EBERHARD KARLS UNIVERSITÄT TÜBINGEN

Gravitational Waves

from Rotational Instabilities of Compact Stars

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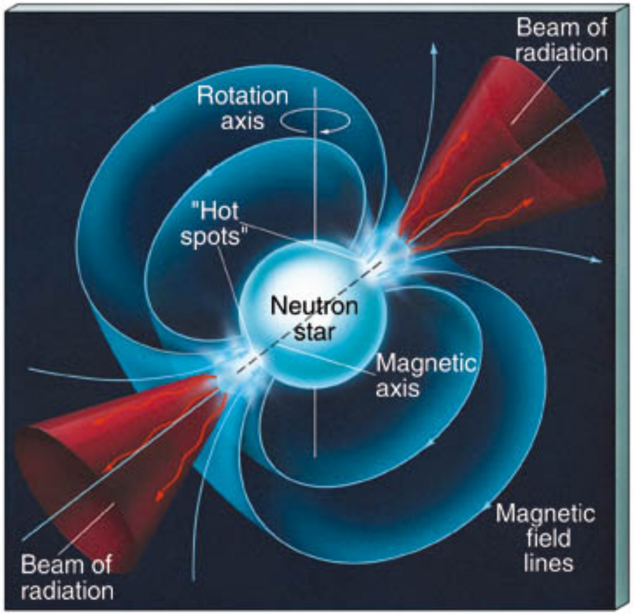
Neutron Stars

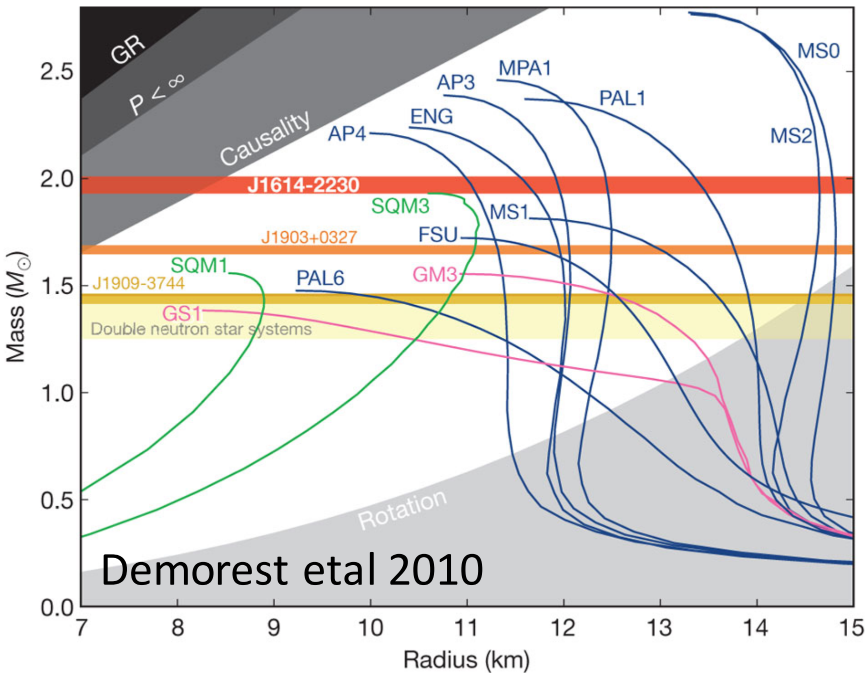
- **Neutron stars** are **stellar remnants** resulting from the gravitational collapse of massive stars in supernova events.
- They are the **most compact stars** known to exist in the universe.
- They have **densities equal to that of the early universe** and **gravity similar to that of a black hole.**

- **Conjectured** 1931
- **Discovered** 1967
- **Known** 2500+
- **Mass** $1.2–2M_{\odot}$
- **Radius** 8-14 km
- **Density** 10^{15} g/cm^3
- **In our Galaxy** ~10^8

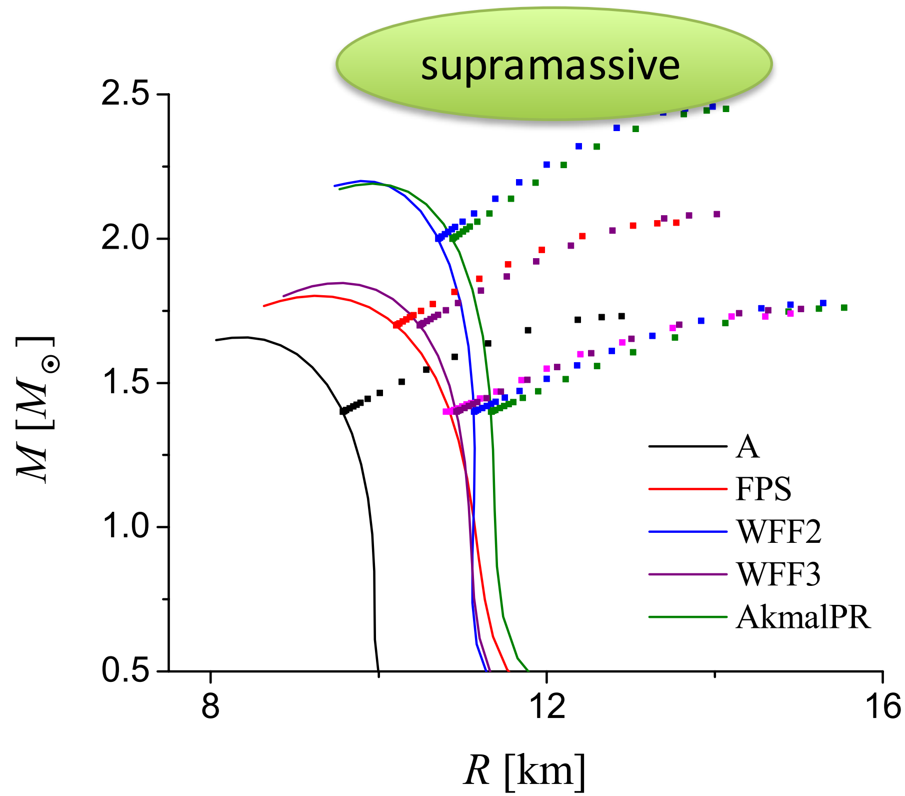


Neutron Stars: Mass vs Radius

Static Models



Rotating Models



$M_{max} \simeq 1.20271 M_{TOV}$

Breu-Rezzolla 2015

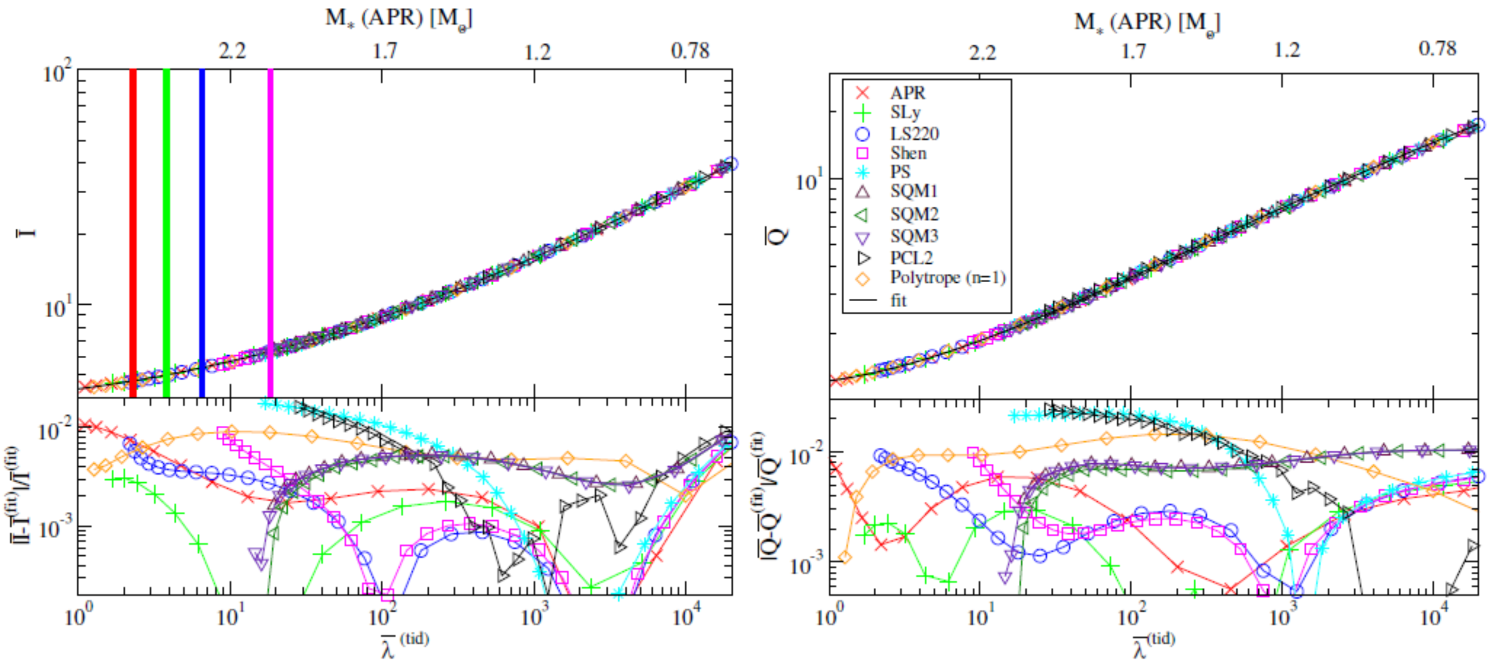
Neutron Stars & "universal relations"

Need for relations between the "observables" and the "fundamentals" of NS physics

Average Density	$\bar{\rho} \sim M / R^3$	
Compactness	$z \sim M / R$	$\eta = \sqrt{M^3 / I}$
Moment of Inertia	$I \sim MR^2$	$I \sim J / \Omega$
Quadrupole Moment	$Q \sim R^5 \Omega^2$	
Tidal Love Numbers	$\lambda \sim I^2 Q$	

I-Love-Q relations

EOS independent relations were derived by Yagi & Yunes(2013) for non-magnetized stars in the slow-rotation and small tidal deformation approximations.

