11th School on Astrophysics and Gravitation
Centre for Astrophysics and Gravitation
Lisbon, IST, September 1-6, 2023

The inflationary universe

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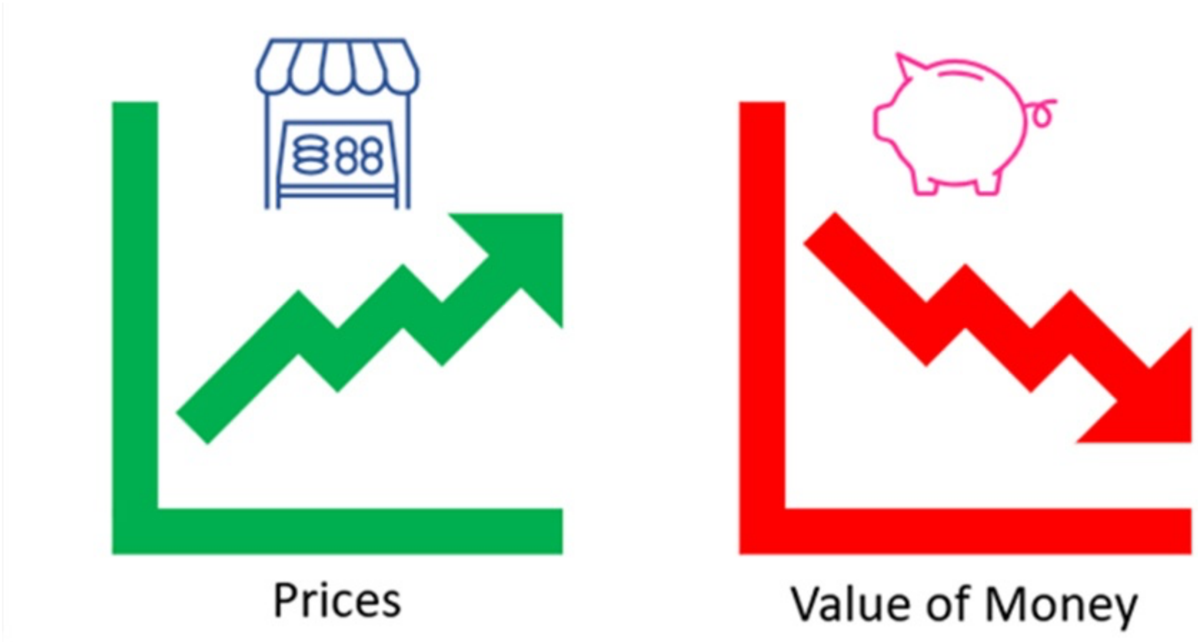
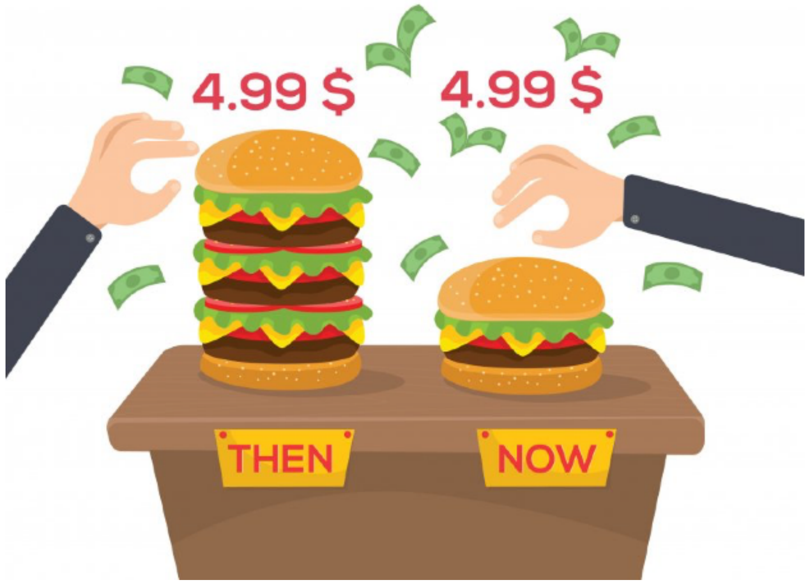
Inflation in Economics

- In economics, **inflation refers to a general rise in the price level** of an economy over a period of time.



- Effectively, it leads to the **reduction in the purchasing power per unit of money**



