Rotating AdS black holes and condensed matter physics

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1. What is AdS/CMP(CFT) correspondence?

2. Recent progress on AdS/CMP(CFT) correspondence.

3. Superfluid flow and rotating AdS Black Holes

4. Critical phenomena on the superfluid flow
I What is AdS/CFT correspondence?

Original AdS (Anti De Sitter spacetime)/CFT (Conformal field theory):

\[ \text{N=4 SYM} \quad \leftrightarrow \quad \text{Type IIB on} \quad \mathbb{A}dS_5 \times S^5 \]

Finite temperature case:

\[ \text{N=4 SYM} \; \text{at finite temp} \quad \leftrightarrow \quad \text{Type IIB on} \; \text{AdS black holes} \]

The **strongly coupled** gauge theory is described by Classical General Relativity in asymptotically AdS spacetime!
The possibility of a connection between string theory and RHIC collisions is unexpected and exhilarating (2005, Press release)
Dissipation and AdS black holes

How to describe dissipation in QGP by AdS BHs?

Perturbed energy is absorbed by BHs

Ripples dissipate by viscosity

The relaxation time is described by Quasi-Normal modes of BHs
Bulk field vs Boundary field

Bulk field fluctuations act as **source term** in Boundary theory

$$S_{\text{int}} \sim \int dx^4 (\delta \phi F_{\mu \nu}^2 + h^{\mu \nu} T_{\mu \nu}^{Y M}) + \cdots$$
GKP-Witten relation

\[ Z[\phi_0] = \exp[-I(\phi)] = \left\langle \exp\left(\int_{\partial M} \phi_0 O(x)\right) \right\rangle \]

Source term for the boundary operator

\[ O(x) \]

: BD operator

CFT

\[ (F^a_{\mu\nu})^2 \]

EM tensor \( T_{\mu\nu} \)

R-current \( J_\mu \)

Entropy (density) \( S \)

Ex)

Bulk

Dilaton \( \phi \)

Perturbed metric \( h_{\mu\nu} \)

U(1) gauge potential \( A_\mu \)

BH Entropy (density) \( S \)

\[ \ldots \]
Scalar field case

AdS metric

$$ds^2 = \frac{r_0^2}{z^2}(\eta_{ij} \, dx^i \, dx^j + dz^2)$$

Solution of Eq:

$$\nabla^2 \phi - m^2 \phi = 0$$

$$\phi \approx b_+(x) \phi_+ + b_-(x) \phi_-$$

$$= b_+(x) z^{\lambda_+} + b_-(x) z^{\lambda_-}$$

Boundary operator VEV $$\langle \mathcal{O} \rangle$$

(Normalizable mode)

Bulk Source term

(Non-Normalizable mode)
Can we apply AdS/CFT correspondence to Condensed matter physics (CMP)?

Motivation: Conventional approach to strongly correlated High Tc Superconductor or quantum phase transitions is difficult.

Our Hope: Strongly coupled gauge theory or GR in AdS (via AdS/CFT) could describe some aspect of strongly correlated CMP, just as QGP.
Recent progress on AdS/CMP correspondence

Holographic superconductor model (Hartnoll, Herzog, Horowitz (2008))

\[ \mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(|\psi|) - |\nabla \psi - iq A\psi|^2 \]

\[ D_\mu = \nabla_\mu - iq A_\mu \]

Metric Ansatz:

\[ ds^2 = -f(r)e^{-\delta(r)}dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2) \]

Field Ansatz:

\[ \Psi = \Psi(r), \quad A_\mu = \Phi(r)(dt)_\mu \]
Gubser (2008): Near the extremal black hole, such charged hairy BHs could exist in AdS spacetime!

Asymptotic behavior of $\psi$:

$$\Psi \sim a_+ r^{-\lambda_+} + a_- r^{-\lambda_-}, \quad \lambda_+ > \lambda_-$$

Normalizable mode (Order parameter)

Superconducting state

$$\alpha_- = 0 \quad \alpha_+ \neq 0$$

Effective mass is arbitrary negative

$$m_{eff}^2 = m^2 - \frac{q^2 A_t^2(r)e^\delta}{f(r)} < 0$$

Many hairy BHs could exist in AdS spacetime!
Maxwell perturbations and the conductivity

Perturbation of \( A_x \sim e^{-i\omega t} \)

\[ A_x \approx \alpha_x + \frac{J_x}{r} \]

\[ \sigma(\omega) = \frac{J_x}{E_x} = -\frac{J_x}{\dot{\alpha}_x} = -\frac{iJ_x}{\omega \alpha_x} \]

- Delta function appears instead of Drude peak due to no dissipation
- Large energy gap suggests strongly coupled effect of boundary theory

PRL 101.031601 Hartnoll, Herzog, Horowitz
However...

Delta function $\delta(\omega)$ always appears even in RN-AdS BH!

Momentum conservation is always satisfied in translationally invariant (planar) BHs!

We need to explore holographic superconductor model in some broken translationally system

Ex) Lattice structure in condensed matter system
Perturbative model:

- N. Iizuka, K. M. (2012) Confirmation of Delta function $\delta(\omega)$ of conductivity in a superconducting state in a massive U(1) gauge toy model with lattices

Non-perturbative model:

- G. T. Horowitz, J. E. Santos, D. Tong (2012, 2013) The construction of spatially modulated charged AdS BHs and confirmation of Delta function $\delta(\omega)$ of conductivity in a superconducting state
- Need to solve Nonlinearly PD Eqs!