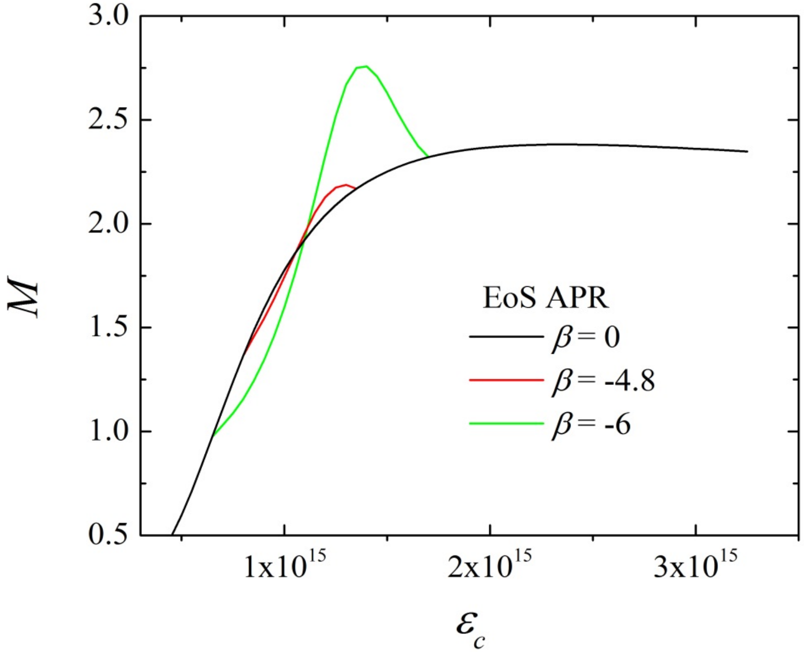


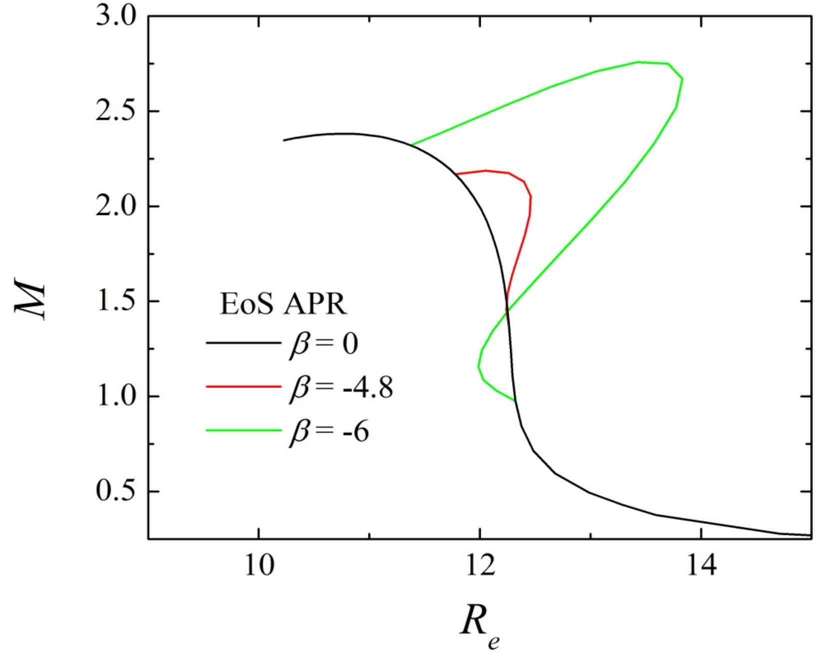
… the relations proved to be valid (*with appropriate normalizations*) even for *fast rotating* and *magnetized* stars

STT of gravity – Neutron Stars

Spontaneous Scalarizarion is possible for β<-4.35
(Damour+Esposito-Farese 1993)

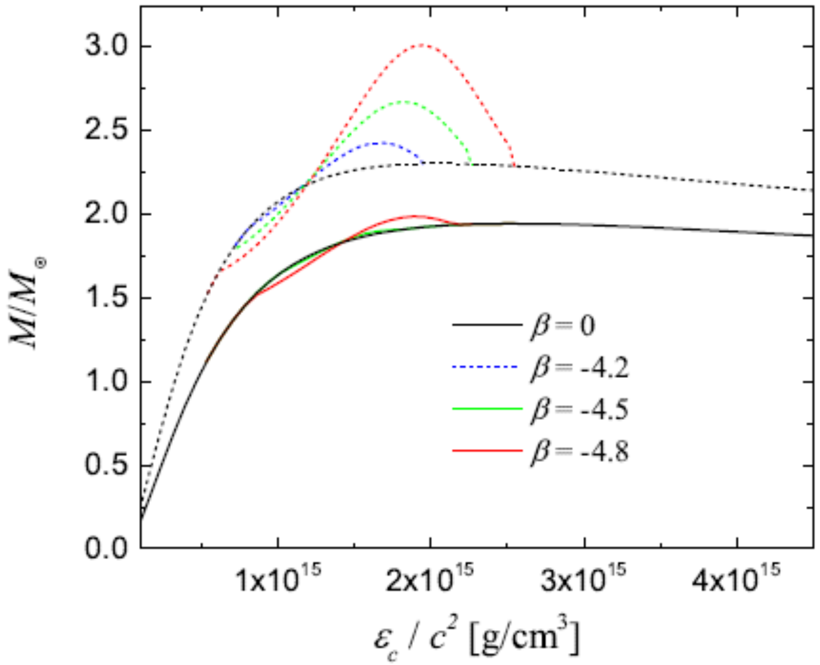
Properties of the static scalarized neutron stars

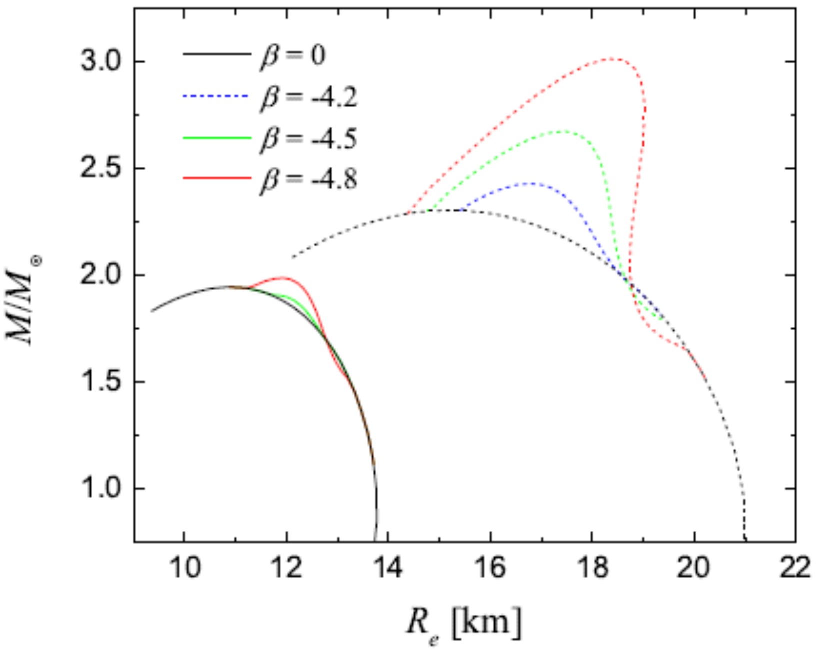




The solutions with nontrivial scalar field are *energetically more favorable* than their GR counterpart (Harada 1997, Harada 1998, Sotani+Kokkotas 2004)**.**

STT of gravity – Fast Rotating Stars



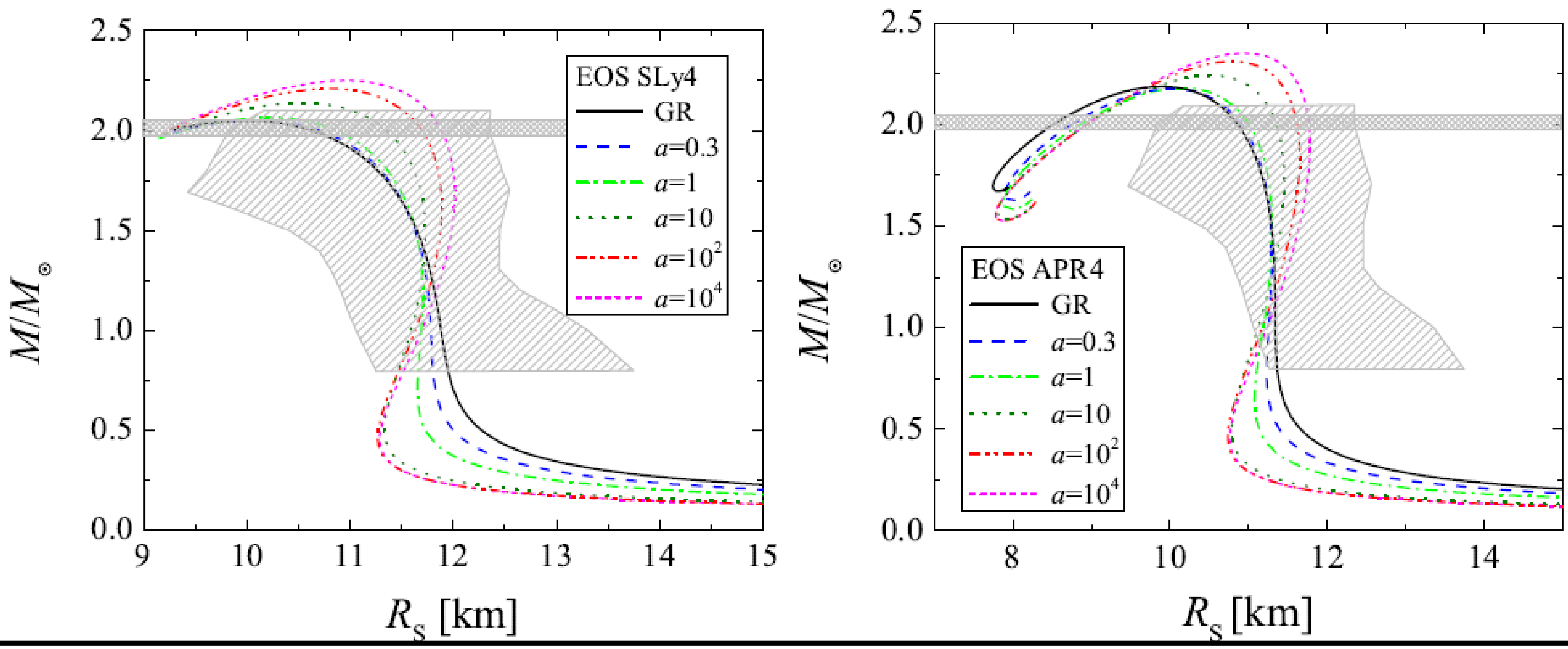


- **The effect of scalarization is *much stronger* for fast rotation.**
- **Scalarized solutions exist for a *much larger range of parameters* than in the static case**

Doneva, Yazadjiev, Stergioulas, Kokkotas 2013

NSs in f(R)-gravity: Static Models

$$f(R) = R + aR^2$$



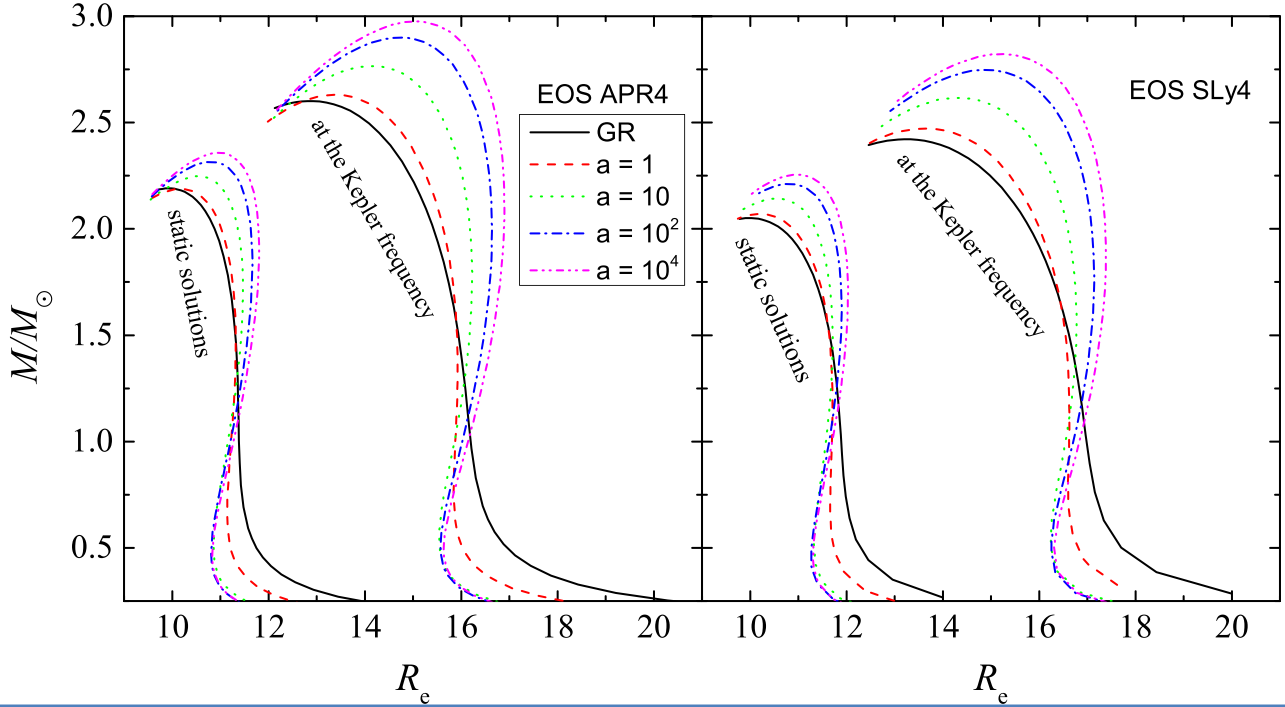
- The differences between the R^2 and GR are comparable with the uncertainties in the nuclear matter equations of state.
- The current observations of the NS masses and radii alone can not put constraints on the value of the parameters a, **unless the EoS is better constrained in the future.**

Yazadjiev, Doneva, Kokkotas, Staykov (2014)

NSs in f(R)-gravity: Fast Rotation

$$f(R) = R + aR^2$$

Mass of radius diagrams for two realistic EOS



Difficult to set constraints on the *f* (*R*) theories using measurement of the neutron star **M** and **R** alone, until the EOS can be determined with smaller uncertainty.