Economic inflation as the villain

But…

Not all kinds of inflation are bad

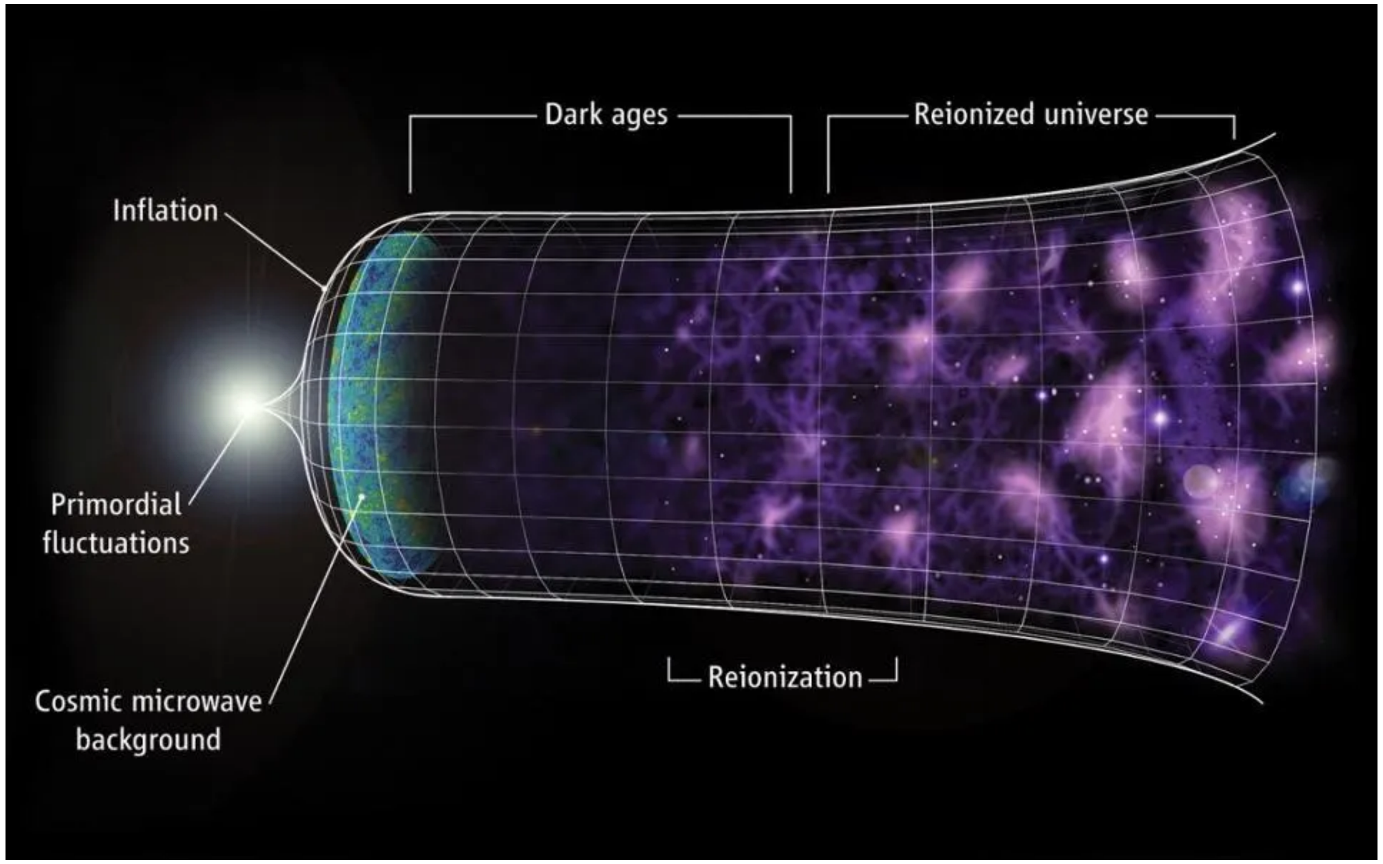
Inflation in Cosmology = cosmic inflation

- In Cosmology, inflation concerns the “size” of the universe (as controlled by the scale factor).

  Recall the scale factor $a(t)$ controls the time-evolution of a spatially homogeneous and isotropic spacetime.

  For instance, in a flat FLRW cosmology, $ds^2 = -dt^2 + a^2(t)\left[\, dr^2 + r^2 d\Omega^2 \,\right]$

- Cosmic inflation is a period of **accelerated** expansion, $\ddot{a} > 0$ (in the early universe).



