

# Physics and Astrophysics of Neutron Stars

NS properties

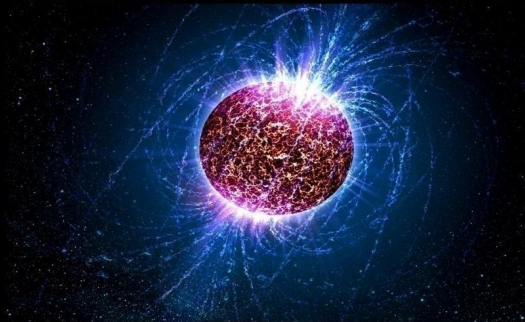
Phase transitions

Phase diagram

Constraints on realistic EoS

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## Lecture 1

SAG11, 1-6 September 2023

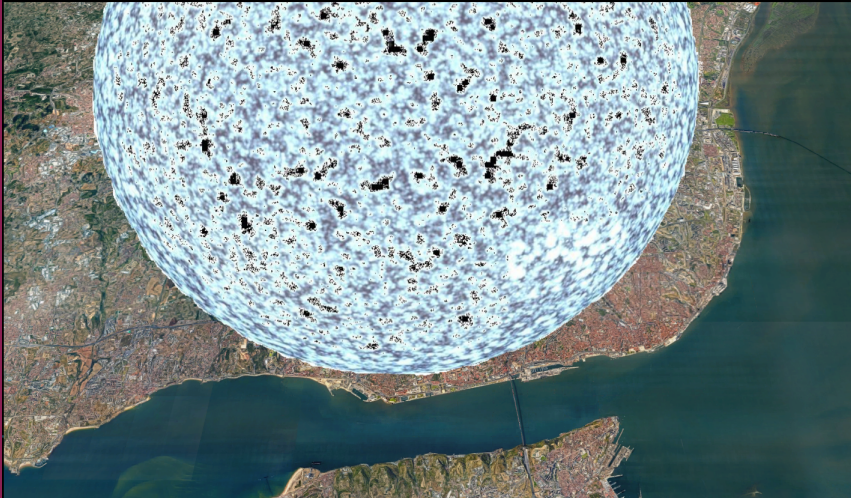
# Neutron star has a size of Lisbon

## NS properties

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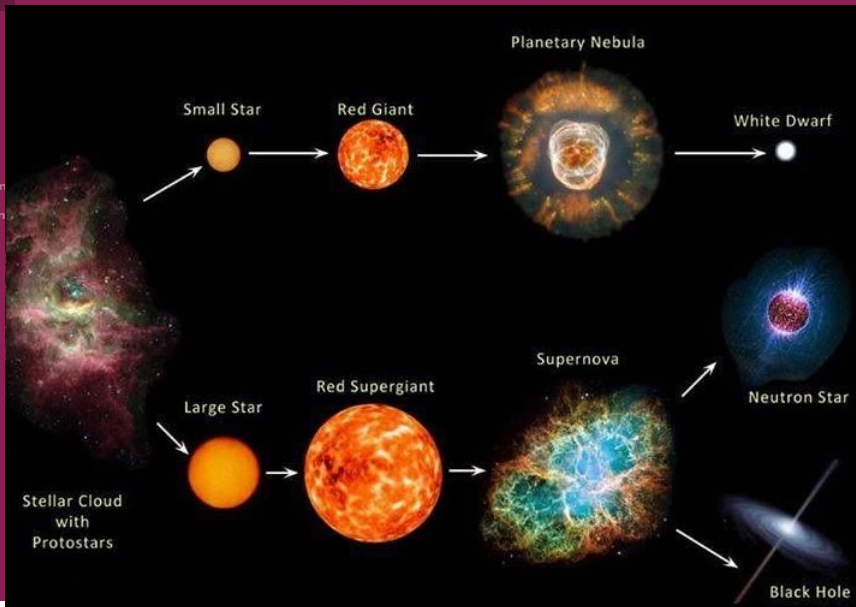
# Stellar evolution

## NS properties

Phase transitions

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# Pulsar - rotating neutron star



**Pulsars can spin up to 1000 times per second!**



NS properties

Phase transitions

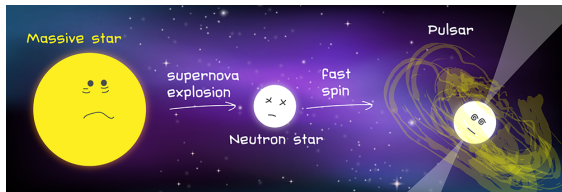
Phase diagram

Constraints on realistic EoS



# Neutron star

- the last stage of massive star evolution, produced in core collapse supernova explosion. Usually detected as a pulsar
- the most compact and exotic astrophysical objects in the universe that are accessible by direct observations
- the most extreme objects in terms of the rotation speed, density, radius, magnetic field, etc.



	Neutron star	White dwarf	Sun
$M_{max}(M_{\odot})$	2	1.44	1
$R$ (km)	11-12	$10^4$	$7 \cdot 10^5$
$n_c$ ( $g/cm^3$ )	$10^{14} - 10^{15}$	$10^7$	$10^2$
rotation speed (s)	$10^{-3} - 1$	100	$2 \cdot 10^6$
$B$ (G)	$10^8 - 10^{16}$	100	1
$T$ (K)	$10^6 - 10^{11}$	$10^3$	$10^5$

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# Outer crust

On the surface  $p = 0$  as there is nothing above the crust

NS temperature  $T \sim 10^9 K$ . The thermal energy is pretty small compared with the Fermi energy, thus the standard NS model uses the zero-temperature approximation, Typical 'excitation energies'  $\sim 10^{10} K \simeq MeV$ . Thus, NS can be regarded as being in the 'lowest energy state' - the ground state of matter

- Nucleons arranged into  $Fe^{56}$  nuclei, the most tightly bound nucleons. Nuclei arranged in a periodic lattice to minimize the energy.
- At density  $10^4 g cm^{-3}$  atoms become completely ionized.
- When the Fermi energy of the electrons exceeds  $\sim 1 MeV$ ,  $Fe^{56}$  is not any more a favoured nucleus due to **inverse beta decay**.



Electrons from the top of the Fermi sea will combine with protons to form neutrons. Neutrinos will escape.

- Due to inverse beta decay nuclei will become more and more neutron rich. Now on  $Ni^{62}$  will become more preferable nuclei. It continues until density becomes equal to  $4.3 \cdot 10^{11} g cm^{-3}$

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# Binding energies of nuclei

NS properties

Phase transitions

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Constraints on realistic EoS

Binding energy per nucleon

$n+n, p+p$ : no binding

$d = n+p$ : 1.1 MeV

$\alpha = 2n + 2p$ : 7.1 MeV

${}^8_4\text{Be} = 2\alpha$ : no binding

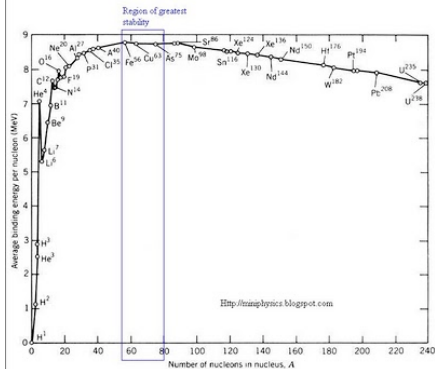
${}^{12}_6\text{C}$ : 7.68 MeV

${}^{16}_8\text{O}$ : 7.98 MeV

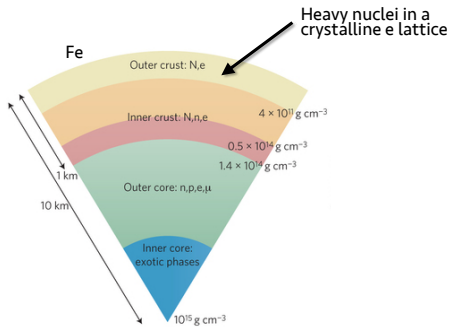
${}^{56}_{26}\text{Fe}$ : 8.79 MeV

${}^{62}_{28}\text{Ni}$ : 8.79 MeV

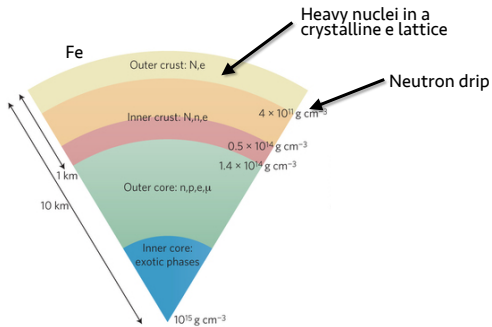
Isotopes: N,Z  
Excited nuclei



## Interior Structure



## Interior Structure



# Neutron drip line defines the boundary between outer and inner parts of the crust

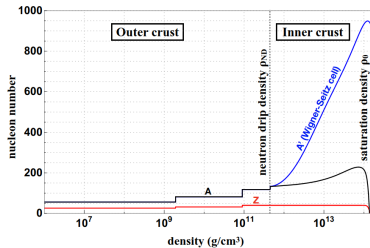
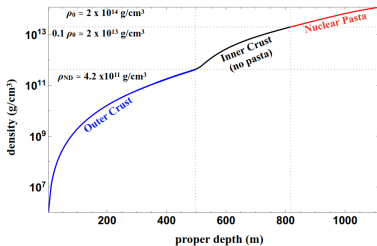
At density  $4.3 \times 10^{11} \text{ g cm}^{-3}$  nuclei become so neutron rich that neutrons begin to 'drip out' of the nuclei. Inner crust starts to consist of lattice of exotic nuclei embedded in a sea of neutrons.