Yazadjiev, Doneva, Kokkotas, (2015)

Neutron Star: "ringing"

p-modes: main restoring force is the **pressure** (**f-mode**) ***(>1.5 kHz)***

$$\sigma \approx \sqrt{\frac{GM}{R^3}}$$

Inertial modes: (r-modes) main restoring force is the **Coriolis force**

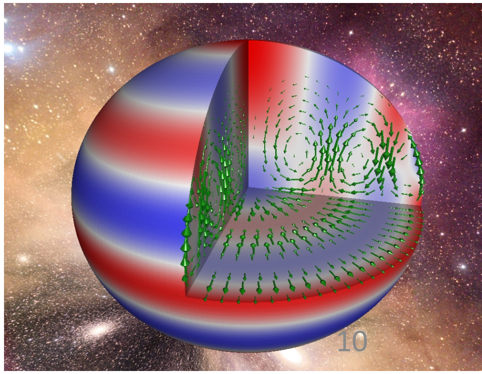
$$\sigma \approx \Omega$$

w-modes: pure **space-time modes** (only in GR) ***(>5kHz)***

$$\sigma \approx \frac{1}{R}\left(\frac{GM}{Rc^2}\right)$$

Torsional modes (t-modes) ***(>20 Hz)*** shear deformations. **Restoring force, the weak Coulomb force of the crystal ions.**

$$\sigma \approx \frac{v_S}{R} \sim 16\ \ell\ \text{Hz}$$



… and many more

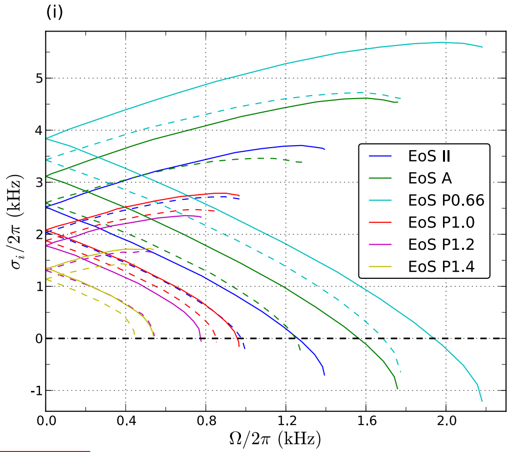
shear, g-, Alfven, interface, … modes

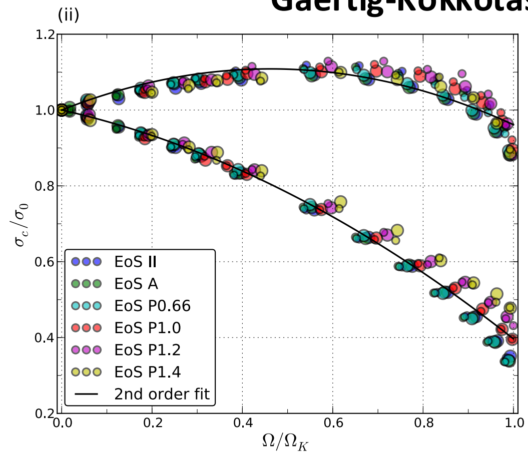
f-modes: Asteroseismology

We can produce **empirical relation** relating the parameters of the *rotating neutron stars* to the observed frequencies.

Gaertig-Kokkotas 2008, 2010, 2011

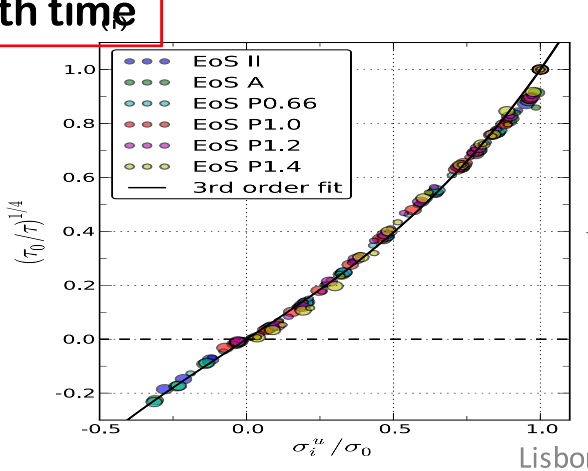
Frequency

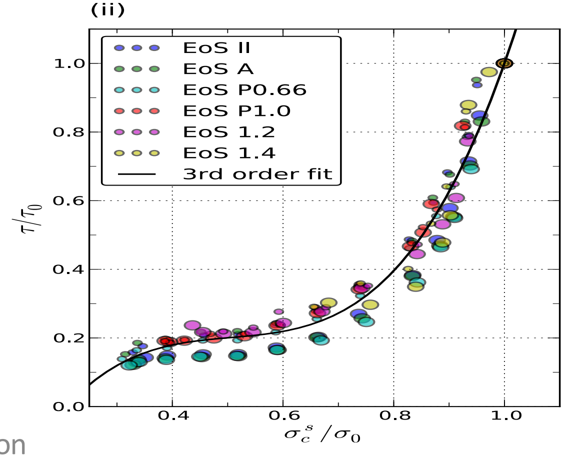




Cowling Approximation

Damping/Growth time





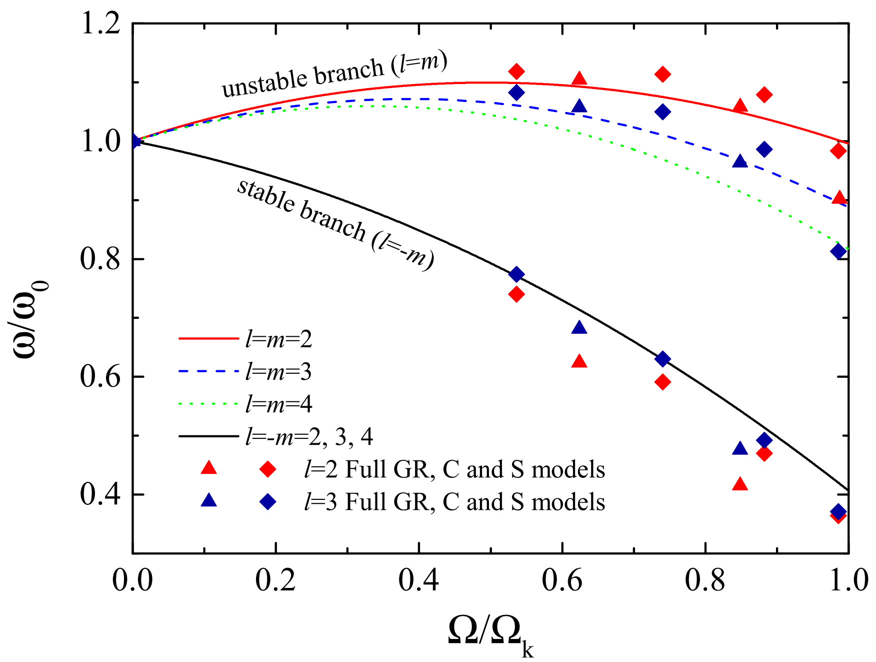
Asteroseismology: Realistic EoS

Doneva, Gaertig, KK, Krüger (2013)

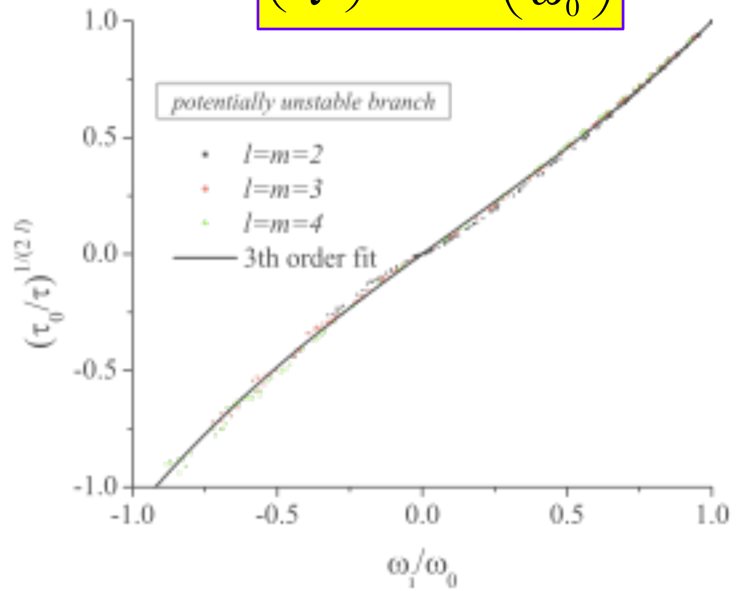
$$\left(\frac{\omega_c}{\omega_0}\right)_{\ell=2,3,4} \approx f\left(\frac{\Omega}{\Omega_K}\right)$$

Nearly "universal" fitting formulae for :
- **the frequencies**
- **the damping times**
- **Independent of GR or Cowling**

$$\left(\frac{\tau_0}{\tau}\right)^{1/2\ell} \approx f\left(\frac{\omega_i}{\omega_0}\right)$$



Oscillation frequencies



Damping/Growth Times

Asteroseismology

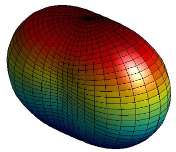
Stable Branch

$\ell = 2, 3, 4$

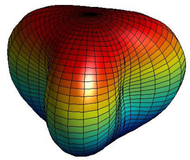
$$\frac{\omega_c^s}{\omega_0} = 1 - 0.235\left(\frac{\Omega}{\Omega_K}\right) - 0.358\left(\frac{\Omega}{\Omega_K}\right)^2$$

Unstable Branch

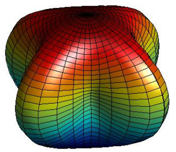
$$\frac{\omega^u_{c\ l=2}}{\omega_0} = 1 + 0.402\left(\frac{\Omega}{\Omega_K}\right) - 0.406\left(\frac{\Omega}{\Omega_K}\right)^2$$



$$\frac{\omega^u_{c\ l=3}}{\omega_0} = 1 + 0.373\left(\frac{\Omega}{\Omega_K}\right) - 0.485\left(\frac{\Omega}{\Omega_K}\right)^2$$



$$\frac{\omega^u_{c\ l=4}}{\omega_0} = 1 + 0.360\left(\frac{\Omega}{\Omega_K}\right) - 0.543\left(\frac{\Omega}{\Omega_K}\right)^2$$