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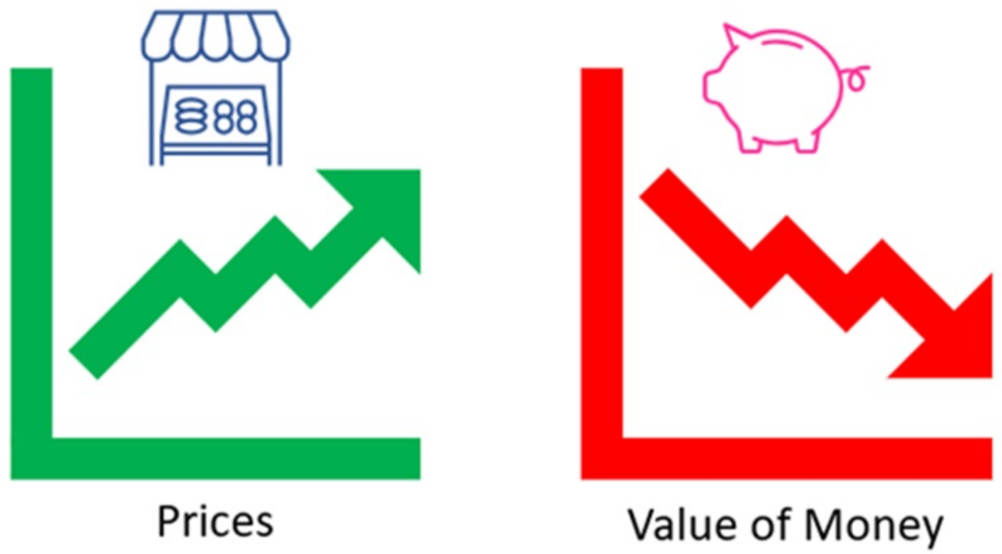
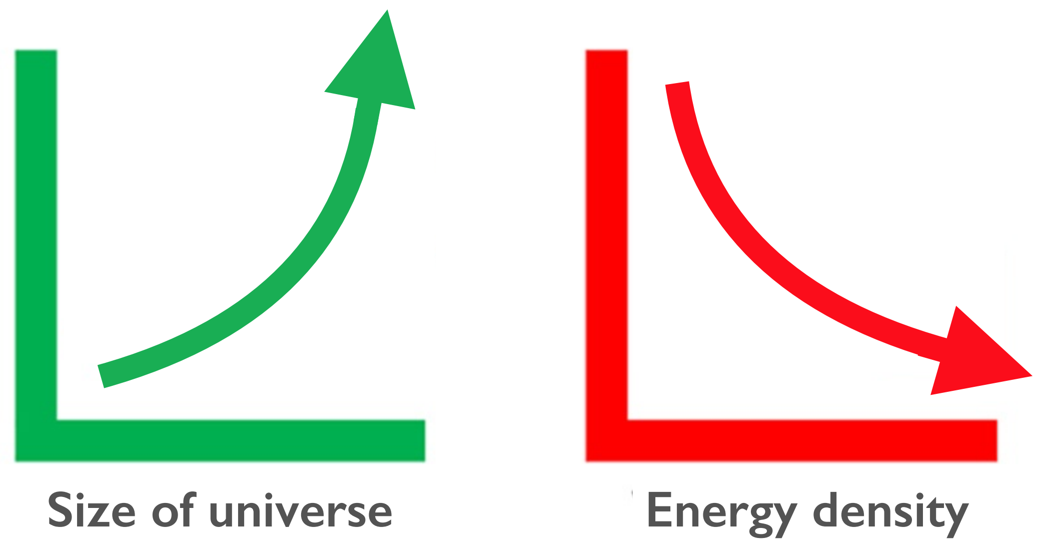
Economic inflation vs. Cosmic inflation





In Cosmology, inflation is the hero.

Cosmic inflation ⊂ Cosmology

- Cosmic inflation is closely connected with the subject of other lectures in this summer school:

  - **Cosmology** (by Herdeiro) — standard cosmology has several shortcomings that cosmic inflation can solve.

  - **Cosmic Microwave Background** (by Carloni) — cosmic inflation leaves distinctive imprints on the CMB.

The idea of Inflation plays a major role in modern Cosmology

- It is a cornerstone of the theory of the Big Bang. (It completes the theory.)

- Its importance was has been officially recognized at the highest level:

2014 KAVLI PRIZE LAUREATES
IN ASTROPHYSICS kavliprize.org

Awarded to Alan H. Guth, Andrei D. Linde, Alexei A. Starobinsky

"for pioneering the theory of cosmic inflation."

Was not awarded (yet) the Nobel prize because it is not experimentally confirmed beyond doubt…

…but its supporting body of evidence is very convincing.

The birth of Inflation

SV (5)
Dec 7, 1979

SPECTACULAR REALIZATION:

This kind of supercooling can explain why the universe today is so incredibly flat — and therefore ~~why~~ resolve the fine-tuning paradox pointed out by Bob Dicke in his Einstein day lectures.

Let me first rederive the Dicke paradox. He relies on the empirical fact the the deacceleration parameter today q_0 is of order 1.

$$q_0 \equiv - \ddot{R} \frac{R}{\dot{R}^2}$$

Use the eqs of motion

$$3\ddot{R} = -4\pi G(\rho + 3p)R$$
$$\dot{R}^2 + k = \frac{8\pi G}{3}\rho R^2 .$$

so

Outline

- Review of standard Cosmology
- Shortcomings of standard Cosmology
- The inflationary paradigm (as a solution to problems of standard Cosmology)
- How to obtain inflation?
- How to end inflation? (graceful exit)
- How to confirm or refute inflation? (imprints on CMB, …)
- Open problems

Standard Cosmology (review)

- The Friedmann-Lemaitre-Robertson-Walker metrics,

$$ds^2 = -dt^2 + a(t)^2\left[\frac{dr^2}{1-\kappa r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right] \qquad \text{with} \quad \kappa = -1, 0, +1\,,$$

are suitable for spatially isotropic and homogeneous spacetimes

- Feeding the Einstein equations, $R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$, with such an ansatz for the metric yields the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2}$$

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p\, g_{\mu\nu}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3\,p)$$

Question 1: Show that the 1st Friedmann equation can be obtained by purely Newtonian considerations, namely the conservation of energy governing the isotropic dynamics of a sphere of mass M with radius *a(t)*, if the total energy (kinetic plus potential) is denoted by $-\kappa/2$.