NS properties

Phase transitions

Phase diagram

Constraints on realistic EoS



Credits: J. Bramante & N. Raj

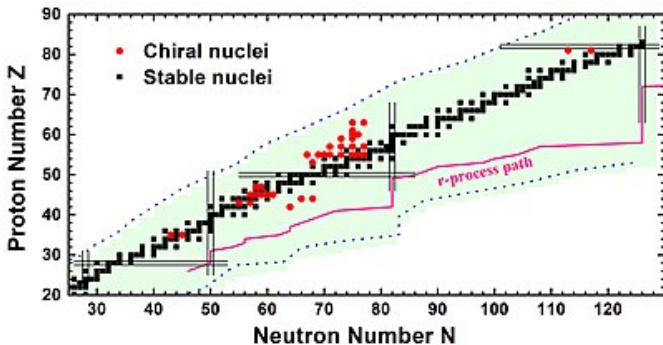
# Chart of nuclei

## NS properties

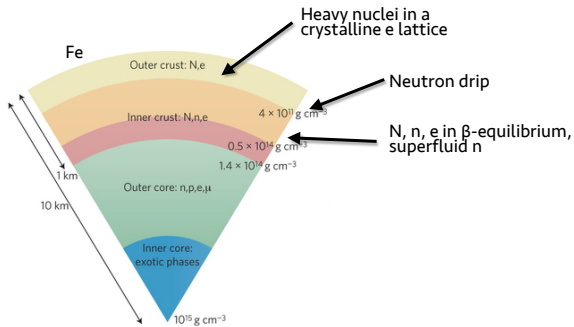
Phase transitions

Phase diagram

Constraints on realistic EoS



## Interior Structure





# Fluid core

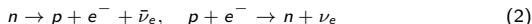
## NS properties

Phase transitions

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Constraints on realistic EoS

- Above  $2.5 \cdot 10^{14} \text{ g cm}^{-3}$  the nuclei will 'melt'. A fluid of neutrons, protons and electrons will appear.
- The relative number of neutrons, protons and electrons can be obtained by requiring  $\beta$  equilibrium:



- **Chemical equilibrium** requires chemical potential of neutrons equals to the sum of chemical potentials of proton and electron.

## $\beta$ equilibrium

Not only **neutrons**

Matter consists also on

**protons, electrons**, and further elementary particles

neutron mass  $m_n = 939.56541$  MeV

proton mass  $m_p = 938.2708$  MeV

electron mass  $m_e = 0.511$  MeV

$\beta$  decay:  $n \rightleftharpoons p + e^- + \bar{\nu}_e$

electron antineutrino  $\bar{\nu}_e$

chemical equilibrium

$$\mu_n = \mu_p + \mu_e + \mu_{\bar{\nu}_e}$$

electron antineutrinos escape,

zero density,  $\mu_{\bar{\nu}_e} = 0$ .

charge neutrality:  $n_e = n_p$

## Nuclear material constituents

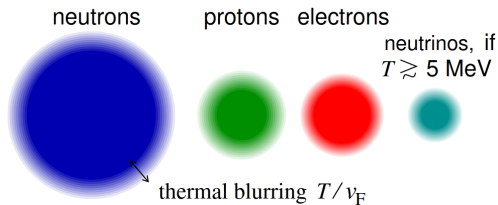
NS properties

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Constraints on realistic EoS

Fermi surfaces:



neutrons:  $\sim 90\%$  of baryons

$\rho_{Fn} \sim 350 \text{ MeV}$

protons:  $\sim 10\%$  of baryons

$\rho_{Fp} \sim 150 \text{ MeV}$

electrons: same density as protons

$\rho_{Fe} = \rho_{Fp}$

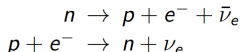
neutrinos: only present if  $\text{mfp} \ll 10 \text{ km}$

i.e. when  $T \gtrsim 5 \text{ MeV}$

## “Cold” beta equilibration

At  $T \lesssim 1 \text{ MeV}$ :

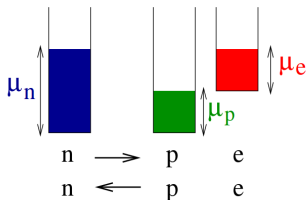
- Fermi surface approximation is valid
- neutrinos escape, so it is the “neutrino-transparent” regime



Neutrino energy  $\sim T$  is negligible compared to  $\mu_n, \mu_p, \mu_e$ .

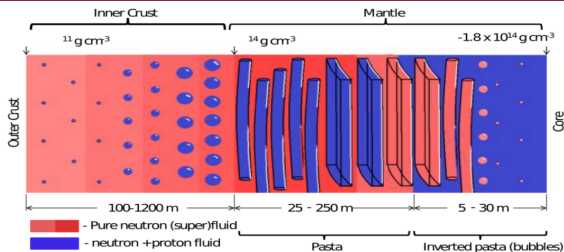
Beta equilibrium condition is

$$\mu_n = \mu_p + \mu_e$$



- ▶ Why does a density change drive the proton fraction out of beta equilibrium?
- ▶ What goes wrong with the Fermi Surface approximation as  $T$  approaches 1 MeV?

# Interior of neutron star



NS properties

Phase transitions

Phase diagram

Constraints on realistic EoS



Gnocchi



Spaghetti

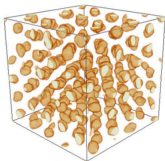


Lasagna

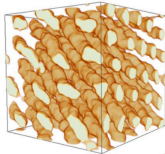


# Nuclear pasta configurations produced in Molecular dynamics simulations

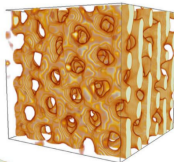
(a) *Gnocchi*



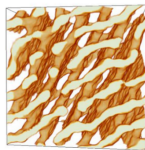
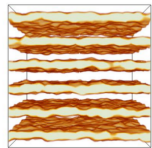
(b) *Spaghetti*



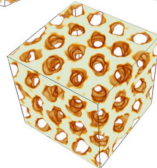
(c) *Waffles*



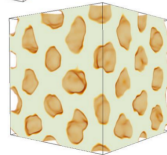
(d) *Lasagna*



(e) *Defects*



(f) *Antispaghetti*



(g) *Antignocchi*

NS properties

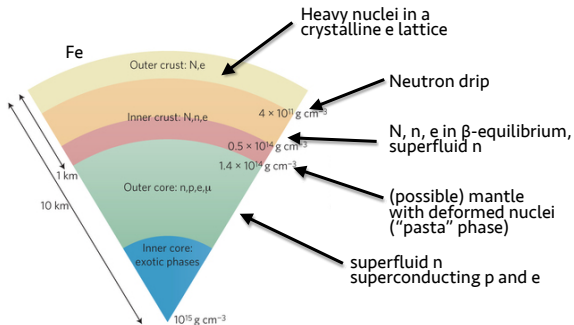
Phase transitions

Phase diagram

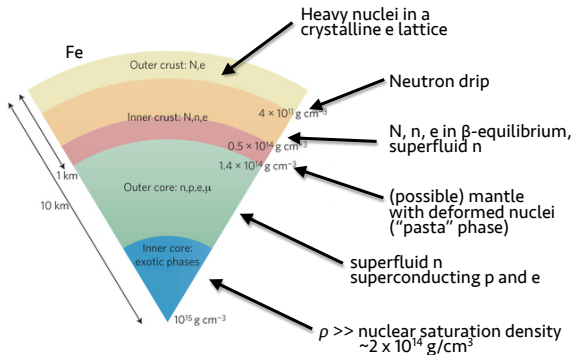
Constraints on realistic EoS

Credits: Horowitz et al., 2015; Schneider et al., 2014, 2013

## Interior Structure

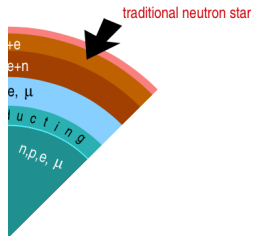


## Interior Structure





## Different Possible Structures



NS properties

Phase transitions

Phase diagram

Constraints on realistic EoS

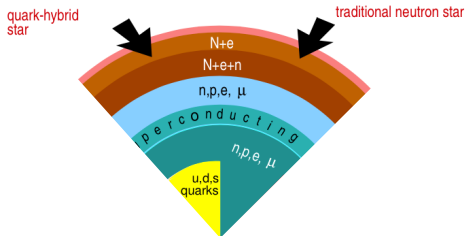
NS properties

Phase transitions

Phase diagram

Constraints on realistic EoS

## Different Possible Structures



[Weber, J. Phys. G 27, 465 (2001)]

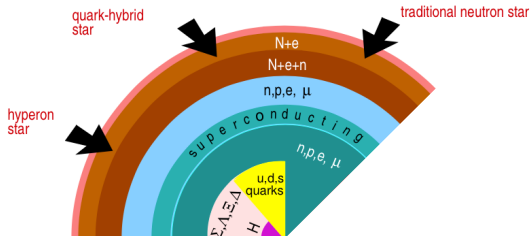
NS properties

Phase transitions

Phase diagram

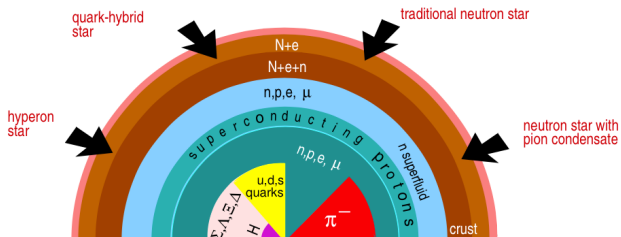
Constraints on realistic EoS

## Different Possible Structures



[Weber, J. Phys. G 27, 465 (2001)]