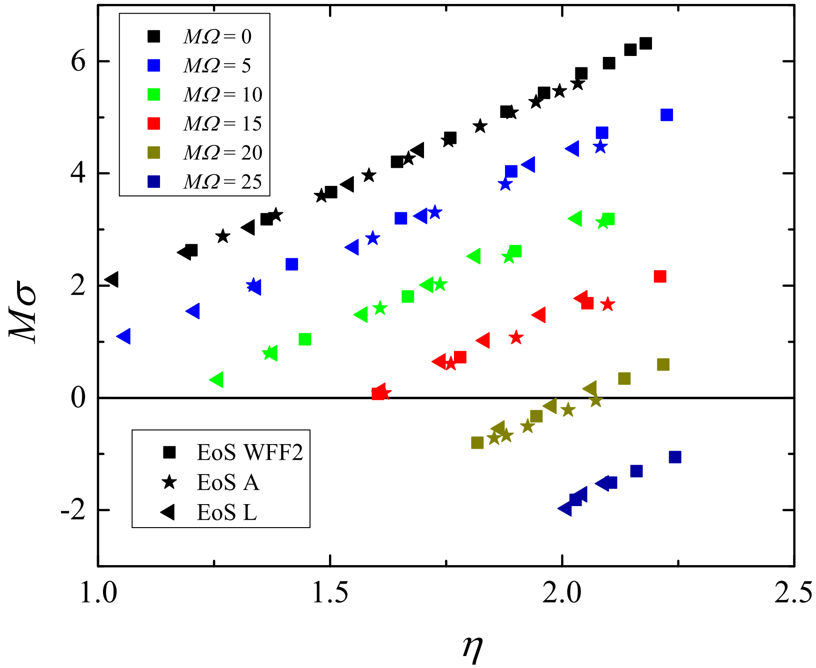
Unstable Branch

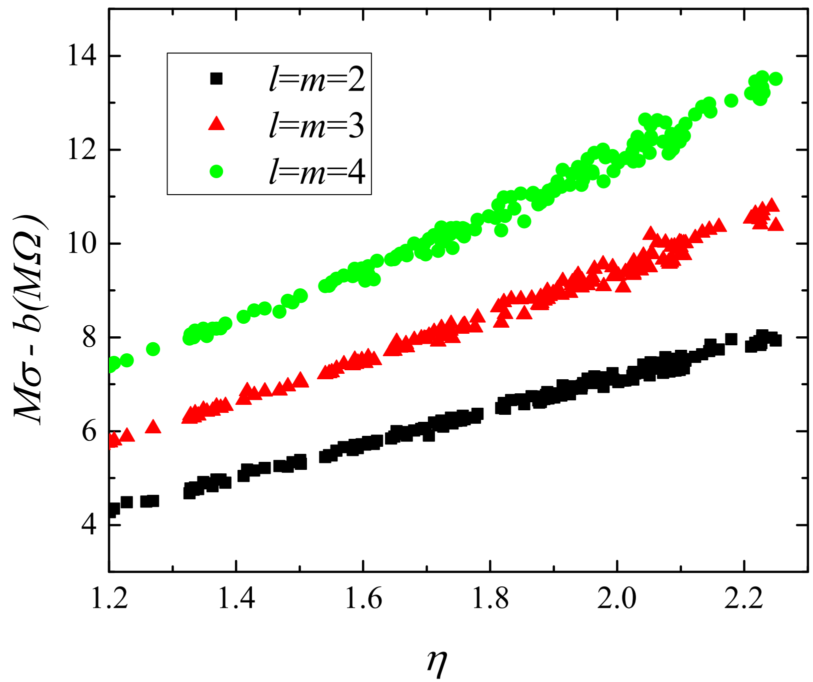
$$\frac{\tau_0}{\tau} = \mathrm{sgn}(\omega_i^u)\left(0.900\left(\frac{\omega_i^u}{\omega_0}\right) - 0.057\left(\frac{\omega_i^u}{\omega_0}\right)^2 + 0.157\left(\frac{\omega_i^u}{\omega_0}\right)^3\right)^{2l}$$

Doneva, Gaertig, KK, Krüger (2013)

Asteroseismology: alternative scalings

$$M\sigma_i^{unst} = \left[(0.56 - 0.94\,\ell) + (0.08 - 0.19\,\ell)M\Omega + 1.2(\ell + 1)\eta\right]$$



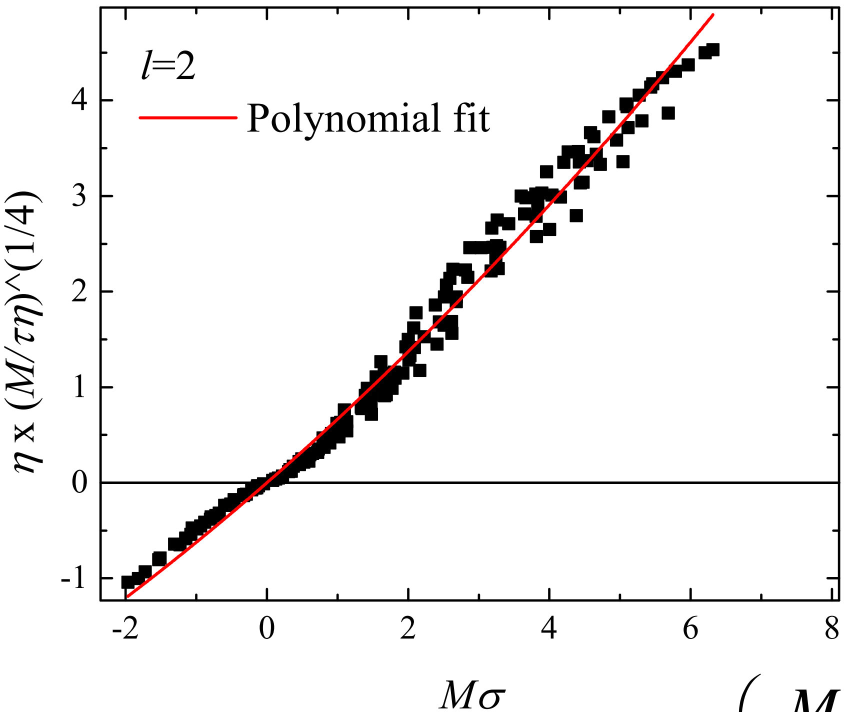


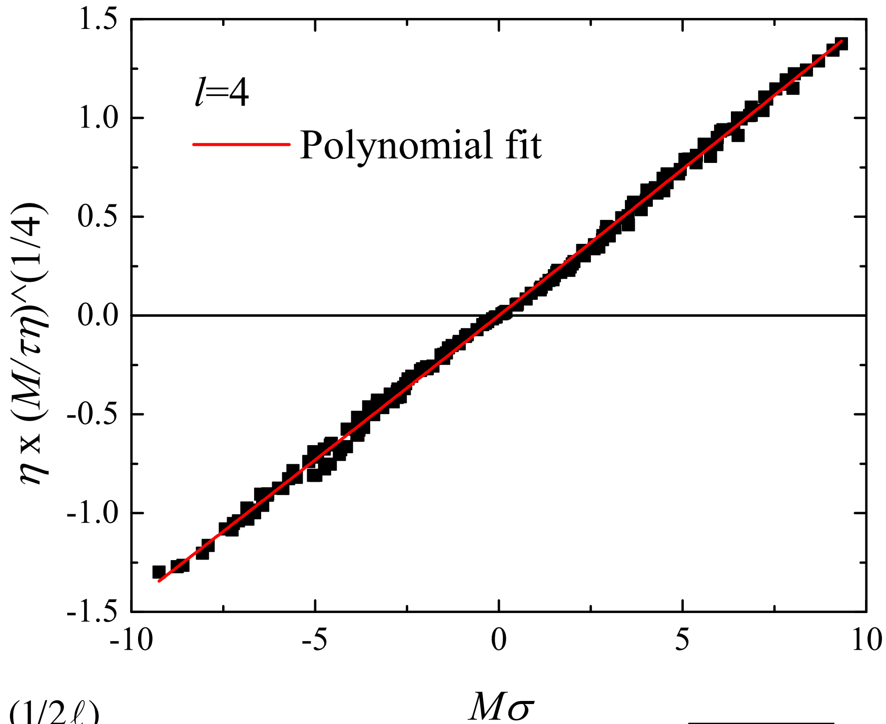
The l = 2 f-mode oscillation frequencies as functions of the parameter η

$$\eta = \sqrt{M^3/I} \approx M/R$$

Doneva-Kokkotas 2015

Asteroseismology: alternative scalings





$$\eta = \sqrt{M^3 / I}$$

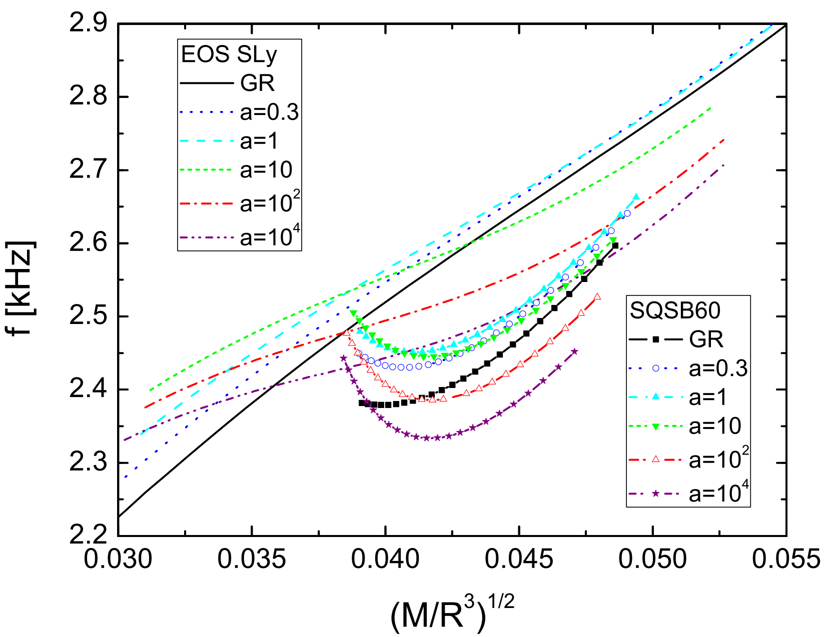
The **normalized damping time** $$\eta\left(\frac{M}{\tau\eta^2}\right)^{(1/2\ell)}$$

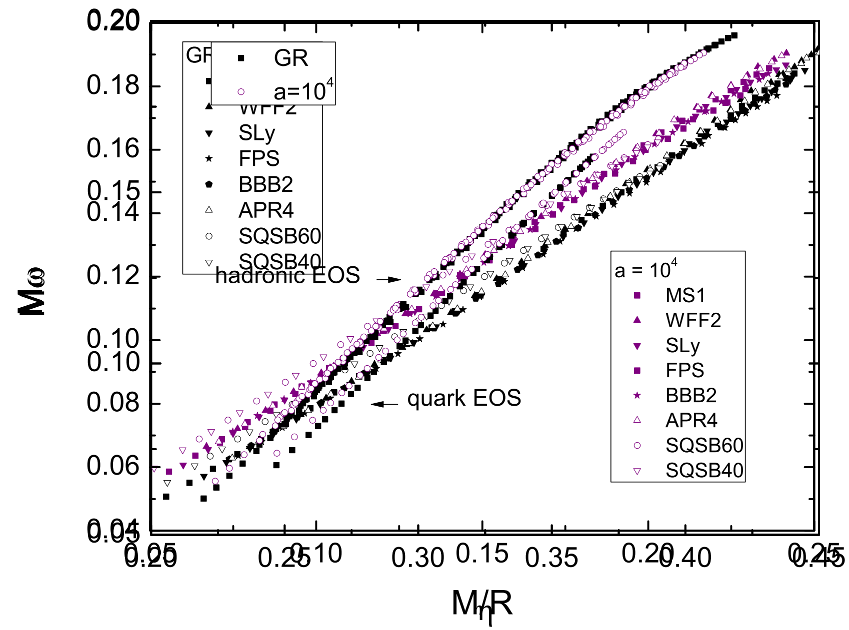
as a function of the normalized oscillation frequency ***Mσ*** for l = m = 2 & l = m = 4 f-modes.

