Fake hair for black holes

E Babichev and CC gr-qc/1312.3204
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General Relativity and gravity modification

- GR is a unique mathematically consistent theory (Lovelock theorem).
- GR has remarkable agreement with weak and strong gravity experiments at local scales
- GR at cosmological scales requires a fine tuned tiny cosmological constant
- Enormous difference in local and cosmological scales. Could it be that gravity is modified at the IR?

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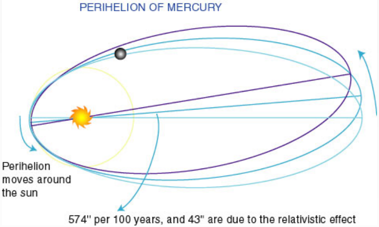

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-A next order correction with one additional parameter was enough to save Newton's laws (at the experimental precision of the time..)
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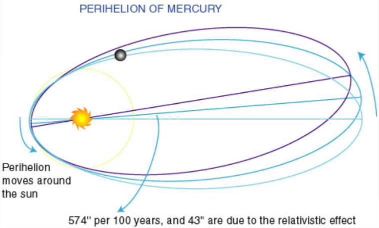

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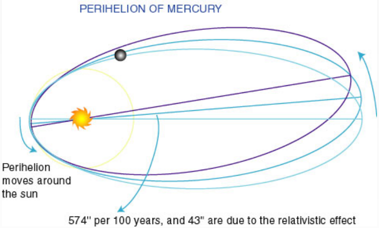

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- Since GR is unique we need to introduce new and genuine gravitational degrees of freedom!
- They must not lead to higher derivative equations of motion. For then additional degrees of freedom are ghosts and vacuum is unstable (Ostrogradski theorem 1850 [Woodard 2006, Rubakov 2014])
- Matter must not directly couple to novel gravity degrees of freedom. Matter sees only the metric and evolves in metric geodesics. As such EEP is preserved and space-time can be put locally in an inertial frame.
- Novel degrees of freedom need to be screened from local gravity experiments. Need a well defined GR local limit (Chameleon [Khoury 2013], Vainshtein [Babichev and Deffayet 2013]).
- Exact solutions essential in modified gravity in order to understand strong gravity regimes and novel characteristics. Need to deal with no hair paradigm.
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Possible modified gravity theories

- Assume extra dimensions : Extension of GR to Lovelock theory with modified yet second order field equations [Deruelle et.al '03, Garraffo et.al. '08, CC '09]. Braneworlds DGP model RS models, Kaluza-Klein compactification
- Graviton is not massless but massive! dRGT theory and bigravity theory. Theories are unique. [C DeRham, 2014]
- 4-dimensional modification of GR: Scalar-tensor theories, $f(R)$, Galileon/Hornedski theories [Sotiriou 2014, CC 2014].
- Lorentz breaking theories: Horava gravity, Einstein Aether theories [Audren, Blas, Lesgourgues and Sibiryakov]
- Theories modifying geometry: inclusion of torsion, choice of geometric connection [Zanelli '08, Olmo 2012]

Scalar-tensor theories

- are the simplest modification of gravity with one additional degree of freedom
- Admit a uniqueness theorem due to Horndeski 1973
- contain or are limits of other modified gravity theories. $F(R)$ is a scalar tensor theory in disguise
- (Can) have insightful screening mechanisms (Chameleon, Vainshtein)
- Include terms that can screen classically a big cosmological constant (Fab 4 [CC, Copeland, Padilla and Saffin 2012])

What is the most general scalar-tensor theory

with second order field equations [Horndeski 1973], [Deffayet et.al.]?

Horndeski has shown that the most general action with this property is

$$S_H = \int d^4x\sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$$

$$L_2 = K(\phi, X),$$
$$L_3 = -G_3(\phi, X)\Box\phi,$$
$$L_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],$$
$$L_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

the G_i are unspecified functions of ϕ and $X \equiv -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX} \equiv \partial G_i/\partial X$.

- In fact same action as covariant Galileons [Deffayet, Esposito-Farese, Vikman]
- Theory screens generically scalar mode locally by the Vainshtein mechanism

Black holes have no hair

During gravitational collapse...

Black holes eat or expel surrounding matter

their stationary phase is characterized by a limited number of charges

and no details

black holes are bald...

