(i) A particular balance between the so-called energy function of the model and the potential energy (Existence of turning point).

(ii) A stationarity condition which demands that a generalization of the Tolman-Oppenheimer-Volkov equilibrium condition be satisfied by the separating shell.

(iii) The latter is an integral form of the generalized Raychaudhuri equation.

THANKS for listening !

Introducing the Misner-Sharp mass [18] and following [9]

$$M' = 4\pi\rho r^2 r' \tag{I.22}$$

it is possible to derive[3]

$$(\mathcal{L}_n r)^2 = \frac{2M}{r} + (1+E)\,(r')^2 - 1 + \frac{1}{3}\Lambda r^2 \tag{I.25}$$

and

$$-\mathcal{L}_n^2 r = \frac{M}{r^2} + 4\pi(P-2\Pi)r - (1+E)\,\frac{\alpha'}{\alpha}r' - \frac{1}{3}\Lambda r\,. \tag{I.26}$$

This allows us to extend the generalization of the TOV function made in [1] to the case where anisotropic stresses are present:

$$\mathrm{gTOV} = -\mathcal{L}_n^2 r\,. \tag{I.27}$$

$$-\frac{\alpha'}{\alpha} = \frac{1}{(\rho+P-2\Pi)}\left[(P-2\Pi)' - 6\Pi\frac{r'}{r}\right]$$

$$\mathrm{gTOV} = -\mathcal{L}_n^2 r = \frac{M}{r^2} + 4\pi(P-2\Pi)r + \frac{(1+E)\,r'}{(\rho+P-2\Pi)}\left[(P-2\Pi)' - 6\Pi\frac{r'}{r}\right] - \frac{1}{3}\Lambda r$$

$$\mathcal{L}_n\left(\frac{\Theta}{3}+a\right) = \epsilon + \frac{1}{3\alpha}D^k D_k\alpha - \left(\frac{\Theta}{3}+a\right)^2 - \left\{\Sigma + \frac{4\pi}{3}\left[\rho + 3\left(P - \Pi\right)\right]\right\} + \frac{\Lambda}{3}, \quad (2.19)$$

$$\mathcal{L}_n a = -\frac{2}{3}a\Theta + a^2 + \epsilon - \left(\Sigma - 4\pi\Pi\right), \quad (2.20)$$

$$\mathcal{L}_n\Sigma = -4\pi\left\{\mathcal{L}_n\Pi + a\left(\rho + P - 2\Pi\right)\right\} - \left(3\Sigma + 4\pi\Pi\right)\left(\frac{\Theta}{3}+a\right), \quad (2.21)$$

$$\Sigma + 4\pi\Pi = q + a\left(\frac{\Theta}{3}+a\right).$$

$$\left(\frac{\Theta}{3}+a\right)^2 = \frac{8\pi\rho}{3} - \frac{{}^3R}{6} + \frac{\Lambda}{3} + 2a\left(\frac{\Theta}{3}+a\right)$$

$$\left(P - 2\Pi\right)' = 6\Pi\frac{r'}{r} - \left(\rho + P - 2\Pi\right)\frac{\alpha'}{\alpha}, \quad (2.24)$$

$$\left(\frac{\Theta}{3}+a\right)' = -3a\frac{r'}{r}, \quad (2.25)$$

$$\frac{4\pi}{3}\rho' + \frac{\left(\left(\Sigma + 4\pi\Pi\right)r^3\right)'}{r^3} = 0\,. \quad (2.26)$$

Familiar LTB form of Einstein equations:

$$ds^2 = -dT^2 + \frac{(\partial_R r)^2}{1+E(T,R)}dR^2 + r^2 d\Omega^2$$

$$\beta = -\dot{r}$$

$$\dot{r}^2 = \alpha^2\left(\frac{2M}{r} + \frac{\Lambda}{3}r^2 + E\right)$$

$$\dot{r}^2 = \alpha^2\left(\frac{2M}{r} + \frac{\Lambda}{3}r^2 + E\right)$$

We find that the separating shell is defined by a generalization of the Tolman-Oppenheimer-Volkoff (TOV) equilibrium condition.

The latter establishes a balance between the pressure gradients, both isotropic and anisotropic, and the strength of the fields induced by the Misner-Sharp mass inside the separating shell and by the pressure fluxes.