Doneva-KK 2015

Asteroseismology:

Alternative Theories of Gravity





- **The maximum deviation between the f-mode frequencies in GR and R^2 gravity is up to 10% and depends on the value of the R^2 gravity parameter *a*.**
- **Alternative normalizations show nicer relations**

$$\eta = \sqrt{M^3 / I}$$

The CFS instability

Chandrasekhar 1970: Gravitational waves lead to a secular instability

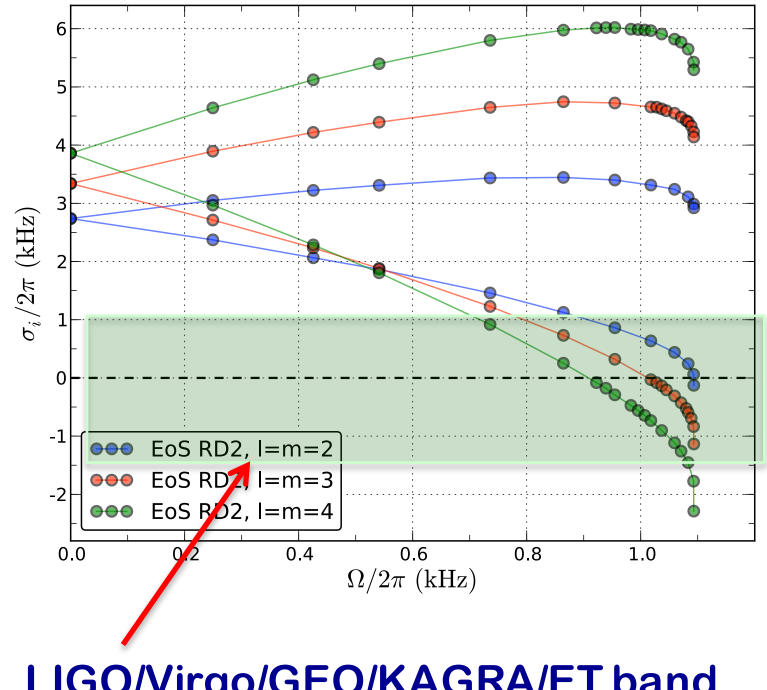
Friedman & Schutz 1978: The instability is generic, modes with sufficiently large *m* are unstable.

A neutral mode of oscillation signals the onset of CFS instability

Gaertig+Kokkotas 2008

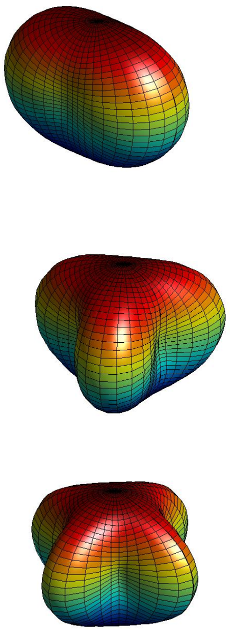
- ✓ Radiation drives a mode unstable if the mode pattern moves backwards according to an observer on the star *($J_{rot}<0$)*, but forwards according to someone far away *($J_{rot}>0$)*.
- ✓ They radiate positive angular momentum, thus in the rotating frame the angular momentum of the mode increases leading to an increase in mode's amplitude.

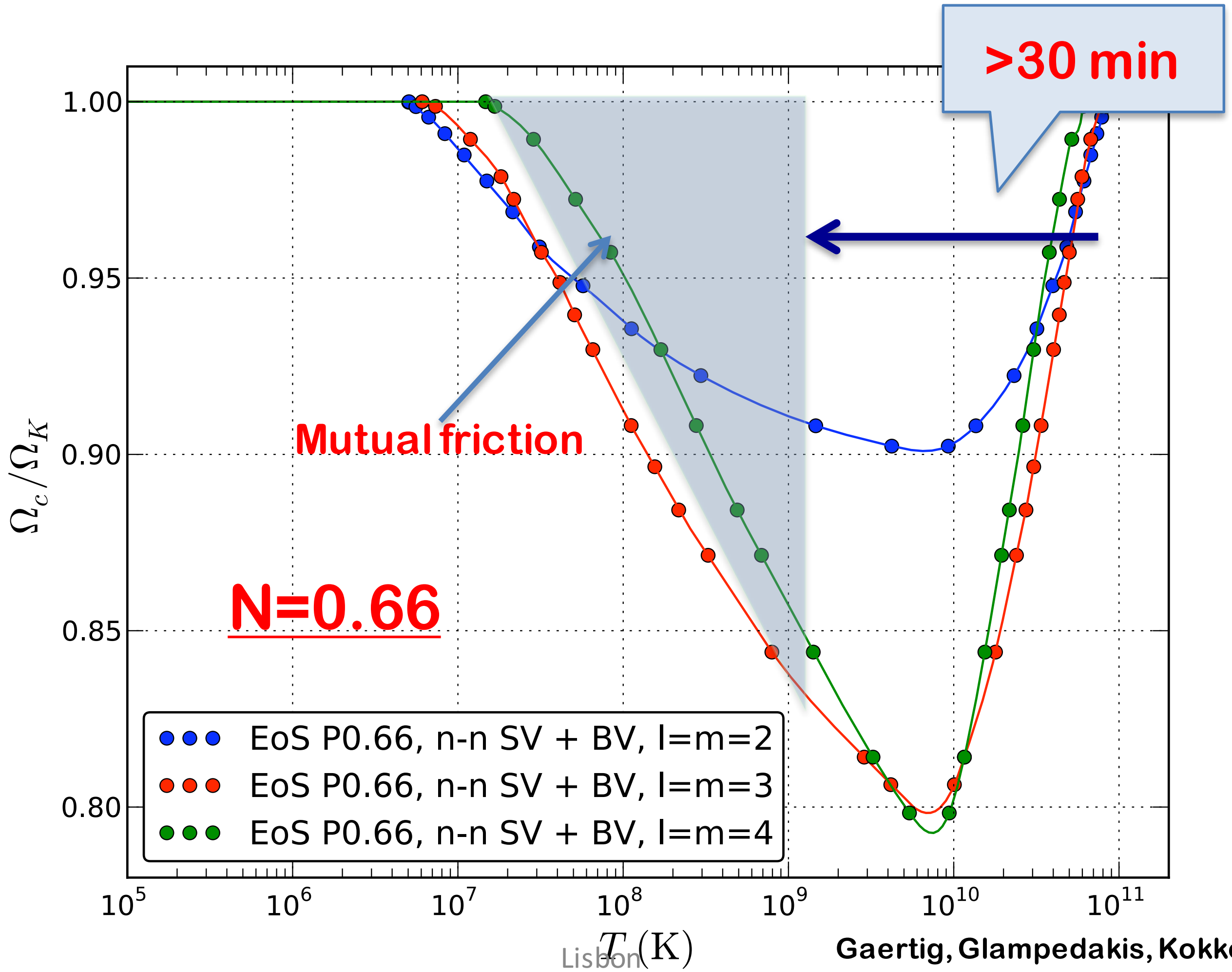
$$\frac{\omega_{\rm in}}{m} = -\frac{\omega_{\rm rot}}{m} + \Omega$$



Instability Window

- ✓ For the 1st time we have the window of f-mode instability in GR
- ✓ Newtonian: (l=m=4) Ipser-Lindblom (1991)





Gaertig, Glampedakis, Kokkotas, Zink (2011)

Saturation of the Instability

Parametric Resonance

$$\dot{Q}_\alpha = \gamma_\alpha Q_\alpha + i\omega_\alpha \mathcal{H}\, Q_\beta Q_\gamma\, e^{-i\Delta\omega t}$$
$$\dot{Q}_\beta = \gamma_\beta Q_\beta + i\omega_\beta \mathcal{H}\, Q_\gamma^* Q_\alpha\, e^{i\Delta\omega t}$$
$$\dot{Q}_\gamma = \gamma_\gamma Q_\gamma + i\omega_\gamma \mathcal{H}\, Q_\alpha Q_\beta^*\, e^{i\Delta\omega t}$$

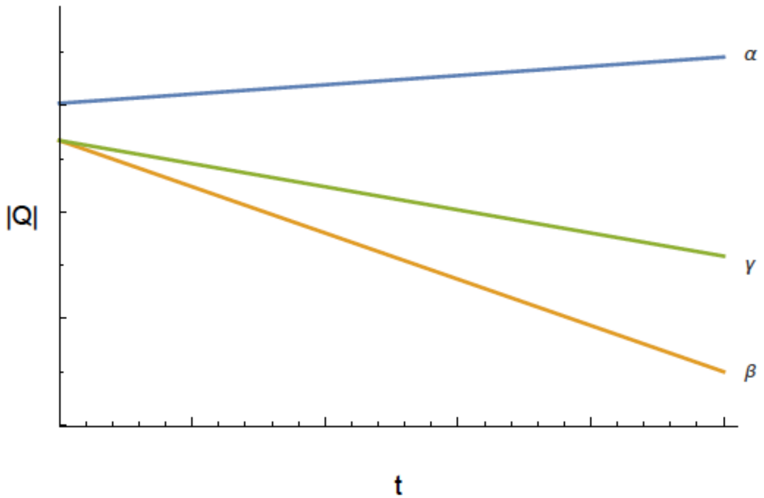
Detuning $\Delta\omega$

Coupling coefficient $\mathcal{H}$

Growth/damping rates γ_i

Detuning $\Delta\omega \equiv \omega_\alpha - \omega_\beta - \omega_\gamma \approx 0$ **resonance condition**

No mode coupling: $\mathcal{H} = 0$ or $\Delta\omega \gg 0$



- Modes evolve independently
- No non-linear interaction

$$\dot{Q}_\alpha = \gamma_\alpha Q_\alpha$$
$$\dot{Q}_\beta = \gamma_\beta Q_\beta$$
$$\dot{Q}_\gamma = \gamma_\gamma Q_\gamma$$

Pnigouras, Kokkotas (2015)

Saturation of the Instability

Parametric Resonance

$$\dot{Q}_\alpha = \gamma_\alpha Q_\alpha + i\omega_\alpha \mathcal{H}\, Q_\beta Q_\gamma\, e^{-i\Delta\omega t}$$
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$$\dot{Q}_\gamma = \gamma_\gamma Q_\gamma + i\omega_\gamma \mathcal{H}\, Q_\alpha Q_\beta^*\, e^{i\Delta\omega t}$$

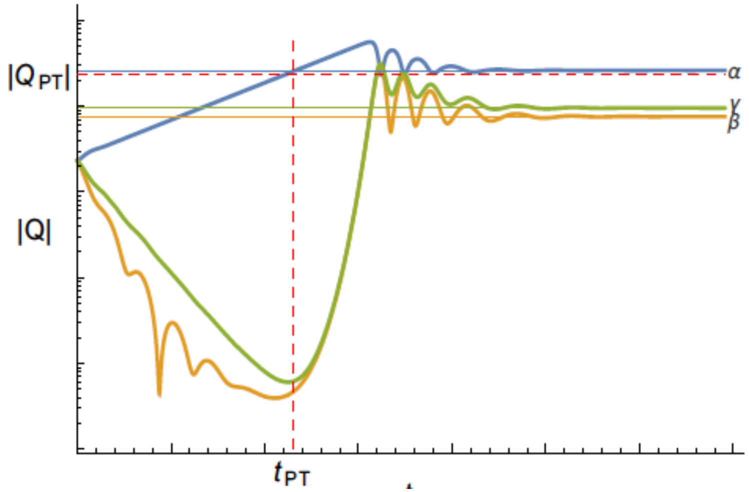
Detuning $\Delta\omega$

Coupling coefficient $\mathcal{H}$

Growth/damping rates γ_i

Detuning $\Delta\omega \equiv \omega_\alpha - \omega_\beta - \omega_\gamma \approx 0$ resonance condition