Standard Cosmology (review)

- The equation of state: $$\boxed{p = (\gamma - 1)\,\rho}$$

- Conservation of energy: $$0 = \nabla_\mu T^{\mu 0} = -\dot{\rho} - 3\frac{\dot{a}}{a}(\rho + p) = -\dot{\rho} - 3\gamma\frac{\dot{a}}{a}\rho$$

$$\boxed{\frac{\dot{\rho}}{\rho} = -3\gamma\frac{\dot{a}}{a}} \Rightarrow \boxed{\rho \propto a(t)^{-3\gamma}} \quad \text{if } \gamma = \text{ const.}$$

- Special cases of perfect fluids:

Dust (pressureless matter)	Radiation (relativistic)	Cosmological constant
$\rho_M \propto a(t)^{-3}$	$\rho_R \propto a(t)^{-4}$	$\rho_\Lambda \propto a(t)^0$

Standard Cosmology (review)

- Hubble parameter: $$\boxed{H(t) := \frac{\dot{a}(t)}{a(t)}}$$ $$H_0 := \frac{\dot{a}(\text{now})}{a(\text{now})} \simeq 70\ \text{km/s/Mpc}$$

- Density parameter(s): $$\boxed{\Omega := \frac{8\pi G}{3H^2}\rho}$$

- With the definitions above, the 1st Friedmann eq. can be written as

$$\boxed{\Omega - 1 = \frac{\kappa}{H^2 a^2}}$$

- This relates density parameter with spatial curvature of universe:

$$\Omega < 1 \ \Rightarrow \ \kappa < 0 \qquad \Omega = 1 \ \Rightarrow \ \kappa = 0 \qquad \Omega > 1 \ \Rightarrow \ \kappa > 0$$

Observations indicate this is the case…

Standard Cosmology (review)

- Distinct contributions to the energy density, $\rho = \rho_M + \rho_R + \rho_\Lambda$

leads to alternative cosmological models (should keep $\Omega_M + \Omega_R + \Omega_\Lambda = 1$)