Within linear response theory, strongly correlated superconductor model is well described by the holographic model

Question

Another simple model?!

What happens \textit{beyond linear response theory}?
III. Superfluid flow and rotating AdS Black Holes

To answer this question, we apply Bianchi type anisotropic model!
Bianchi type VII$_0$ space – Helical structure –

Three Killing vectors:

\[
\xi_1 = \partial_y, \quad \xi_2 = \partial_z, \quad \xi_3 = \partial_x - z\partial_y + y\partial_z
\]

Invariant one-form:

\[
\omega^1 = \cos(x) \, dy + \sin(x) \, dz, \quad \omega^2 = -\sin(x) \, dy + \cos(x) \, dz,
\]

\[
\omega^3 = dx
\]

Translationally invariance is violated along x direction!

Helical structure is naturally incorporated in the Bianchi VII$_0$ space!
Holographic superconductor model:

\[ \mathcal{L} = \left( R + \frac{12}{L^2} - \frac{1}{4} F^2 - \frac{1}{4} W^2 - |D\psi|^2 - m^2 |\psi|^2 \right), \]

\[ F = dA, \quad W = dB, \quad A, B : U(1) \text{ gauge field} \]

Metric ansatz:

\[ ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + e^{2\nu_3(r)} (\omega^3 - \Omega(r) dt)^2 + e^{2\nu_1(r)} (\omega^1)^2 + e^{2\nu_2(r)} (\omega^2)^2, \]

5-dim AdS Bianchi VII\(0\) Black brane

Einstein Eqs.

Ordinary differential equations
Asymptotic behavior of $\Omega$

$$\Omega \approx \frac{P}{r^4}$$

Momentum of superconducting persistent current

Q. $P$ is carried by BH or matters outside of BH?

A. $P$ cannot be carried by BH according to regularity condition at the horizon

Complex scalar field $\psi$ carry the whole angular momentum $P$

No contradiction with BH rigidity theorem

Non-trivial relation:

$$P = -\mu \times J$$

$\mu$: chemical potential

$J$: Supercurrent

N. Iizuka, A. Ishibashi, K. M. PRL(2014)
Landau and Tisza's two fluid model

\( T_{\mu\nu} = (\epsilon + P)u_{\mu}u_{\nu} + P\eta_{\mu\nu} + \mu\rho_s v_{\mu}v_{\nu}, \)

\( j_{\mu} = \rho_n u_{\mu} + \rho_s v_{\mu}, \)

\( \mu : \) Velocity of normal component \hspace{1cm} \( v_{\mu} : \) Velocity of superfluid component

**Josephson Eq.**

\[ v_{\mu} u^{\mu} = -1 \]

When \( u_x = 0 \)

We find \( \frac{T_{tx}}{\mu j_x} = v_t = -(u^t)^{-1} = -1 \)

\[ \partial_{\mu} T^{\mu\nu} = 0 \]
\[ \partial_{\mu} j^{\mu} = 0 \]

Precise agreement with numerical calculation
IV. Critical phenomena on the superfluid flow

Rotating AdS black hole is unstable?

- Some AdS BHs is **unstable** against **superradiant instability**
  
  V. Cardoso, O. J. C. Dias, J. P. S. Lemos, S. Yoshida (2004)

- Possible final state should be less symmetric BHs with **only one Killing vector**
  

- Many **less symmetric** AdS BH solutions:
  


**Metric possesses only one Killing vector**

*High frequency modes are continued to be generated (Cascade behavior)*
Q. Under what conditions BH turbulence occurs?

Maybe, AdS/CMP correspondence predicts the conditions and help to understand the nature of turbulence based on the knowledge of condensed matter physics.
Normal fluid case

\[ \text{Re} \approx \frac{\rho v L}{\eta} \]

\( \eta \): shear viscosity

High Re \rightarrow turbulence

• S. R. Green, F. Carrasco, L. Lehner (2014) 2-dim. Relativistic hydrodynamics shows turbulence

Superfluid case

Superfluid velocity \( v > V_c \) (Critical velocity) \rightarrow turbulence

\( V < V_c \)

\( V \approx V_c \)

\( V > V_c \)

Vortex appears!

Vortex tangle
As a starting point, it may be interesting to know how superfluid steady flow state is broken.

The simplest model: 1-dim. **Non-linear Schrödinger model**

\[
i \partial_t \phi - i v \partial_x \phi = -\partial_{xx} \phi - \phi + |\phi|^2 \phi + U(x) \phi
\]

**Weakly interacting Bose-Einstein superfluid model**

Hakim (1997) \[ U(x) = g \delta(x), \quad g > 0 \] **repulsive potential**
Hakim (1997)

When \( g < g_c \) two steady flow solutions appear.

When \( g = g_c \) the two solutions coalesce and disappear.

\[ g > g_c \text{ the gray soliton solutions are created by the obstacle.} \]

Saddle node bifurcation

Gray Soliton
What happens in holographic model?

A. Ishibashi, K. M., T. Okamura (work in progress)

\[ \mathcal{L} = -|\nabla \psi - iA\psi|^2 - m^2|\psi|^2 - V(x, u)|\psi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \]

We analytically solve the equations and confirmed that saddle node bifurcation always occurs.

In detail, we give a talk in VIII BH workshop!

Onset of turbulence!
Rotating AdS BHs with momentum

Superradiance instability

Obstacle instability

Rotating BH turbulence!

Gray Soliton

Thank you!