Parametric resonance: $\mathcal{H} \neq 0$ and $\Delta\omega \approx 0$



- Parent feeds daughters and makes them grow
- *Parametric threshold:* daughters grow when

$$|Q_\alpha|^2 > |Q_{\rm PT}|^2 \equiv \frac{\gamma_\beta \gamma_\gamma}{\omega_\beta \omega_\gamma \mathcal{H}^2}\left[1 + \left(\frac{\Delta\omega}{\gamma_\beta + \gamma_\gamma}\right)^2\right]$$

- $|Q_\alpha^{\rm sat}| \approx |Q_{\rm PT}|$

Pnigouras, Kokkotas (2015)

Saturation of the Instability

Parametric Resonance

$$\dot{Q}_\alpha = \gamma_\alpha Q_\alpha + i\omega_\alpha \mathcal{H}\, Q_\beta Q_\gamma\, e^{-i\Delta\omega t}$$

$$\dot{Q}_\beta = \gamma_\beta Q_\beta + i\omega_\beta \mathcal{H}\, Q_\gamma^* Q_\alpha\, e^{i\Delta\omega t}$$

$$\dot{Q}_\gamma = \gamma_\gamma Q_\gamma + i\omega_\gamma \mathcal{H}\, Q_\alpha Q_\beta^*\, e^{i\Delta\omega t}$$

Detuning $\Delta\omega$

Coupling coefficient $\mathcal{H}$

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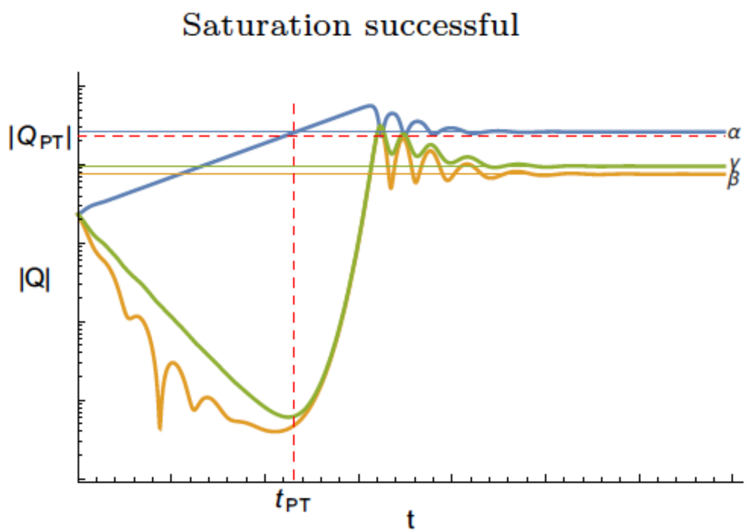
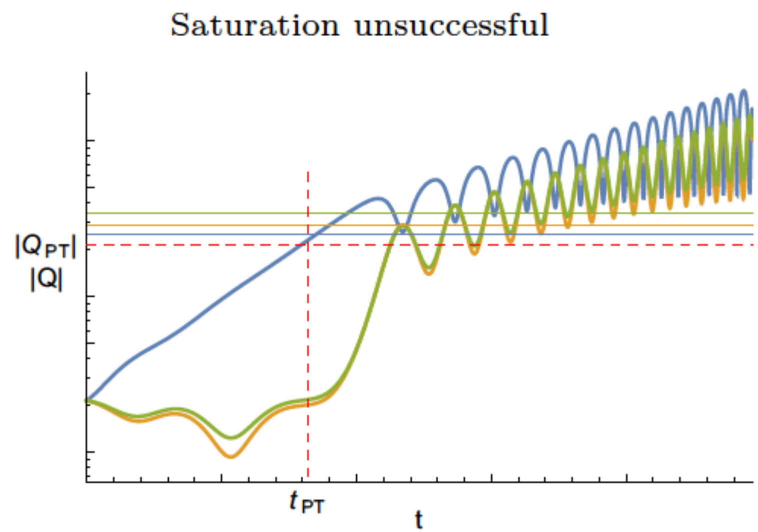
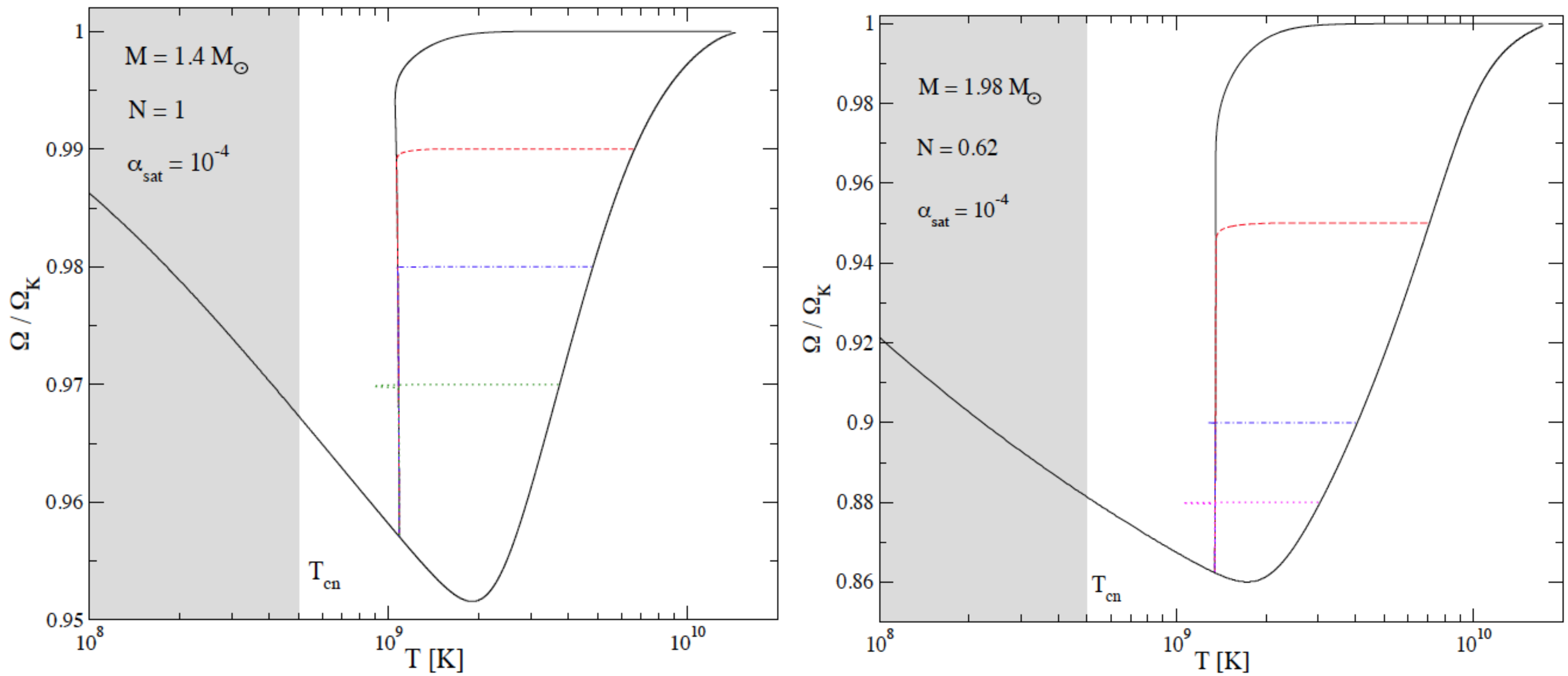




Pnigouras, Kokkotas (2015)

Evolution of a nascent (unstable) NS

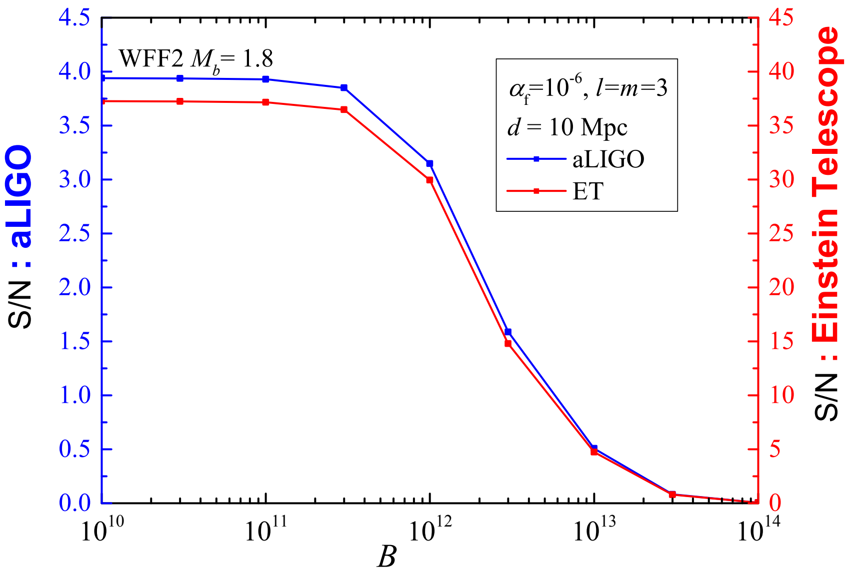


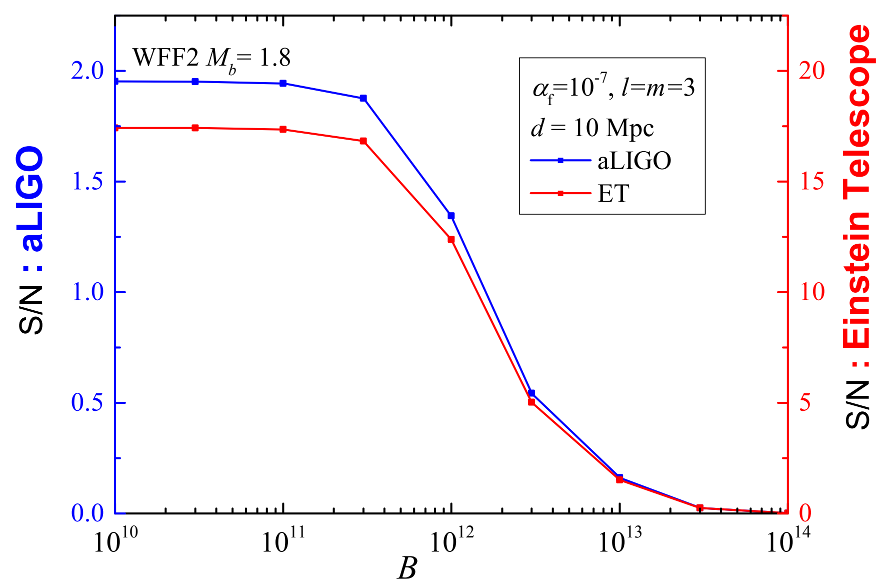
Mutual Friction plays NO ROLE for the f-mode instability

Procedure as described in Owen etal 1998 & Anderson, Jones, KK 2002

Passamonti-Gaertig-KK-Doneva (2013)