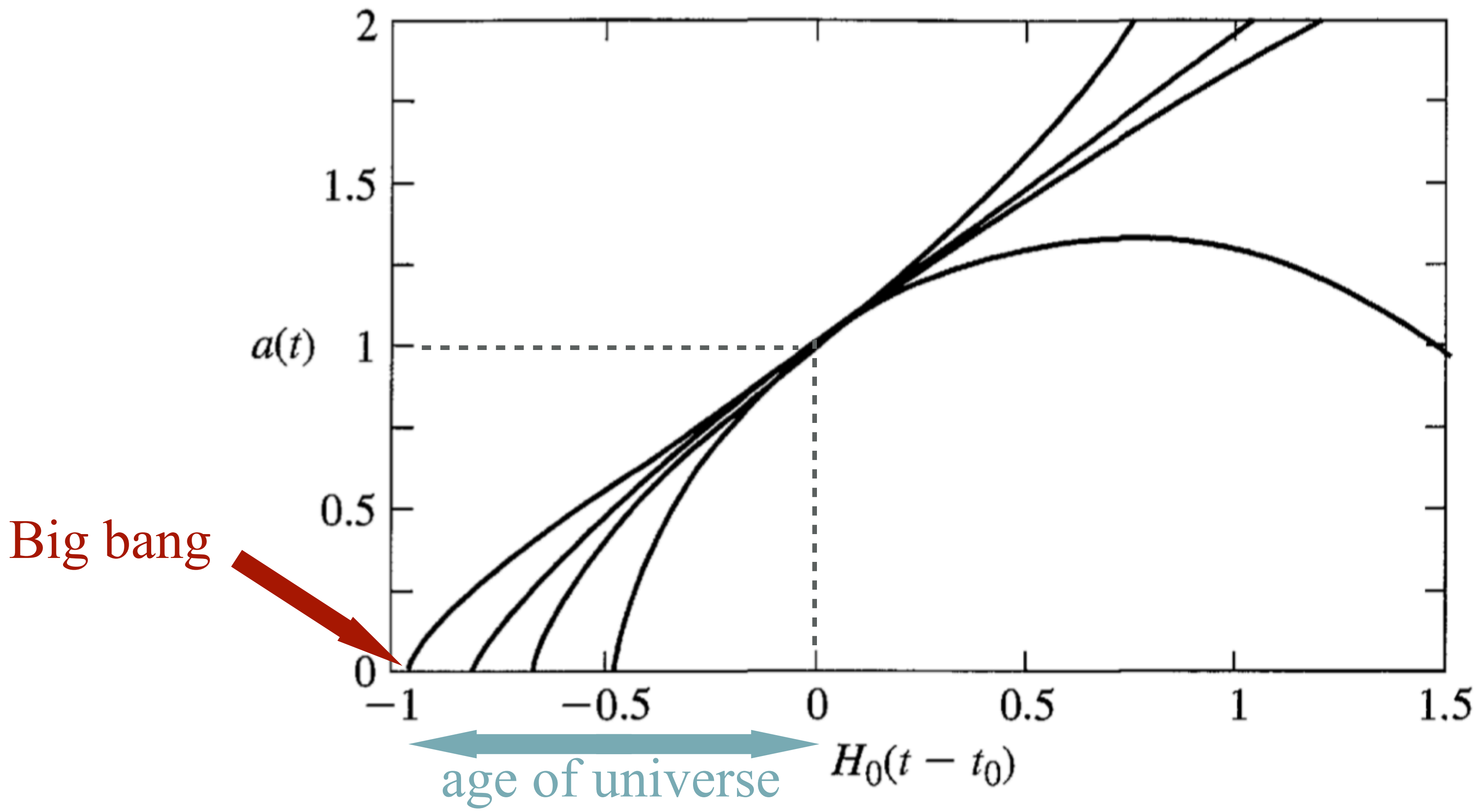


FIGURE 8.3 Expansion histories for different values of Ω_M and Ω_Λ. From top to bottom, the curves describe $(\Omega_M, \Omega_\Lambda) = (0.3, 0.7)$, $(0.3, 0.0)$, $(1.0, 0.0)$, and $(4.0, 0.0)$.

from Carroll's "Spacetime and geometry"

Standard Cosmology (review)

- Since $\Omega_\Lambda \propto \Omega_M\, a(t)^3$ $\Rightarrow$ matter dominates at early times

  $\Rightarrow$ cosmological constant dominates at late times (assuming an eternally expanding universe)

- In a flat, **matter-dominated universe** (with no cosmological constant):

$$\kappa = 0\,, \qquad \Omega = \Omega_M = 1 \qquad \Rightarrow\ \rho \propto a(t)^{-3}$$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho \ \propto\ a^{-3} \qquad \Rightarrow \boxed{a(t) \propto t^{2/3}}$$

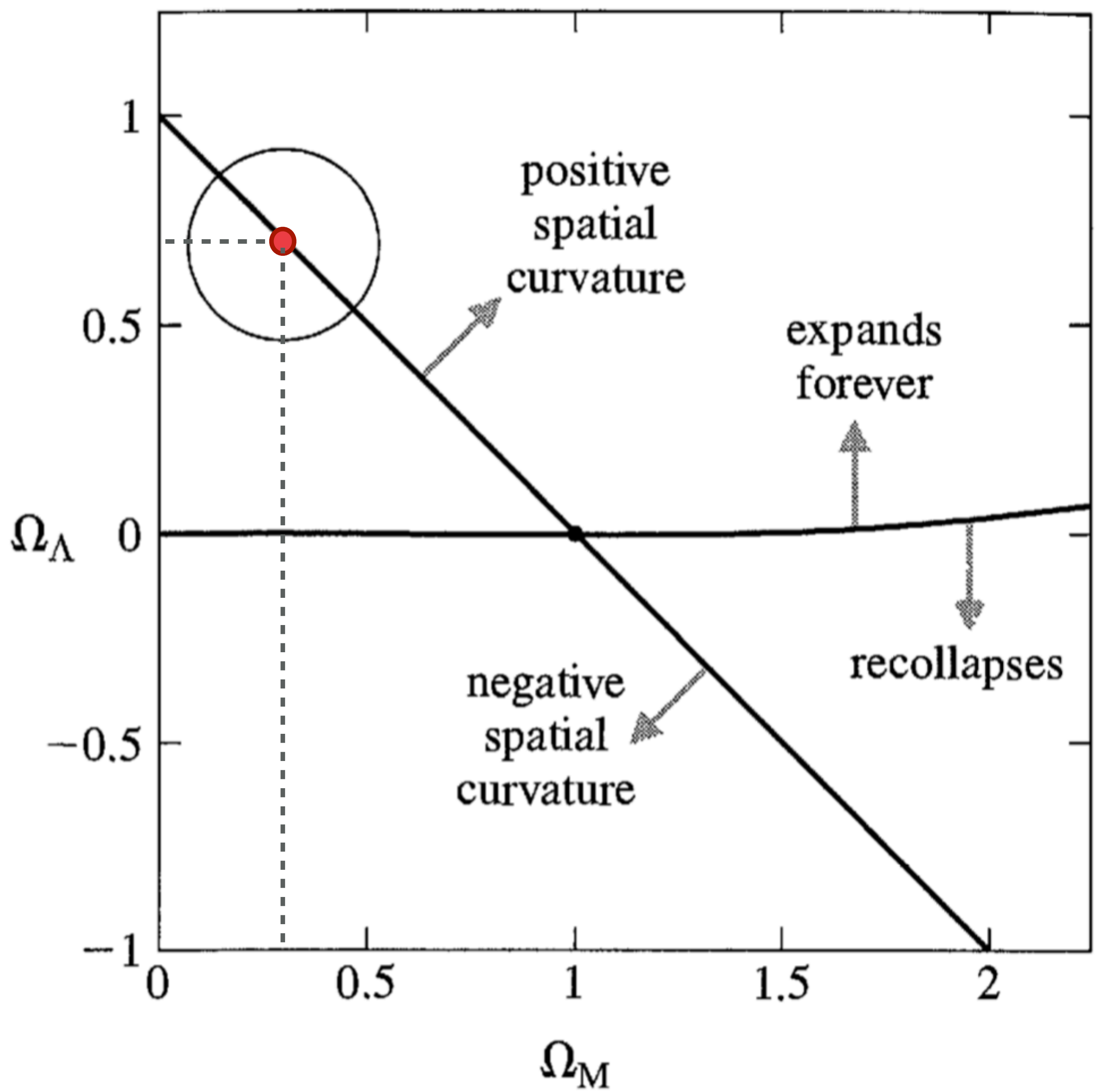
- In a flat **universe dominated by the cosmological constant**:

$$\kappa = 0\,, \qquad \Omega = \Omega_\Lambda = 1 \qquad \Rightarrow\ \rho \propto a(t)^0$$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho \ = \text{const.} = \frac{\Lambda}{3} \qquad \Rightarrow \boxed{a(t) \propto e^{\sqrt{\Lambda/3}\ t}}$$

exponential growth!

- Our universe



from Carroll's "Spacetime and geometry"

Why do we (think we) need inflation?

- Inflation is an idea to solve several theoretical problems in cosmology simultaneously: (But also supporting evidence from CMB observations.)

  - Flatness problem
  - Horizon problem
  - Monopole problem
  - Isotropy problem

- "The inflationary universe picture [...] offers the possibility of killing several of these problematic birds with one not too unreasonable stone."

John D. Barrow in "The Inflationary Universe: Modern Developments" (1988)

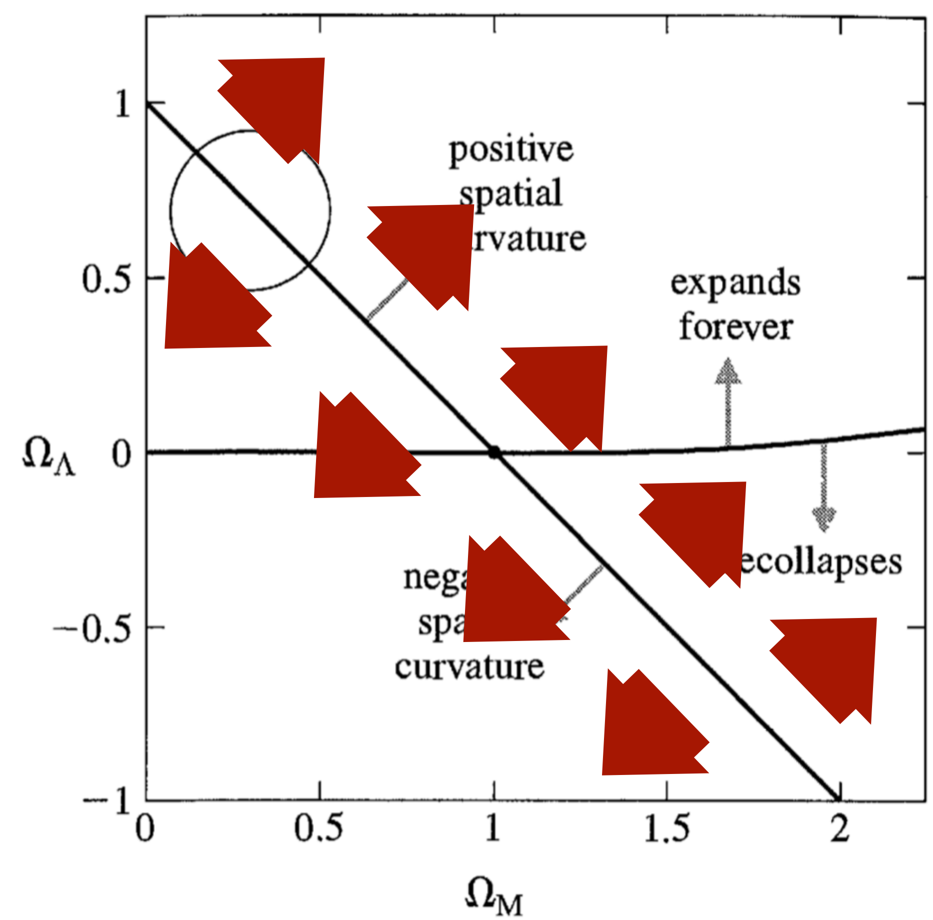
The flatness problem

- As seen, the present universe has $\Omega \simeq 1$.

  However, this is an unstable point of dynamical equations. (At least for $\gamma > 2/3$.)

  Question 2: Show that the Friedmann eqs. imply $\dfrac{d\Omega}{dt} = (3\gamma - 2)\, H\, \Omega\, (\Omega - 1)$.