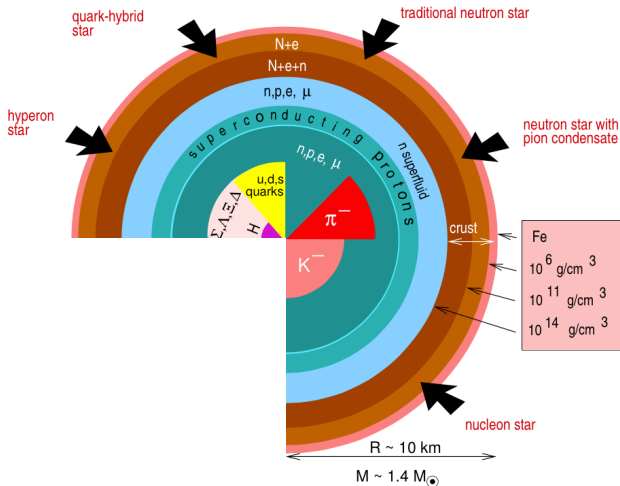
# Different Possible Structures



[Weber, J. Phys. G 27, 465 (2001)]

NS properties  
 Phase transitions  
 Phase diagram  
 Constraints on realistic EoS

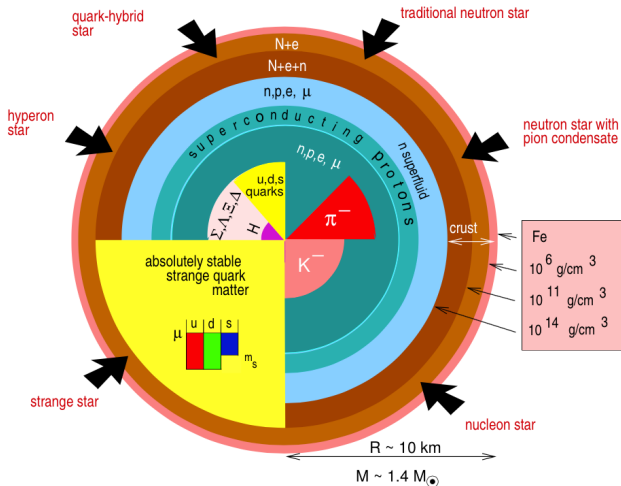
# Different Possible Structures



[Weber, J. Phys. G 27, 465 (2001)]

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# Different Possible Structures



[Weber, J. Phys. G 27, 465 (2001)]

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# Phase diagrams - map of matter properties

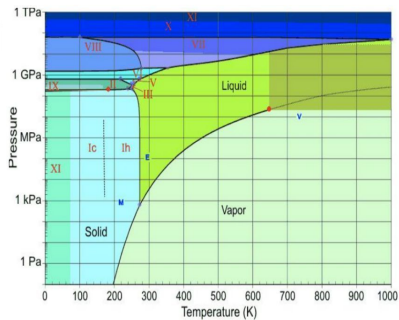
NS properties

Phase transitions

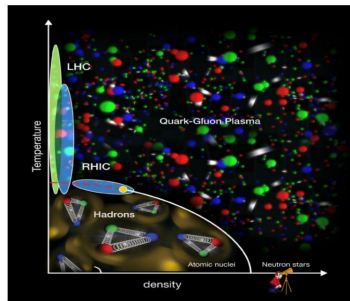
Phase diagram

Constraints on realistic EoS

## Phase diagram of water

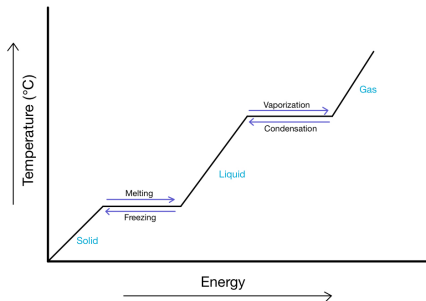


## Phase diagram of strongly interacting matter

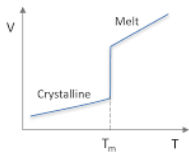


# Types of phase transitions

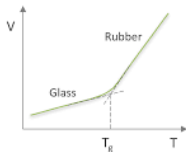
Phase Change Diagram



First Order Transition



Second Order Transition



NS properties

Phase transitions

Phase diagram

Constraints on realistic EoS



# Phase transitions

A phase transition is a change in state from one phase to another

NS properties

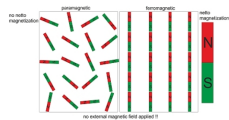
Phase transitions

Phase diagram

Constraints on realistic EoS

Examples of phase transitions:

- Quantum condensation of bosonic fluids (Bose-Einstein condensation);
- The breaking of symmetries in the laws of physics during the early history of the universe as its temperature cooled;
- Water freezing, evaporation, condensation, sublimation;
- The transition between the ferromagnetic and paramagnetic phases of magnetic materials at the Curie point;
- and many more.



paramagnetic to ferromagnetic phase transition  
from zero to non-zero magnetic moment

# Phase transition in champagne

NS properties

**Phase  
transitions**

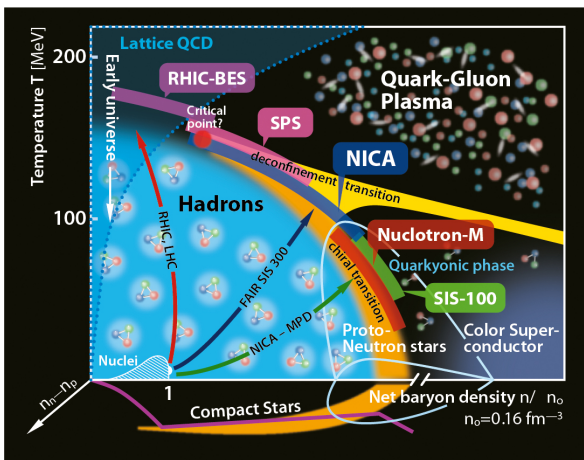
Phase diagram

Constraints on  
realistic EoS



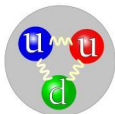
# Strongly Interacting Matter Phase Diagram

NS properties  
Phase transitions  
Phase diagram  
Constraints on realistic EoS

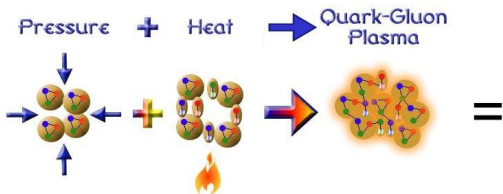


# What is Quark-Gluon Plasma?

## Proton structure



**Proton consists of two u and one d quarks**  
**Neutron consists of two d and one u quarks**



NS properties  
Phase transitions  
Phase diagram  
Constraints on realistic EoS

# Want to know what is inside? What to do?

NS properties

Phase  
transitions

**Phase diagram**

Constraints on  
realistic EoS



To break it apart!!!

NS properties

Phase  
transitions

Phase diagram

Constraints on  
realistic EoS



# Large Hadron Collider (LHC) in CERN

NS properties  
Phase transitions  
Phase diagram  
Constraints on realistic EoS

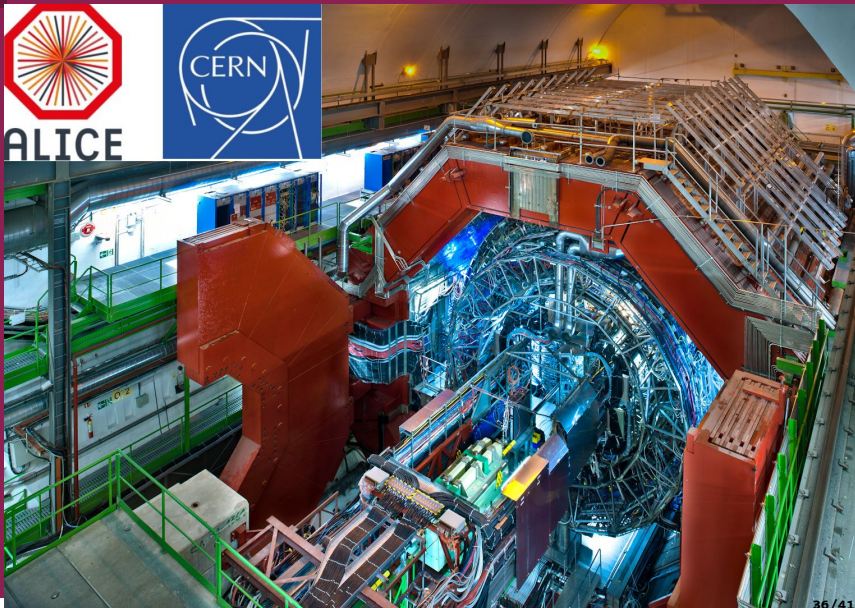




# ALICE experiment at the LHC



ALICE



NS properties

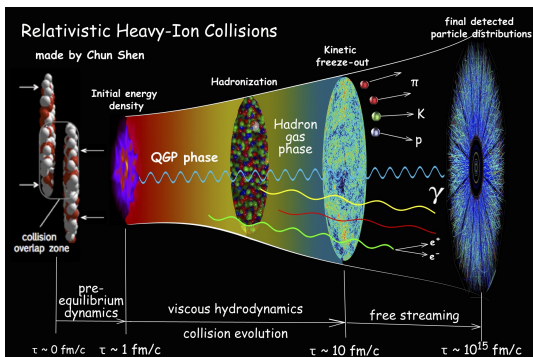
Phase transitions

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Constraints on realistic EoS