Evolution of a nascent (unstable) NS





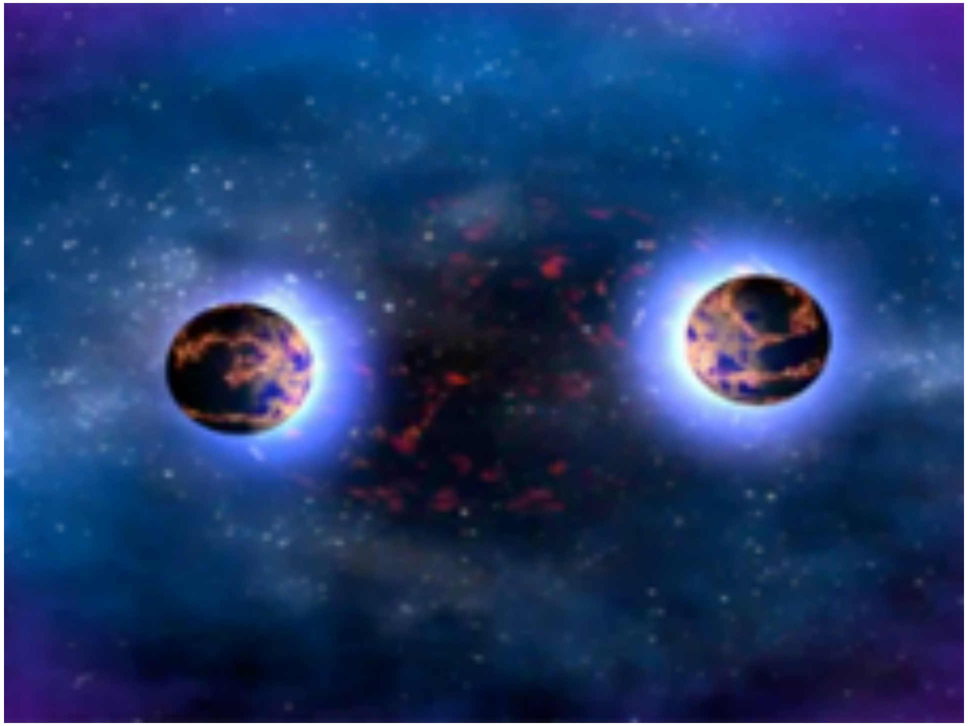
The instability can be potentially observed by events in Virgo cluster

BUT

- Event rate is unknown
- Competiton with r-mode and magnetic field slow-down
- Saturation amplitude is varying during the procces

Passamonti-Gaertig-Kokkotas-Doneva (2013)

A GRAVITATIONAL WAVE AFTERGLOW IN BINARY NEUTRON STAR MERGERS



Binary Neutron Star Mergers
the standard scenario

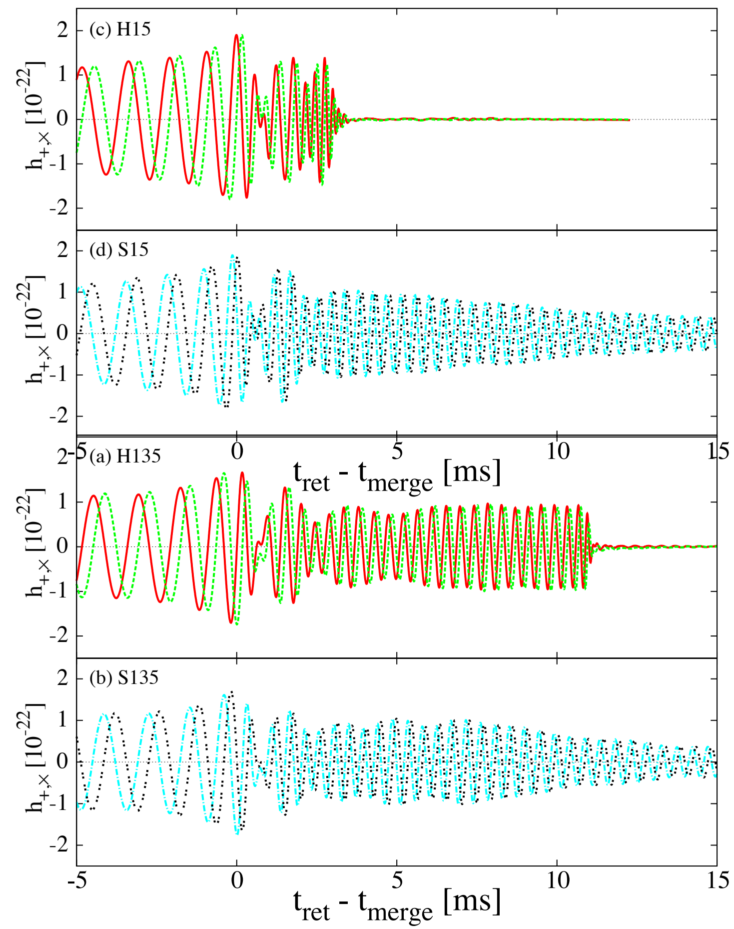
I. After the merging the final body most probably will be a supramassive NS (2.5-3 $M_{\odot}$)

II. The body will be differentially rotating

III. The "averaged" magnetic field will amplified due to MRI (up to 3-4 orders of magnitude)

IV. The strong magnetic field and the emission of GWs will drain rotational energy

V. This phase will last only a few tenths of msecs and can potentially provide information for the Equation of State (EOS)

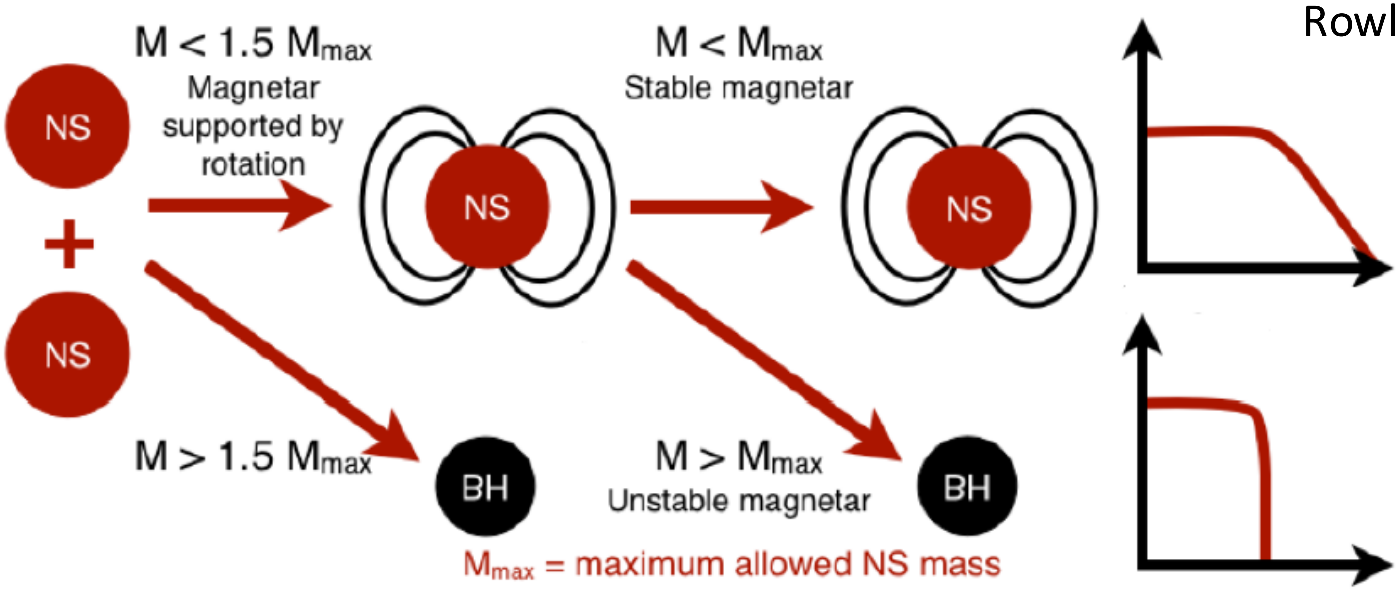


Kiuchi, Sekiguchi, Kyutoku, Shibata2012

Post-Merger Scenario

Three different outcomes of the merger of a BNS merger

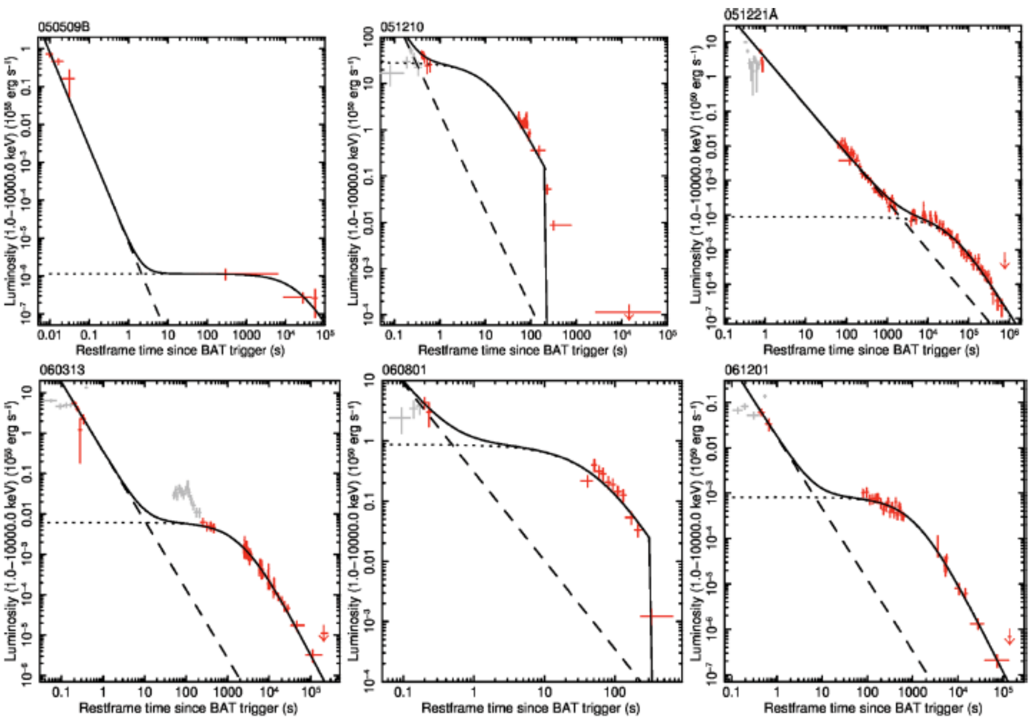
Rowlinson 2013



- ✓ The outcome is dependent upon the mass (M) of the central object formed and the maximum possible mass of a neutron star (M_{max}).
- ✓ On the right are sketches of the expected light-curves if a stable (top) or an unstable magnetar (bottom) is formed.

Short γ-ray light curves

- The favored progenitor model for SGRBs is the merger of two NSs that triggers an explosion with a burst of collimated γ-rays.
- Following the initial prompt emission, some SGRBs exhibit a plateau phase in their X-ray light curves that indicates additional energy injection from a central engine, believed to be a rapidly rotating, highly magnetized neutron star.
- The collapse of this "protomagnetar" to a black hole is likely to be responsible for a steep decay in X-ray flux observed at the end of the plateau.