No hair arguments/theorems dictate that adding degrees of freedom lead to singular solutions...

For example in vanilla scalar-tensor theories black hole solutions are GR black holes with constant scalar.

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Conformally coupled scalar field

- Consider a conformally coupled scalar field ϕ:

$$S[g_{\mu\nu}, \phi, \psi] = \int_{\mathcal{M}} \sqrt{-g} \left(\frac{R}{16\pi G} - \frac{1}{2}\partial_\alpha \phi \partial^\alpha \phi - \frac{1}{12} R\phi^2 \right) \mathrm{d}^4 x + S_m[g_{\mu\nu}, \psi]$$

- **Invariance of the EOM of ϕ under the conformal transformation**

$$\begin{cases} g_{\alpha\beta} \mapsto \tilde{g}_{\alpha\beta} = \Omega^2 g_{\alpha\beta} \\ \phi \mapsto \tilde{\phi} = \Omega^{-1} \phi \end{cases}$$

- There exists a black hole geometry with non-trivial scalar field and secondary black hole hair.
The BBMB solution [N. Bocharova et al.-70 , J. Bekenstein-74]

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- Static and spherically symmetric solution

$$ds^2 = -\left(1-\frac{m}{r}\right)^2 dt^2 + \frac{dr^2}{\left(1-\frac{m}{r}\right)^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

with **secondary** scalar hair

$$\boxed{\phi = \sqrt{\frac{3}{4\pi G}}\frac{m}{r-m}}$$

- Geometry is that of an extremal RN.
  Problem:The scalar field is **unbounded** at ($r=m$).
- Controversy on the stability [Bronnikov et al.-78, McFadden et al.-05]
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Scalar-tensor theories and black holes

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- Is it possible to have non-trivial and regular scalar-tensor black holes for an asymptotically flat space-time?
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Higher order scalar-tensor theory

Construct black hole solutions for,

- Higher order scalar tensor theory: Horndeski/Galileon theory (Lovelock/Lanczos theory)
- Shift symmetry for the scalar
- Spherically symmetric and static space-time.

Example theory

Consider the action,

$$S = \int d^4x\sqrt{-g}\left[\zeta R - 2\Lambda - \eta\left(\partial\phi\right)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right],$$

- Metric field equations read,

$$\zeta G_{\mu\nu} - \eta\left(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2\right) + g_{\mu\nu}\Lambda$$
$$+\frac{\beta}{2}\left((\partial\phi)^2 G_{\mu\nu} + 2P_{\mu\alpha\nu\beta}\nabla^\alpha\phi\nabla^\beta\phi\right.$$
$$\left.+g_{\mu\alpha}\delta^{\alpha\rho\sigma}_{\nu\gamma\delta}\nabla^\gamma\nabla_\rho\phi\nabla^\delta\nabla_\sigma\phi\right) = 0,$$

- Scalar field has translational invariance :$\phi \to \phi$+const.,
- Scalar field equation can be written in terms of a current

$$\nabla_\mu J^\mu = 0, \quad J^\mu = \left(\eta g^{\mu\nu} - \beta G^{\mu\nu}\right)\partial_\nu\phi.$$

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$(\eta g^{rr} - \beta G^{rr})\sqrt{g}\phi' = c$
- but current is singular $J^2 = J^\mu J^\nu g_{\mu\nu} = (J^r)^2 g_{rr}$ unless $J^r = 0$ at the horizon...
Generically $\phi = constant$ everywhere [Hui and Nicolis] and we have again the appearance of a no-hair theorem...
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  In scalar equation, $\eta g^{\mu\nu} - \beta G^{\mu\nu} \to$ metric EoM
  $R \to G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, $\Lambda \to g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

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Time dependent scalar field

- Set $\beta G^{rr} - \eta g^{rr} = 0$ rendering the scalar equation "redundant"...
- Consider $\phi = \phi(t, r)$ with static space-time,

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

- (tr)-component of EoM is non trivial and reads, $\frac{\beta\phi'}{r^2}\left(\frac{rfh'}{h}+\left(f-1-\frac{\eta r^2}{\beta}\right)\dot{\phi}-2rf\dot{\phi}'\right)=0$
- General solution, $\phi(t, r) = \psi(r) + q_1(t)e^{X(r)}$ with $X(r) = \frac{1}{2}\int dr\left(\frac{1}{r}-\frac{1}{rf}-\frac{\eta r}{\beta f}+\frac{h'}{h}\right)$ and $\ddot{q}_1(t) = C_1 q_1(t) + C_2$
- Simplest solution softly breaking translational invariance $q_1(t) = q\, t$ and thus $\phi(t, r) = q\, t + \psi(r)$