# Stages of Heavy-Ion Collisions



- Non-equilibrium dynamics - the formation of local equilibrium
- The hydrodynamic expansion - local equilibrium
- Hadronisation - formation of full equilibrium
- Hadron freeze-out – full equilibrium, **formation of particle yields**
- Free streaming

NS properties  
Phase transitions  
Phase diagram  
Constraints on realistic EoS

# Constraints on the EoS

NS properties  
Phase transitions  
Phase diagram  
Constraints on realistic EoS

## HEP

### - proton flow

anisotropic expansion is caused by gradient of pressure, which gives an access to EoS

F. Donato et al., Science 298, 1283 (2002)

### - hadron multiplicities

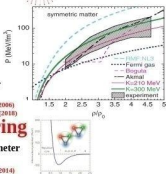
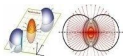
hard core radii of hadrons control the rate of their production in thermal medium:  $R = 0.3 - 0.5 \text{ fm}$

A. Andronic et al., Nucl. Phys. A 772, 187 (2006)  
K. A. Bugay et al., Nucl. Phys. A 970, 133 (2018)

### - nucleon-nucleon scattering

hard core radius of nucleons extracted as a parameter of microscopic interaction potential:  $R = 0.5 \text{ fm}$

M. Naghd, Phys. Part. Nucl. 5, 924 (2014)



## Nucl. Phys.

### - nuclear matter ground state

binding energy per nucleon at saturation density  $n_0$ :

$$n_0 = 0.16 \pm 0.01 \text{ fm}^{-3}, E(n_0)/A = -16.0 \pm 1.0 \text{ MeV}$$

incompressibility at  $n_0$ :

$$K_0^{(0)} = 200 - 260 \text{ MeV}$$

symmetry energy at  $n_0$ :

$$S(n_0) = J = 30 \pm 4 \text{ MeV}$$

symmetry energy slope at  $n_0$ :  $L \equiv 3n_0 \left( \frac{\partial S(n_B)}{\partial n_B} \right)_{n_B=n_0} = 20 - 115 \text{ MeV}$

E. Khan, Phys. Rev. C, 80, 011307 (2009)  
M. Dutra et al., Phys. Rev. C, 85, 035201 (2012)

Zhang, Z., Chen, L.-W., Phys. Lett. B, 726, 234 (2013)



## Chiral effective theory

- up to  $\sim 1.1 n_0$

Drischler, et al. PRC 102, 054315 (2020)  
Tews, et al. APJ 860, 149 (2018)

## Perturbative QCD

- from  $\sim 40 n_0$

Komoltsev & Kurkela, PRL 128 (2022)

## Astro

-  $\sim 2 M_{\text{sun}}$

PSR J0348+0432:  $M = 2.01^{+0.04}_{-0.04} M_{\odot}$

J. Antoniadou et al., Science, 348, 449 (2013)

PSR J0740+6620:  $M = 2.08^{+0.07}_{-0.07} M_{\odot}$

E. Fonseca et al., APJ, 915, L12 (2021)

PSR J1810+1744:  $M = 2.13^{+0.04}_{-0.04} M_{\odot}$

R. W. Ross et al., APJ, 908, L48 (2021)

PSR J0952-0607:  $M = 2.35^{+0.17}_{-0.17} M_{\odot}$

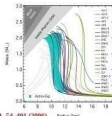
R. W. Ross et al., APJ, 916, 2 (2022)

- NICER results

- NSs cooling



## M-R relation



F. Özel, P. Freire, A&A, 54, 401 (2005)

## Grav. Phys.

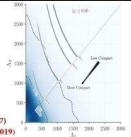
GW170817 + kilonova

NS+NS and NS+BH mergers



Love numbers and tidal polarizability are highly sensitive to EoS

LIGO and Virgo collaborations, PRL 119, 161101 (2017)  
LIGO and Virgo collaborations, arXiv:2001.01761 (2019)



## Supernova explosions

simulations of supernova explosions predict the lightest possible NS to be  $M = 1.17 M_{\odot}$

Suwa Y., et al., MNRAS 481,434 3305 (2018)

# The nuclear Equation of State (EoS) at T=0

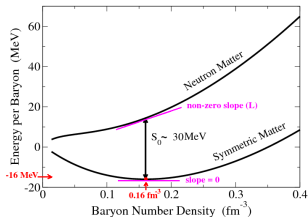
$$\frac{E(n_B, \delta)}{A} = \frac{E(n_B, 0)}{A} + S(n_B)\delta^2 + O(\dots^4)$$

symmetric matter (a) asymmetric matter

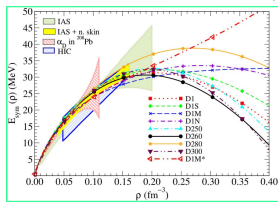
$$\delta = \frac{n_n - n_p}{n_B}$$

$$n_B = n_n + n_p$$

$$\epsilon = \frac{n_B - n_0}{n_0}$$



$$\frac{E(n_B, 0)}{A} = \frac{E(n_0)}{A} + \frac{1}{18} K_0 \epsilon^2$$



Credits: C. Gonzalez-Boquera

$$S(n_B) = S(n_0) + \frac{1}{3} L \epsilon + \frac{1}{18} K_{\text{Sym}} \epsilon^2$$

$$\frac{E(n_0)}{A} \equiv \frac{E_0}{A} = -16 \pm 1 \text{ MeV binding energy per nucleon at saturation density } n_0 = 0.16 \pm 0.01 \text{ fm}^{-3}$$

$$S(n_0) \equiv S_0 \equiv J = 30 \pm 4 \text{ MeV}$$

$$S(n_0) \equiv \frac{1}{2} \left( \frac{\partial^2 E}{\partial \delta^2} \right)_{n_B=n_0, \delta=0} \quad \text{symmetry energy at } n_0$$

$$K_0 \equiv 9n_0^2 \left( \frac{\partial^2 E}{\partial n_B^2} \right)_{n_B=n_0, \delta=0} = 200 - 260 \text{ MeV}$$

incompressibility at  $n_0$

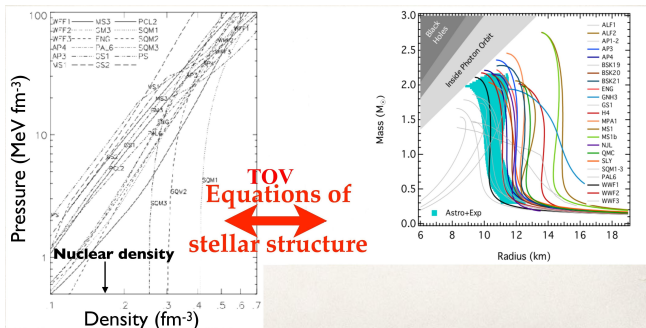
$$L \equiv 3n_0 \left( \frac{\partial S(n_B)}{\partial n_B} \right)_{n_B=n_0, \delta=0} = 20 - 115 \text{ MeV}$$

symmetry energy slope at  $n_0$

NS properties  
Phase transitions  
Phase diagram  
Constraints on realistic EoS

# EoS $\Leftrightarrow$ M-R diagram

NS properties  
Phase transitions  
Phase diagram  
Constraints on realistic EoS



The Tolman-Oppenheimer-Volkoff (TOV) equation constrains the structure of a spherically symmetric body of isotropic material which is in static gravitational equilibrium. The equation is derived by solving the Einstein equations for a general time-invariant, spherically symmetric metric.

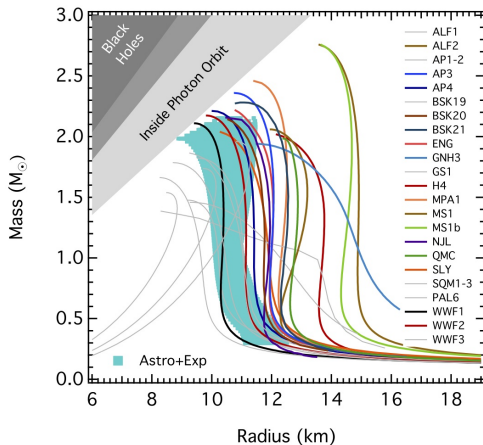
$$\frac{dp}{dr} = -(\rho + \epsilon) \frac{M + 4\pi r^3 p}{r(r - 2M)}, \quad (3)$$

$$\frac{dM}{dr} = 4\pi \epsilon r^2, \quad (4)$$

TASK: Derive the TOV Eq.