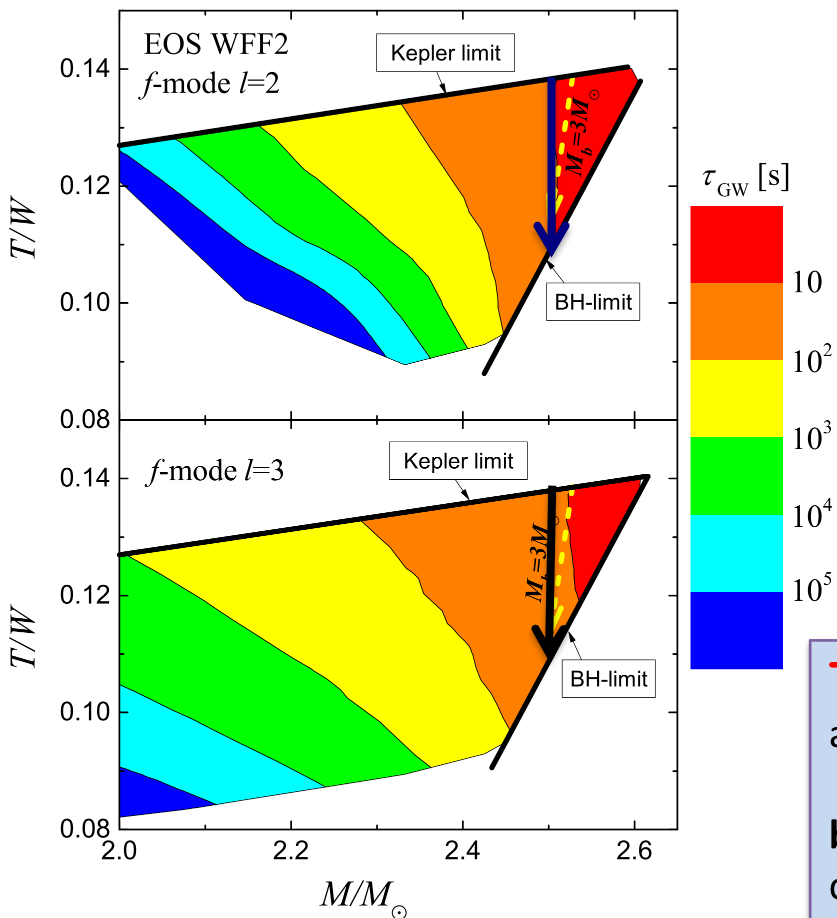


Rowlinson, O'Brien, Metger, Tanvir, Levan 2013

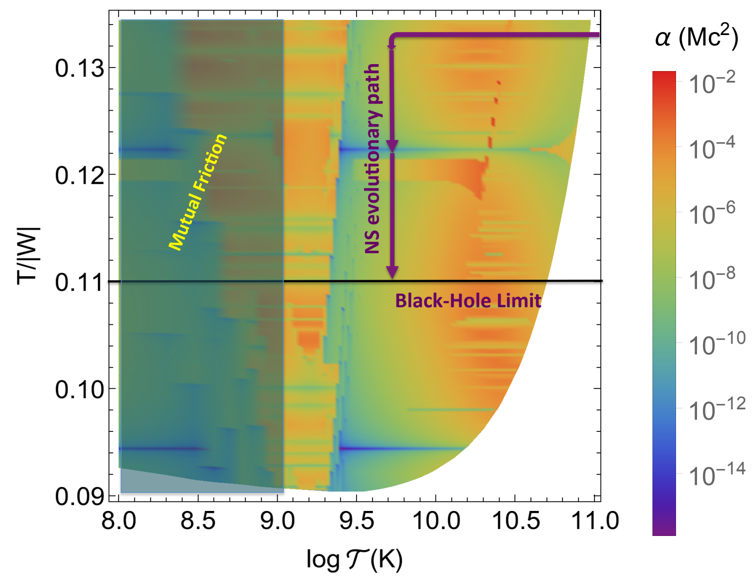
Post-Merger NS: secular instability

Doneva-KK-Pnigouras 2015

The post-merger object **is still stable** and rotates at nearly Kepler **periods < 1ms**



The evolution into the instability window



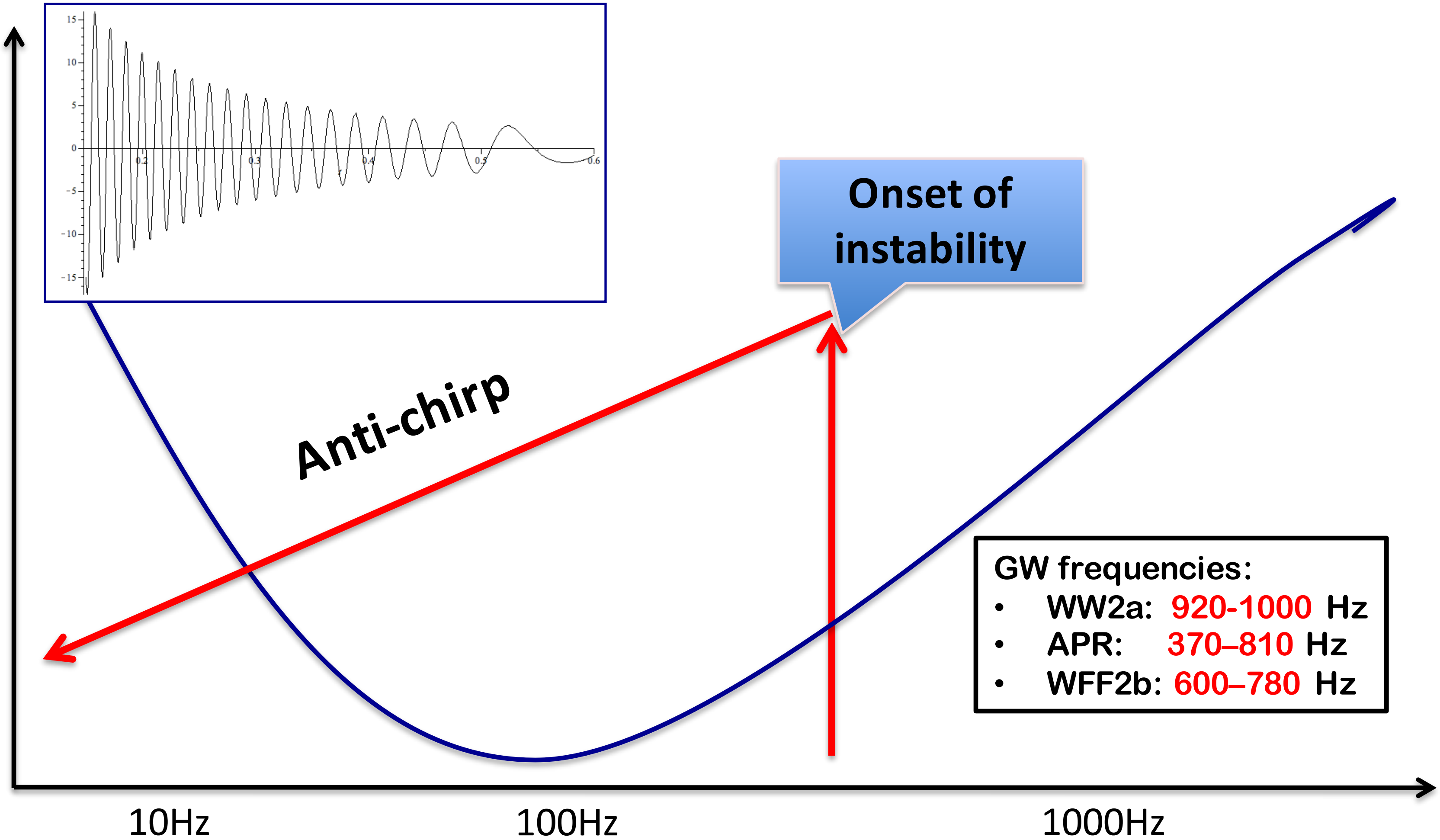
The detailed evolution depends:

a) Strength of the **magnetic field** (averaged may reach $10^{15\text{-}16}$ G !)

b) Equation of state of the post-merger neutron star

c) Fine details of **the non-linear dynamics** (three mode coupling, shock waves, wave breaking)

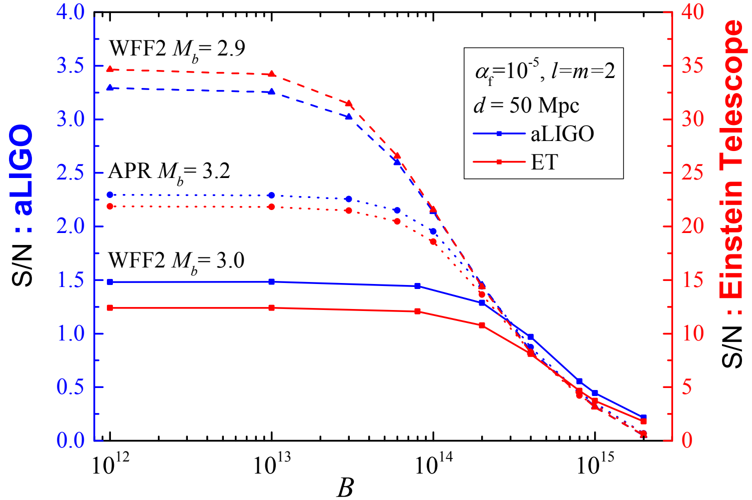
F-mode instability: Detectability

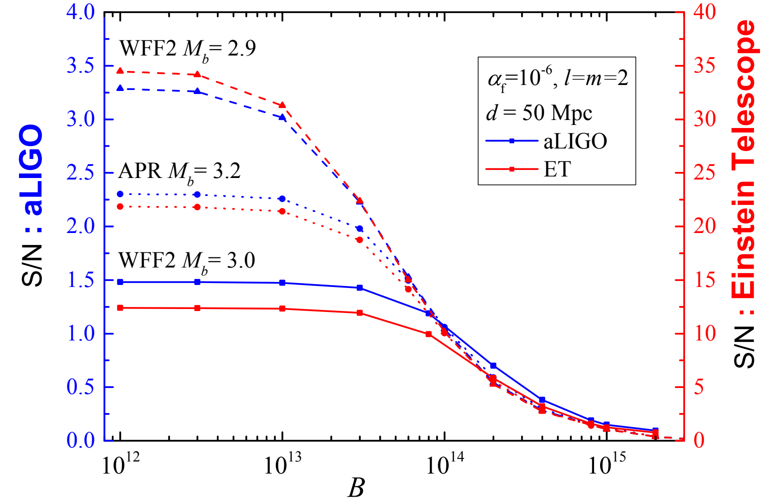


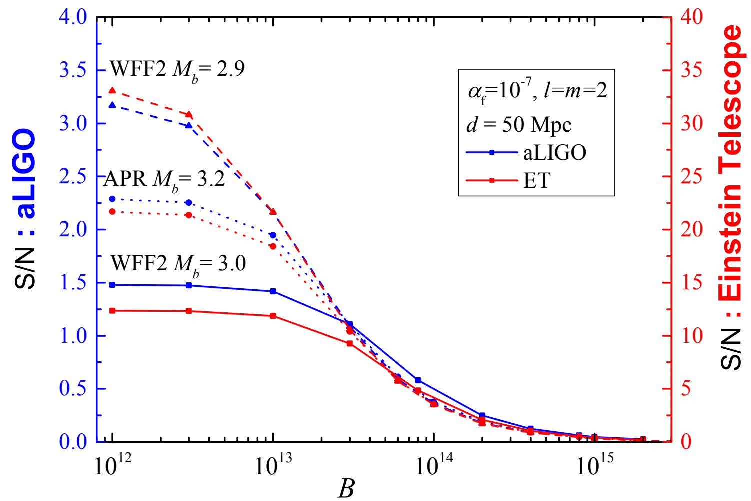
GW frequencies:

- WW2a: 920-1000 Hz
- APR: 370–810 Hz
- WFF2b: 600–780 Hz

Post-Merger NS: GW Afterglow







Competition between the B-field and the secular instability

GW frequencies:

WW2a: 920-1000 Hz

APR: 370–810 Hz

WFF2b: 600–780 Hz

Doneva-KK-Pnigouras 2015

Conclusions

- ✓ The influence of alternative/extended theories of gravity on NS parameters is much more pronounced for fast rotation.
- ✓ Difficult to set constraints on theories using measurement of the neutron star M and R alone, until the EOS can be determined with smaller uncertainty.

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Conclusions

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- ✓ Difficult to set constraints on theories using measurement of the neutron star M and R alone, until the EOS can be determined with smaller uncertainty.

- ✓ Asteroseismology for fast rotating stars is possible
- ✓ Asteroseismology for magnetars is promising

- ✓ f-mode instability can be potentially a good source for GWs for supramassive NS
- ✓ The efficincy depends on the saturation amplitude and strength of B-field.