No time derivatives present in the field equations

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No time derivatives present in the field equations

Time dependent scalar field

- Set $\beta G^{rr} - \eta g^{rr} = 0$ rendering the scalar equation "redundant"...
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- Hypotheses: $\beta G^{rr} - \eta g^{rr} = 0$ and $\phi(t,r) = q\,t + \psi(r)$,
- $-\partial_r[(\beta G^{rr} - \eta g^{rr})\partial_r \psi] - \partial_t[(\beta G^{tt} - \eta g^{tt})\partial_t(qt)] = 0$
- no scalar charge, current ok, $\phi \neq 0$, and (tr)-eq satisfied
- Geometric constraint, $f = \frac{(\beta+\eta r^2)h}{\beta(rh)'}$, fixing spherically symmetric gauge.
- $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$
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Solving the remaining EoM

- From (rr)-component get ψ'

$$\psi' = \pm \frac{\sqrt{r}}{h(\beta + \eta r^2)} \left(q^2 \beta(\beta + \eta r^2) h' - \frac{\lambda}{2} (h^2 r^2)' \right)^{1/2} .$$

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$$\lambda \equiv \zeta\eta + \beta\Lambda$$

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- For $\eta = \Lambda = 0$ time dependence is essential!!
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- Consider $S = \int d^4x \sqrt{-g} \left[\zeta R + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$
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- in Eq of scalar $\beta G^{\mu\nu}$ $\rightarrow$ Einstein equation
- $G^{rr} = 0 \rightarrow f = \frac{h}{(rh)'}$ and $\phi(t,r) = q\,t + \psi(r)$
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- Scalar looks singular for $r \to r_h$ but $t_h \to \infty$!
- Consider $v = t + \int (fh)^{-1/2} dr$ then $ds^2 = -hdv^2 + 2\sqrt{h/f}\, dvdr + r^2 d\Omega^2$ Regular chart for horizon, EF coordinates ([Jacobson], [Ayon-Beato, Martinez & Zanelli])
- $\phi_+ = q \left[v - r + 2\sqrt{\mu r} - 2\mu \log \left(\sqrt{\frac{r}{\mu}} + 1 \right) \right] + \text{const}$
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- Need to solve:

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- Example: Black hole in an Einstein static universe $(\zeta\eta + \beta\Lambda = 0)$
- $h = 1-\frac{\mu}{r}, \;\; f = \left(1-\frac{\mu}{r}\right)\left(1+\frac{\eta r^2}{\beta}\right),$
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Can we get de Sitter asymptotics?

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- Solution is regular at the horizon for de Sitter asymptotics

de Sitter black hole

- Consider $S = \int d^4x\sqrt{-g}\left[\zeta R - 2\Lambda - \eta\left(\partial\phi\right)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]$
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Self tuned de Sitter Schwarzschild

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- The effective cosmological constant is not the vacuum cosmological constant. In fact,
- $q^2\eta = \Lambda - \Lambda_{eff} > 0$
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Conformally coupled scalar field

- Consider a conformally coupled scalar field ϕ:

$$S[g_{\mu\nu},\phi,\psi]=\int_{\mathcal{M}}\sqrt{-g}\left(\frac{R}{16\pi G}-\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi-\frac{1}{12}R\phi^2\right)\mathrm{d}^4x+S_m[g_{\mu\nu},\psi]$$

- **Invariance of the EOM of ϕ under the conformal transformation**

$$\begin{cases} g_{\alpha\beta}\mapsto\tilde{g}_{\alpha\beta}=\Omega^2 g_{\alpha\beta} \\ \phi\mapsto\tilde{\phi}=\Omega^{-1}\phi \end{cases}$$

- There exists a black hole geometry with non-trivial scalar field and secondary black hole hair.
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BBMB completion [CC, Kolyvaris, Papantonopoulos and Tsoukalas]