# Mass-Radius diagram

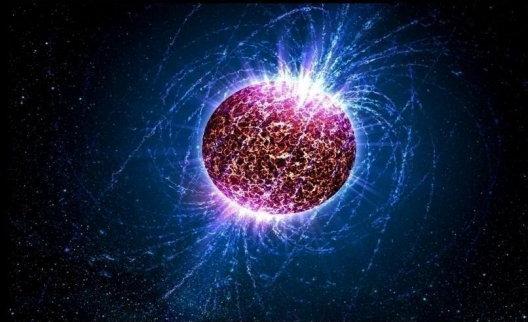
NS properties  
Phase transitions  
Phase diagram  
Constraints on realistic EoS



# Physics and Astrophysics of Neutron Stars

Violetta Sagun

CFisUC, Department of Physics, University of Coimbra, Portugal



## Lecture 2

SAG11, 1-6 September 2023

NS families

Strange stars

M and R  
measurements

Double NS  
system

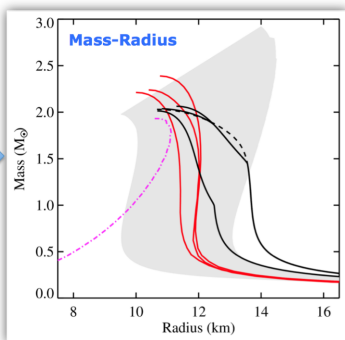
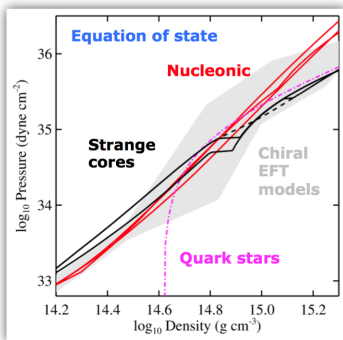
Millisecond  
pulsars

Magnetars

NS glitches

Superfluidity/  
superconductivity

# Equation of state (EoS) $\iff$ Mass-Radius diagram



Watts et al. 2015 SKA Science Book

EoS is an input to the Tolman-Oppenheimer-Volkoff (TOV) equation

$$\frac{dp}{dr} = -\frac{(\epsilon + p)(M + 4\pi r^3 p)}{r^2(1 - 2M/r)}, \quad (1)$$

- NS families
- Strange stars
- M and R measurements
- Double NS system
- Millisecond pulsars
- Magnetars
- NS glitches
- Superfluidity/superconductivity

# Third family of compact stars

## NS families

Strange stars

M and R measurements

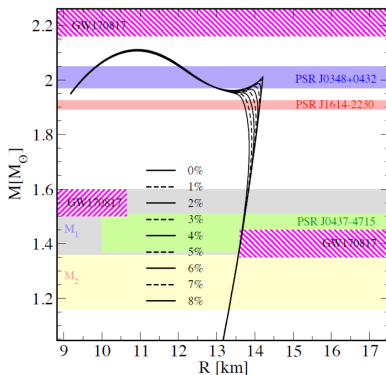
Double NS system

Millisecond pulsars

Magnetars

NS glitches

Superfluidity/  
superconductivity



Credits: David Alvarez-Castillo, David Blaschke, arXiv:1807.03258 [nucl-th]



# Hybrid Stars

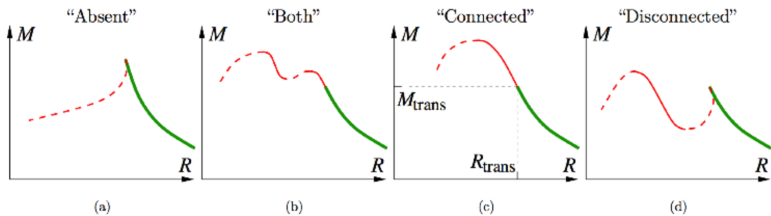


FIG. 2: Four possible topologies of the mass-radius relation for hybrid stars. The thick (green) line is the hadronic branch. Thin solid (red) lines are stable hybrid stars; thin dashed (red) lines are unstable hybrid stars. In (a) the hybrid branch is absent. In (c) there is a connected branch. In (d) there is a disconnected branch. In (b) there are both types of branch. In realistic neutron star  $M(R)$  curves, the cusp that occurs in cases (a) and (d) is much smaller and harder to see [13, 14]

Alford, Han, and Prakash (2013)

NS families

Strange stars

$M$  and  $R$   
measurements

Double NS  
system

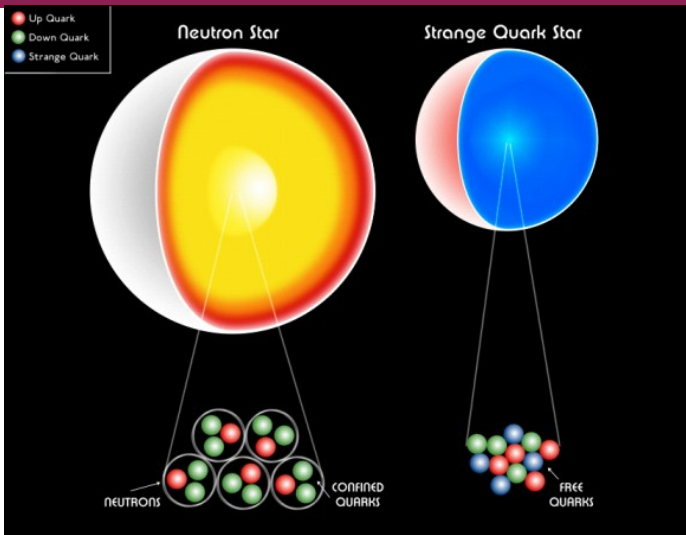
Millisecond  
pulsars

Magnetars

NS glitches

Superfluidity/  
superconductivity

# Strange stars



NS families

**Strange stars**

M and R  
measurements

Double NS  
system

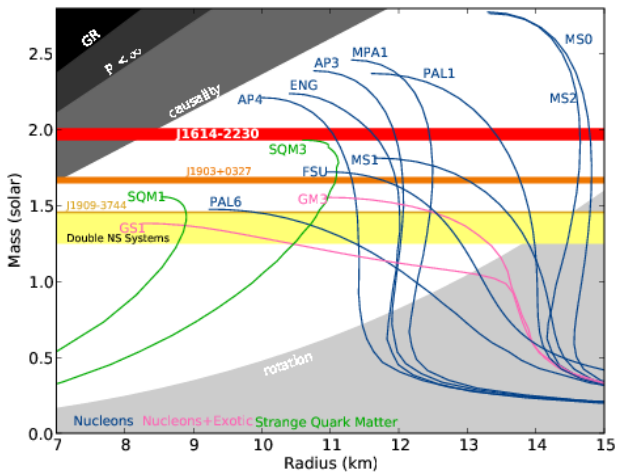
Millisecond  
pulsars

Magnetars

NS glitches

Superfluidity/  
superconductivity

# Strange star M-R diagram



NS families

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M and R measurements

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Superfluidity/  
superconductivity

# NS masses

- > 2500 pulsars known

About 90% of radio pulsars are isolated → Mass cannot be measured

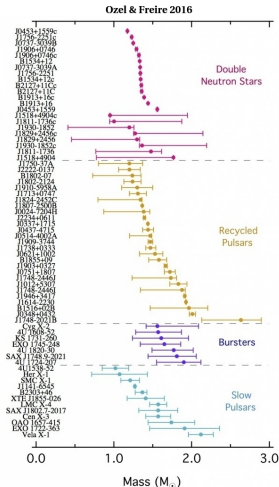
The remaining 250 pulsars are located in binary systems → info on mass



**PSR J0348+0432**  
 $M = (2.01 \pm 0.04) M_{\text{sun}}$  (Antoniadis et al '13)

**PSR J0740+6620**  
 $M = 2.14^{+0.20}_{-0.18} M_{\text{sun}}$  (Cromartie et al. '19)

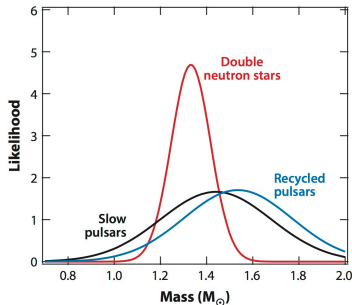
- High precision determined masses:  
**Hulse-Taylor pulsar**  
 $M = 1.4414 \pm 0.0002 M_{\text{sun}}$   
**Hulse-Taylor Nobel Prize 1994**



- Hulse-Taylor binary system (PSR B1516-60) is the first system for which reliable mass measurements were made. Consist of pulsar and an NS. 1993 Nobel Prize in Physics "for the discovery of a new type of pulsar, a discovery that has opened up new possibilities for the study of gravitation".

NS families  
 Strange stars  
 M and R measurements  
 Double NS system  
 Millisecond pulsars  
 Magnetars  
 NS glitches  
 Superfluidity/  
 superconductivity

# Neutron star masses



inferred mass distributions  
for the different populations of neutron stars.