- To have $\Omega \simeq 1$ today, we must assume that Ω was **fine-tuned** to within $|1-\Omega| \leq 10^{-58}$ at the Planck time…



The horizon problem

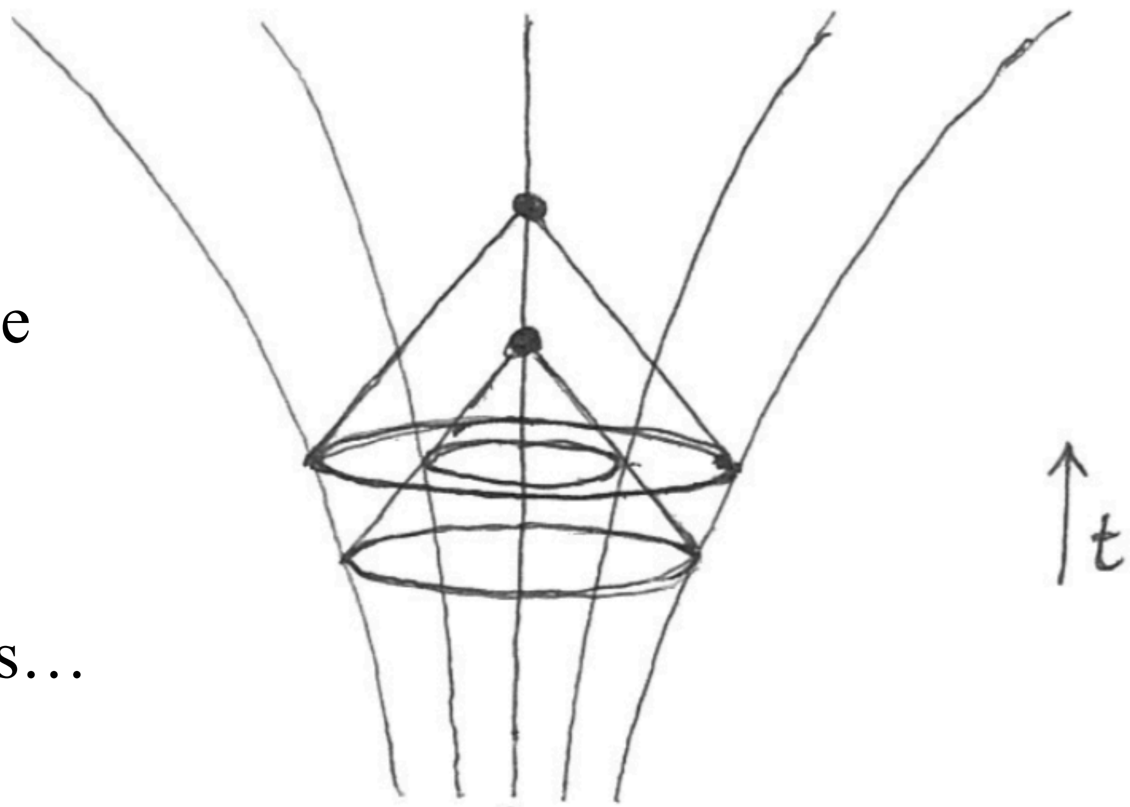
- FLRW have **cosmological horizons**.
  A given point can be in causal contact only with points within a distance

$$d_H(t) = a(t)\int_0^t \frac{dt'}{a(t')}$$

For power-law expansion this grows linearly in time,
but for exponential expansion this grows exponentially in time!

- Looking at opposite directions in the sky we can observe portions of spacetime that supposedly were never in causal contact.

Nevertheless, they show very similar properties…



The horizon problem

- At early times $a(t) \propto t^{\frac{2}{3\gamma}}$.
  So, once again, the value $\gamma = 2/3$ seems to be special.

- If $\gamma > \dfrac{2}{3}$, then at early times $d_H(t) \ll a(t)$ and the universe expands faster than the horizon.
  If $\gamma < \dfrac{2}{3}$, then at early times $d_H(t) \gg a(t)$ and no such problem arises.

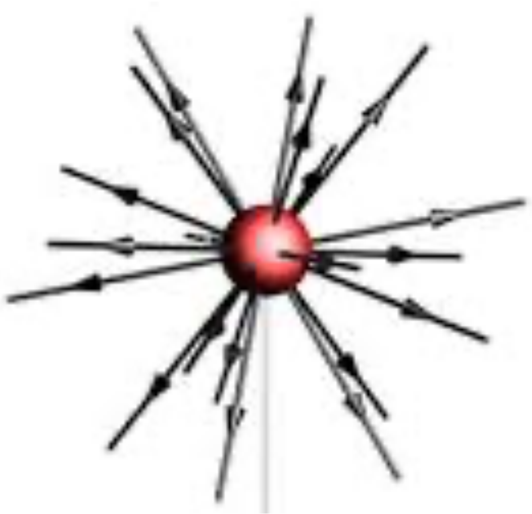
Postulating a period of expansion dominated by some form of matter with $\gamma < 2/3$ solves both the flatness and the horizon problems.

$$\Leftrightarrow \boxed{\rho + 3p < 0}$$

The monopole problem

- In Grand Unified Theories (GUT) the appearance of **monopoles** is inevitable.

  These are a type of **topological defect** originated by symmetry breaking events as the universe cools down.