- We would like to combine the above properties in order to obtain a hairy black hole.
- Consider the following action, $S(g_{\mu\nu},\phi,\psi) = S_0 + S_1$ where
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  and
- $$S_1 = \int dx^4 \sqrt{-g}\ \left(\beta G_{\mu\nu}\nabla^\mu\Psi\nabla^\nu\Psi - \gamma T^{BBMB}_{\mu\nu}\nabla^\mu\Psi\nabla^\nu\Psi\right)\ ,$$
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Black hole with primary hair

- Solve as before assuming linear time dependence for Ψ
- Scalar ϕ has an additional branch regular at the "horizon"
- A second solution reads,

$$h(r) = 1 - \frac{m}{r}, \qquad f(r) = (1 - \frac{m}{r})\left(1 - \frac{\gamma c_0^2}{12\beta r^2}\right)$$

$$\phi(r) = \frac{c_0}{r},$$

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Adding electromagnetic charge

Following the same idea we can add an EM field

$$I[g_{\mu\nu},\phi,A_\mu] = \int \sqrt{-g}d^4x \;\left[R - \eta\,(\partial\phi)^2 - 2\,\Lambda + \beta\, G_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi \right. \\ \left. -\frac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu} - \gamma\,T_{\mu\nu}\,\nabla^\mu\phi\nabla^\nu\phi\right],$$

where we have defined

$$T_{\mu\nu} := \frac{1}{2}\left[F_{\mu\sigma}F_\nu{}^\sigma - \frac{1}{4}\,g_{\mu\nu}\,F_{\alpha\beta}F^{\alpha\beta}\right].$$

Note that the coupling of the EM field is not trivial. But the scalar field equations defines a current as before

$$\nabla_\mu J^\mu = \nabla_\mu\left[\left(\beta\,G^{\mu\nu} - \eta\,g^{\mu\nu} - \gamma\,T^{\mu\nu}\right)\nabla_\nu\phi\right] = 0,$$

Adding electromagnetic charge

We consider,

$$ds^2 = -h(r)\,dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{1-\theta^2}\,d\theta^2 + r^2\theta^2\,d\chi^2, \qquad \phi(t,r) = \psi(r) + q\,t, \qquad A_\mu dx^\mu = A(r)dt. \qquad (1)$$

We define

$$S\,(r) = \frac{\left(\eta\, r^2 + \beta\right)\left(r^2 B(r)^2\gamma + 4\,\left(r\,h(r)\right)'\,\beta\right)}{4\,\beta}, \qquad B(r) = A'(r), \qquad (2)$$

and the EOM reduce to,

$$q^2\beta\,\left(\eta\, r^2 + \beta\right)^2 + \frac{r^2\left(\eta\, r^2 + \beta\right)^2(\beta-\gamma)\,B(r)^2}{4\,\beta} - S(r)\left[(\eta - \beta\,\Lambda)\,r^2 + 2\,\beta\right] + C_0 S(r)^{3/2} = 0,$$

$$\left(\frac{\beta(\beta-\gamma)(\eta\, r^2+\beta)}{S(r)^{1/2}} + \frac{\beta\gamma C_0}{2}\right) B(r) = \frac{2Q}{r^2}$$

RN like solution

$$F_{rt} = B(r) = \frac{2\,Q}{r^2}. \tag{3}$$

The metric functions take the form

$$h(r) = f(r) = 1 + \frac{\eta r^2}{3\,\beta} - \frac{\mu}{r} + \frac{Q^2}{r^2}, \qquad \psi'^2 = -\frac{(f(r) - 1)\,q^2}{f(r)^2}, \tag{4}$$

while the coupling constants are,

$$\beta = \gamma, \qquad q^2 = \frac{\eta + \Lambda\beta}{\eta\beta} \qquad C_0 = (\eta - \beta\Lambda)\frac{\sqrt{\beta}}{\eta}$$

Conclusions

- Have found GR black holes with a non-trivial and regular scalar field
- Shift symmetry and higher order essential!!
- Rendered scalar field eq redundant and allowed for linear time dependence
- Time dependence essential for regularity on the event horizon
- Solutions are hairy(charge q) and non-hairy (time dependent), hence fake.
- Method can be applied in differing Gallileon context [Kobayashi and Tanahashi], in higher dimensions, including EM and other matter fields.
- Is there a way to find observable for q? Is there a distinction possible?
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