F. Ozel and P. Freire, *Annu. Rev. Astron. Astrophys.* 54, 401 (2016)

NS families

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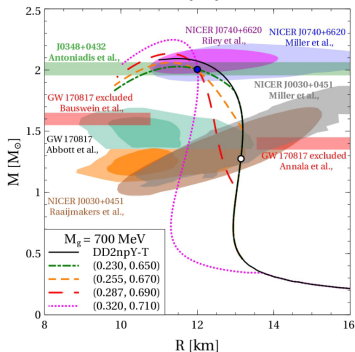
Superfluidity/  
superconductivity

# M and R measurements

## 4 NSs with mass above $2M_{\odot}$

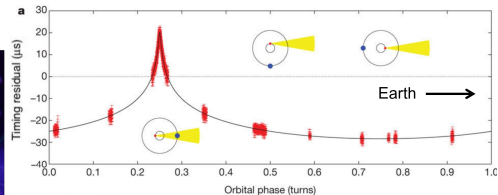
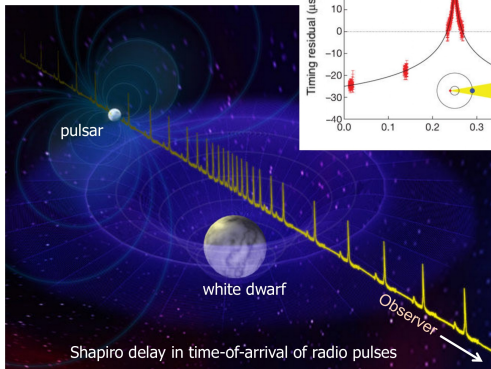
- PSR J0348+0432:  $M = 2.01^{+0.04}_{-0.04} M_{\odot}$  (Antoniadis+ 2013)
- PSR J0740+6620:  $M = 2.08^{+0.07}_{-0.07} M_{\odot}$  (Fonseca+ 2021)
- PSR J1810+1744:  $M = 2.13^{+0.04}_{-0.04} M_{\odot}$  (Romani+ 2021)
- PSR J0952-0607:  $M = 2.35^{+0.17}_{-0.17} M_{\odot}$  (Romani+ 2022)

Radius measurements have big uncertainties that are related to the modelling of the atmosphere composition



# Binary system

- NS families
- Strange stars
- M and R measurements**
- Double NS system
- Millisecond pulsars
- Magnetars
- NS glitches
- Superfluidity/  
superconductivity



Example

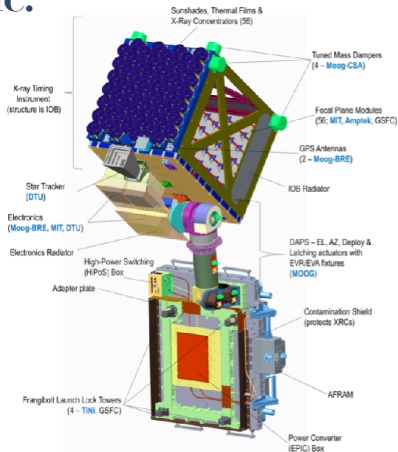
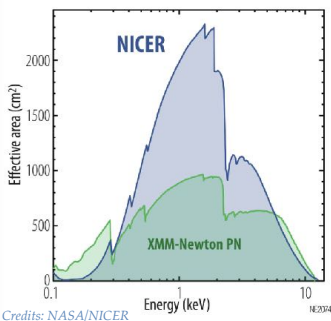
Demorest et al. (2010)

Pulsar mass:	$1.97 \pm 0.04 M_{\text{sun}}$
White dwarf mass:	$0.500 \pm 0.006 M_{\text{sun}}$
Orbital period:	8.69 days
Pulsar spin period:	3.15 ms

# Neutron Star Interior Composition ExploreR (NICER)

## NICER is NASA's fast X-ray photon counting machine.

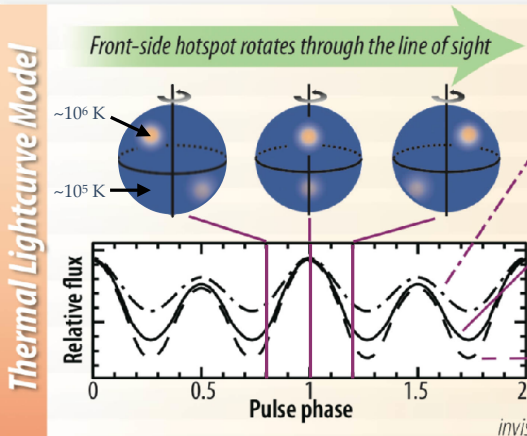
NS families  
Strange stars  
M and R measurements  
Double NS system  
Millisecond pulsars  
Magnetars  
NS glitches  
Superfluidity/  
superconductivity



Credits: Sebastien Guillot



The pulsed emission caused by hot spots on a rotating neutron star can help measure the compactness.



NS families

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M and R  
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# NICER results

NS families

Strange stars

**M and R measurements**

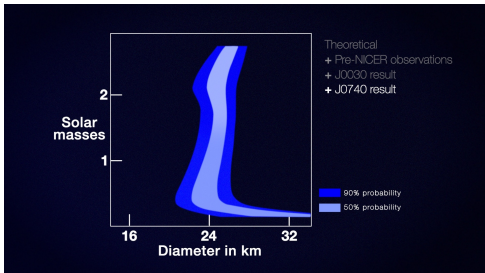
Double NS system

Millisecond pulsars

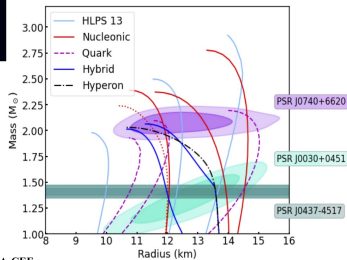
Magnetars

NS glitches

Superfluidity/  
superconductivity



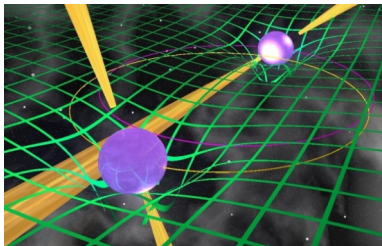
Credits: NASA



Credits: JINA-CEE

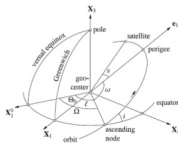
# Double pulsar system

- PSR J0737-3039 (PSR J0737-3039A ( $1.337 M_{\odot}$ ) & PSR J0737-3039B ( $1.250 M_{\odot}$ )) is the only known system containing two pulsars – thus a 'double pulsar' system.
- The orbit has decayed since the binary system was initially discovered, in precise agreement with the loss of energy due to gravitational waves described by Einstein's general theory of relativity.



## Keplerian orbital parameters

Keplerian motion defined by six orbital parameters



Parameter	Notation
$\Omega$	Right ascension of ascending node
$i$	Inclination of orbital plane
$\omega$	Argument of perigee
$a$	Semimajor axis of orbital ellipse
$e$	Numerical eccentricity of ellipse
$T_0$	Epoch of perigee passage

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superconductivity

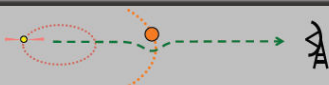
# Post-Keplerian parameters

Nice (2013)



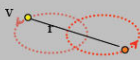
Precession

$$\dot{\omega} = 3 \frac{G^{2/3}}{c^2} \left( \frac{P_b}{2\pi} \right)^{-5/3} \frac{1}{1-e^2} \left[ (m_1 + m_2) \right]^{2/3}$$



Shapiro Delay

$$\Delta t = 2 \frac{G}{c^3} m_2 \ln [ 1 - \sin i \sin(\varphi - \varphi_0) ]$$



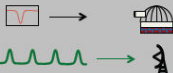
Grav Redshift/Time Dilation

$$\gamma = \frac{G^{3/2}}{c^2} \left( \frac{P_b}{2\pi} \right)^{1/2} e \frac{m_2 (m_1 + 2m_2)}{(m_1 + m_2)^{3/2}}$$



Gravitational Radiation

$$\dot{P}_b = - \left( \frac{192\pi}{5} \right) \frac{G^{5/2}}{c^5} \left( \frac{P_b}{2\pi} \right)^{-5/2} \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) \frac{1}{(1-e^2)^2} \frac{m_1 m_2}{(m_1 + m_2)^3}$$



Second Orbit

$$\frac{m_1}{m_2} = \frac{a_1 \sin i}{a_2 \sin i}$$

Any PK measurement yields a line in the  $(m_1, m_2)$ -plane.  
Hence, two PK parameters determines  $m_1$  and  $m_2$  uniquely.

## Post-Keplerian Parameters

Expressions for post-Keplerian parameters depend on theory of gravity. For general relativity:

$\dot{\omega}$ : Periastron precession

$\gamma$ : Time dilation and grav. redshift

$r$ : Shapiro delay “range”

$s$ : Shapiro delay “shape”

$\dot{P}_b$ : Orbit decay due to GW emission

$\Omega_{\text{geod}}$ : Frequency of geodetic precession resulting from spin-orbit coupling

PSR B1913+16:

$\dot{\omega}$ ,  $\gamma$ ,  $\dot{P}_b$  measured

PSR J0737-3039A/B

$\dot{\omega}$ ,  $\gamma$ ,  $r$ ,  $s$ ,  $\dot{P}_b$  measured

$$\dot{\omega} = 3T_{\odot}^{2/3} \left(\frac{P_b}{2\pi}\right)^{-5/3} \frac{1}{1-e^2} (m_p + m_c)^{2/3}$$

$$\gamma = T_{\odot}^{2/3} \left(\frac{P_b}{2\pi}\right)^{1/3} e \frac{m_c(m_p+2m_c)}{(m_p+m_c)^{4/3}}$$

$$r = T_{\odot} m_c$$

$$s = \sin i = T_{\odot}^{-1/3} \left(\frac{P_b}{2\pi}\right)^{-5/3} x \frac{(m_p+m_c)^{2/3}}{m_c}$$

$$\dot{P}_b = -\frac{192\pi}{5} T_{\odot}^{5/3} \left(\frac{P_b}{2\pi}\right)^{-5/3} f(e) \frac{m_p m_c}{(m_p+m_c)^{1/3}}$$

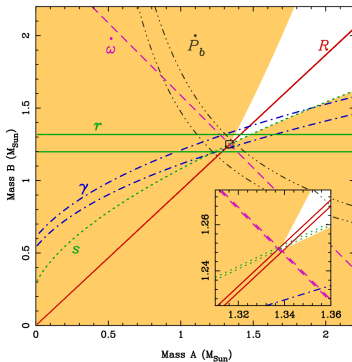
$$\Omega_{\text{geod}} = \left(\frac{2\pi}{P_b}\right)^{5/3} T_{\odot}^{2/3} \frac{m_c(4m_p+3m_c)}{2(m_p+m_c)^{4/3}} \frac{1}{1-e^2}$$

$$T_{\odot} = GM_{\odot}/c^3 = 4.9254909\mu\text{s}$$

# Confirmation of GR

Mass-mass diagram for the J0737-3039 system. All lines meet at a single point in the diagram, i.e., GR passes the tests posed by these four distinct constraints.

The orbital decay due to the emission of gravitational waves has been measured for six DNS systems: PSR B1913+16, J0737-3039A/B, B1534+12, J1756-2251, J1906+0746 and B2127+11C.



Credits: Kramer et al. 2006

# Millisecond pulsars

- Millisecond pulsars (MSPs) refers to pulsars with spin periods in the range  $1.39 < P < 20$  ms and  $\dot{P} < 10^{-19}$ .
- These systems have been heavily recycled by a long-lived accretion phase in a low-mass X-ray binary (LMXB).
- $\sim 20\%$  are isolated, most of the remaining objects have white dwarf companions and very small orbital eccentricities.
- The fast (and very stable) rotation of MSPs makes their timing significantly more precise than for the pulsars in DNSs. However, their low orbital eccentricities pose a problem for mass determination (and tests of GR), since, in these cases, the PK parameters  $\dot{w}$  and  $\gamma$  cannot be measured accurately.
- MSPs in globular clusters (GCs) have gravitational interactions with nearby stars that, with time, can make an initially circular orbit acquire a substantial eccentricity  $\Rightarrow$  possible to measure  $\dot{w}$  and total mass.



NS families

Strange stars

M and R  
measurements

Double NS  
system

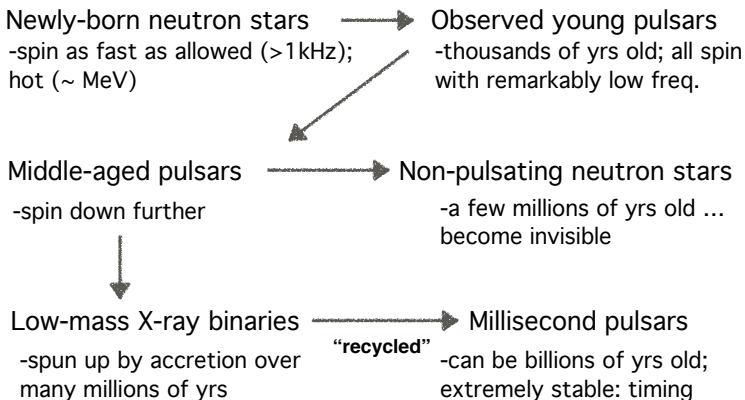
Millisecond  
pulsars

Magnetars

NS glitches

Superfluidity/  
superconductivity

## Generic Evolution of Pulsars



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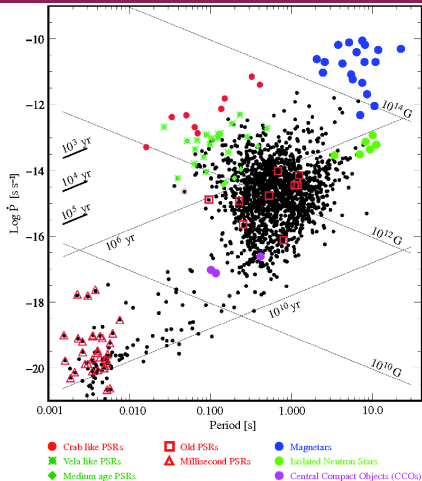
NS glitches

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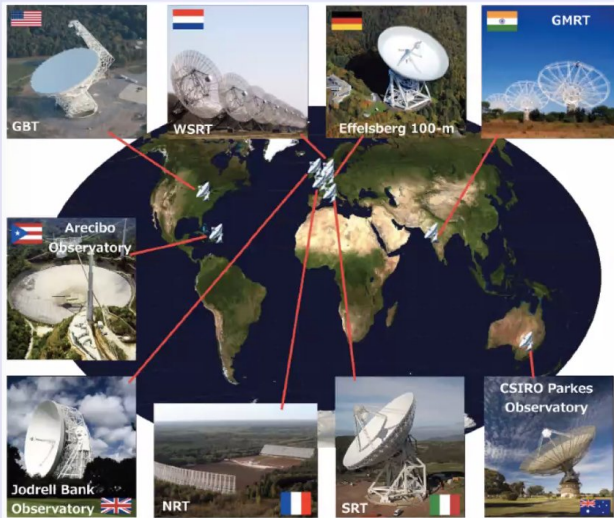
# $P - \dot{P}$ diagram

- NS families
- Strange stars
- M and R measurements
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- Millisecond pulsars
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- Superfluidity/superconductivity



Credits: W.Becker, M.G. Bernhardt, A. Jessner, arXiv:1305.4842

# International Pulsar Timing Array



NS families

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system

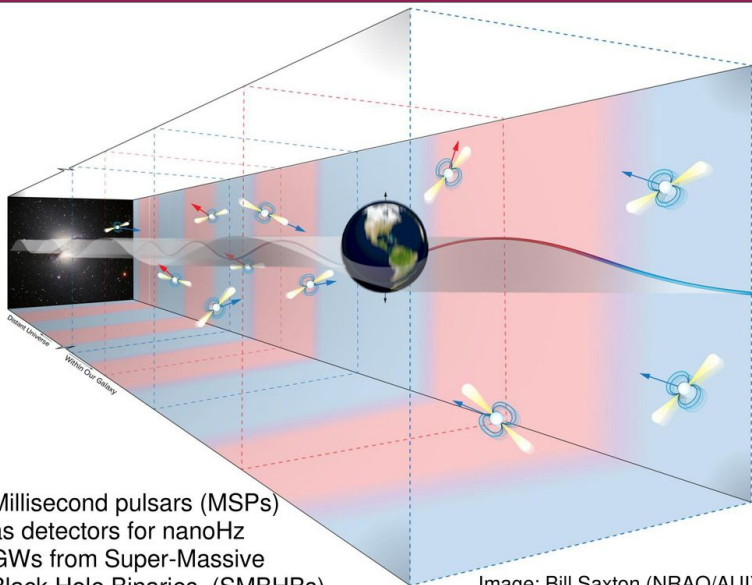
Millisecond  
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Magnetars

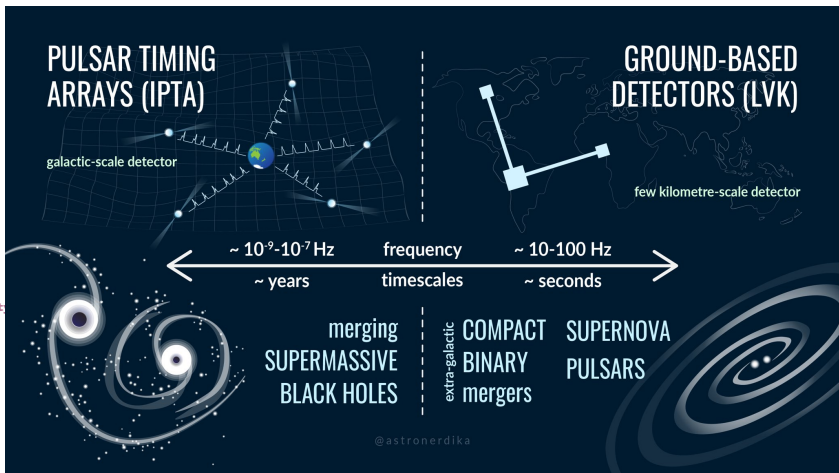
NS glitches

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NS families  
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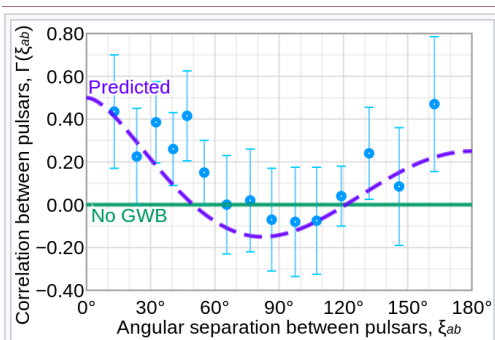


- NS families
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# First results

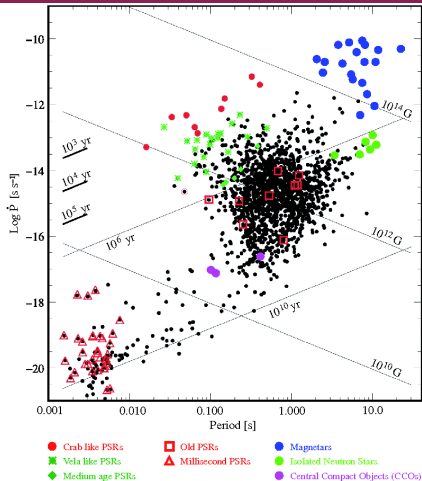
NS families  
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Plot of correlation between pulsars observed by [NANOGrav \(2023\)](#) vs angular separation between pulsars, compared with a theoretical model (dashed purple, or *Hellings-Downs* curve) and if there were no gravitational wave background (solid green)

# $P - \dot{P}$ diagram

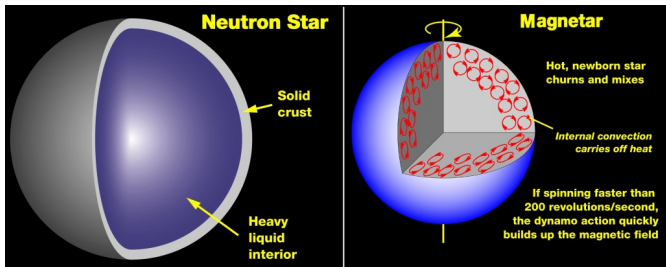
NS families  
Strange stars  
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Millisecond  
pulsars  
**Magnetars**  
NS glitches  
Superfluidity/  
superconductivity



Credits: W.Becker, M.G. Bernhardt, A. Jessner, arXiv:1305.4842

# Magnetars (magnetic star)

- **Ultra-strong magnetic field  $\sim 10^{14} - 10^{15}$  gauss.**  
For comparison: magnetic field strength of Earth  $\sim 0.5$  gauss, normal star  $\sim 100$  gauss, Sunspots  $\sim 3000$  gauss, pulsars  $\sim 10^{12}$  gauss.
- **Young NSs.**
- **Sources of the soft  $\gamma$  repeaters (SGR) and anomalous X-ray pulsars powered by the decay of the magnetic field.**  
SGR is an astronomical object which emits large bursts of  $\gamma$ -rays and X-rays at irregular intervals.
- **Ultra-strong NS field decay, causing extreme heating of crust  $\implies$  high X-ray luminosity.**
- **Starquakes triggered on the surface of the magnetar disturb the magnetic field leading to extremely powerful  $\gamma$ -ray flare emissions.**



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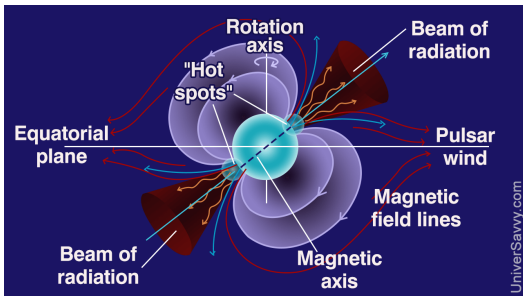
# Magnetars

- During initial formation of NS, rapid spin of core can produce magnetic fields as high as  $10^{15-16}$  G - most NSs do not spin fast upon formation, prohibiting this dynamo from operating; more "typical" pulsar fields result.

Credits: Patric Slane

Strongest magnetic field that could possibly be contained in a NS

$$\frac{B_{max}^2}{8\pi} \sim P \sim \frac{GM^2}{R^4} \implies B_{max} \sim 4 \cdot 10^{18} \text{ G} \quad (2)$$



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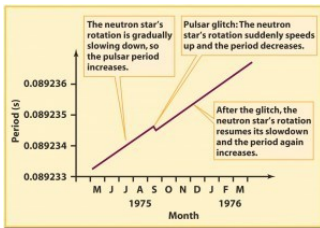


# Glitches

A glitch is a sudden increase in the rotational frequency of a rotation-powered pulsar. Following a glitch is a period of gradual recovery where the observed periodicity slows to a period close to that observed before the glitch. These gradual recovery periods have been observed to last from days to years. Currently, only multiple glitches of the Crab and Vela pulsars have been observed and studied extensively.)

Explanation: Glitches are thought to be the result of a rapid transfer of angular momentum between this inner superfluid and the outer crust, to which the NS magnetosphere is attached and whose radiation we observe.

The process leading to a glitch is believed to be from the interior of the NS, rather than from the magnetosphere which causes the spin down. This is because the structure of pulses observed during glitch events remains unchanged.



Credits: S.A. Abdullah, M.M. Yaseen

NS families

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M and R  
measurements

Double NS  
system

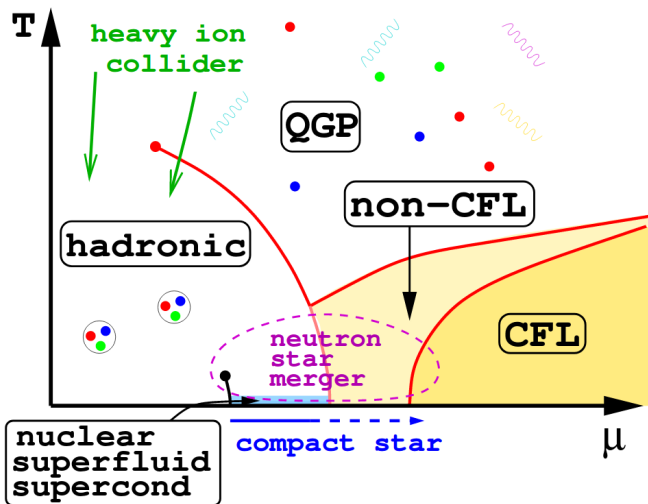
Millisecond  
pulsars

Magnetars

NS glitches

Superfluidity/  
superconductivity

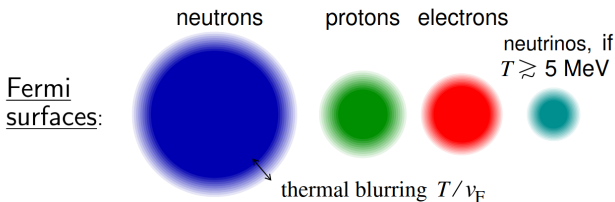
# QCD phase diagram



- NS families
- Strange stars
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# Cooper pairing in NSs

The Cooper pairing appears as a result of the attraction of particles with the anti-parallel momenta, which is expected to occur, at low enough temperature, in any degenerate system of fermions in which there is an attractive interaction between particles whose momenta lie close to the Fermi surface



neutrons:  $\sim 90\%$  of baryons

$$p_{Fn} \sim 350 \text{ MeV}$$

protons:  $\sim 10\%$  of baryons

$$p_{Fp} \sim 150 \text{ MeV}$$

electrons: same density as protons

$$p_{Fe} = p_{Fp}$$

neutrinos: only present if  $\text{mfp} \ll 10 \text{ km}$  i.e. when  $T \gtrsim 5 \text{ MeV}$

Credits: Mark Alford

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# Term symbols

$$2S+1 L_J$$

**S** = total spin quantum number  
**2S+1** = multiplicity (singlet, doublet, triplet, etc.)  
**L** = total orbital quantum number  
**J** = total angular momentum quantum number

<b>L</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>symbol</b>	<b>S</b>	<b>P</b>	<b>D</b>	<b>F</b>	<b>G</b>

Spin-singlet pairs

$$S = 0$$



Spin-triplet pairs

$$S = 1$$

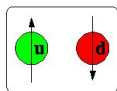


## Example:

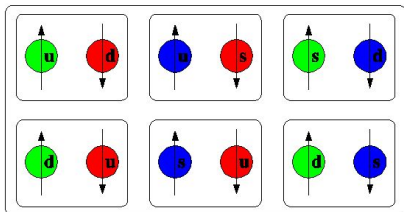
$$\begin{array}{l}
 s_1, s_2, \dots \rightarrow S \\
 l_1, l_2, \dots \rightarrow L \\
 S, L \rightarrow J
 \end{array}
 \left| \Rightarrow \right.
 \begin{array}{l}
 s_1 = +1/2 \quad s_2 = -1/2 \Rightarrow S = 0 \quad \text{- singlet state} \\
 s_1 = +1/2 \quad s_2 = +1/2 \Rightarrow S = 1 \quad \text{- triplet state} \\
 2^{*0+1} S_0 = {}^1 S_0 \qquad 2^{*1+1} P_2 = {}^3 P_2
 \end{array}$$

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## Spin-zero pairings



2-flavor SC (2SC)  
Bailin, Love (1984)



Color-flavor-locking  
(CFL)  
Alford, Rajagopal,  
Wilczek (1984)

# Lecture Tomorrow: Multi-Messenger Astrophysics

## FIRST COSMIC EVENT OBSERVED IN GRAVITATIONAL WAVES AND LIGHT

Colliding Neutron Stars Mark New Beginning of Discoveries

Collision creates light across the entire electromagnetic spectrum. Joint observations independently confirm Einstein's General Theory of Relativity, help measure the age of the Universe, and provide clues to the origins of heavy elements like gold and platinum

Gravitational wave lasted over 100 seconds

On August 17, 2017, 12:41 UTC, LIGO (US) and Virgo (Europe) detect gravitational waves from the merger of two neutron stars, each around 1.5 times the mass of our Sun. This is the first detection of spacetime ripples from neutron stars.

Within two seconds, NASA's Fermi Gamma-ray Space Telescope detects a short gamma-ray burst from a region of the sky overlapping the LIGO/Virgo position. Optical telescope observations pinpoint the origin of this signal to NGC 4993, a galaxy located 130 million light years distant.

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