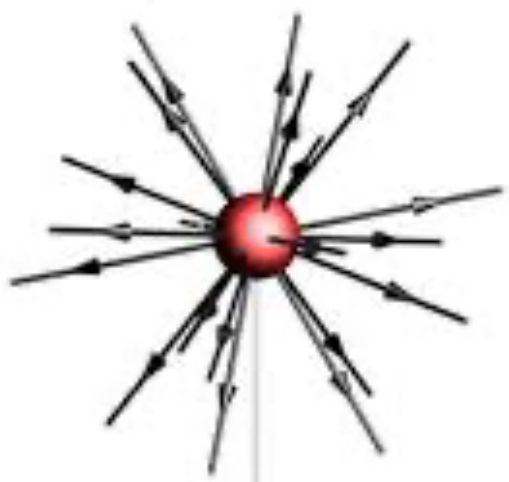
- Their **abundance** is determined by the correlation length of the fundamental field undergoing symmetry breaking in the Grand Unification Scheme, which is bounded (from above) by the cosmological horizon.

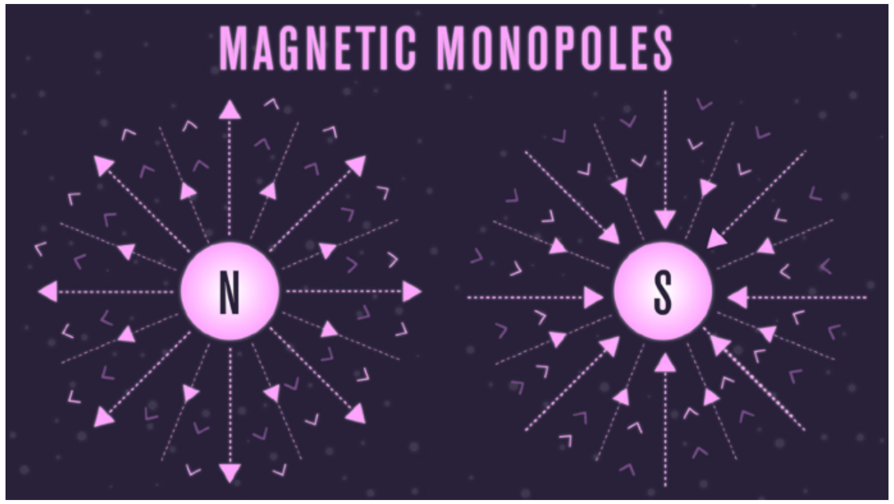
  Upshot: *we generically expect ~1 such exotic relics per Hubble sphere.*

  [A Hubble sphere is a sphere whose radius is $d_H(t)$]

The monopole problem

- Energy considerations imply that these monopoles carry a net magnetic charge.





But unlike Dirac's magnetic monopole, **the mass of GUT monopoles is extraordinarily large**, since it is fixed by the energy scale at which the unification of forces occurs.

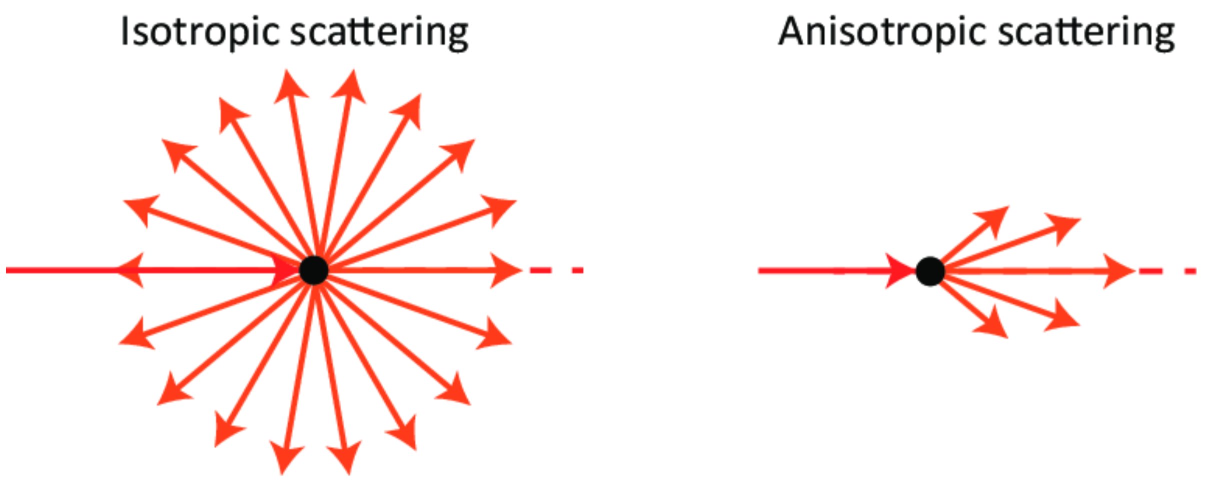
- In standard cosmology they would contribute way more to the total energy density of the universe than what is observed.

Postulating a period of very rapid expansion solves this problem simply by diluting the monopoles.

The isotropy problem

- In a universe dominated by dust or radiation, anisotropies makes the dynamics increasingly anisotropic (no matter how tiny they initially where).

- Yet, we observe a highly isotropic state.



- Once again, a fluid with $\gamma < 2/3$ would solve the problem.
Anisotropies contribute additional terms to the Friedmann equation, which in this case make the anisotropies die off faster than the isotropic matter stress.

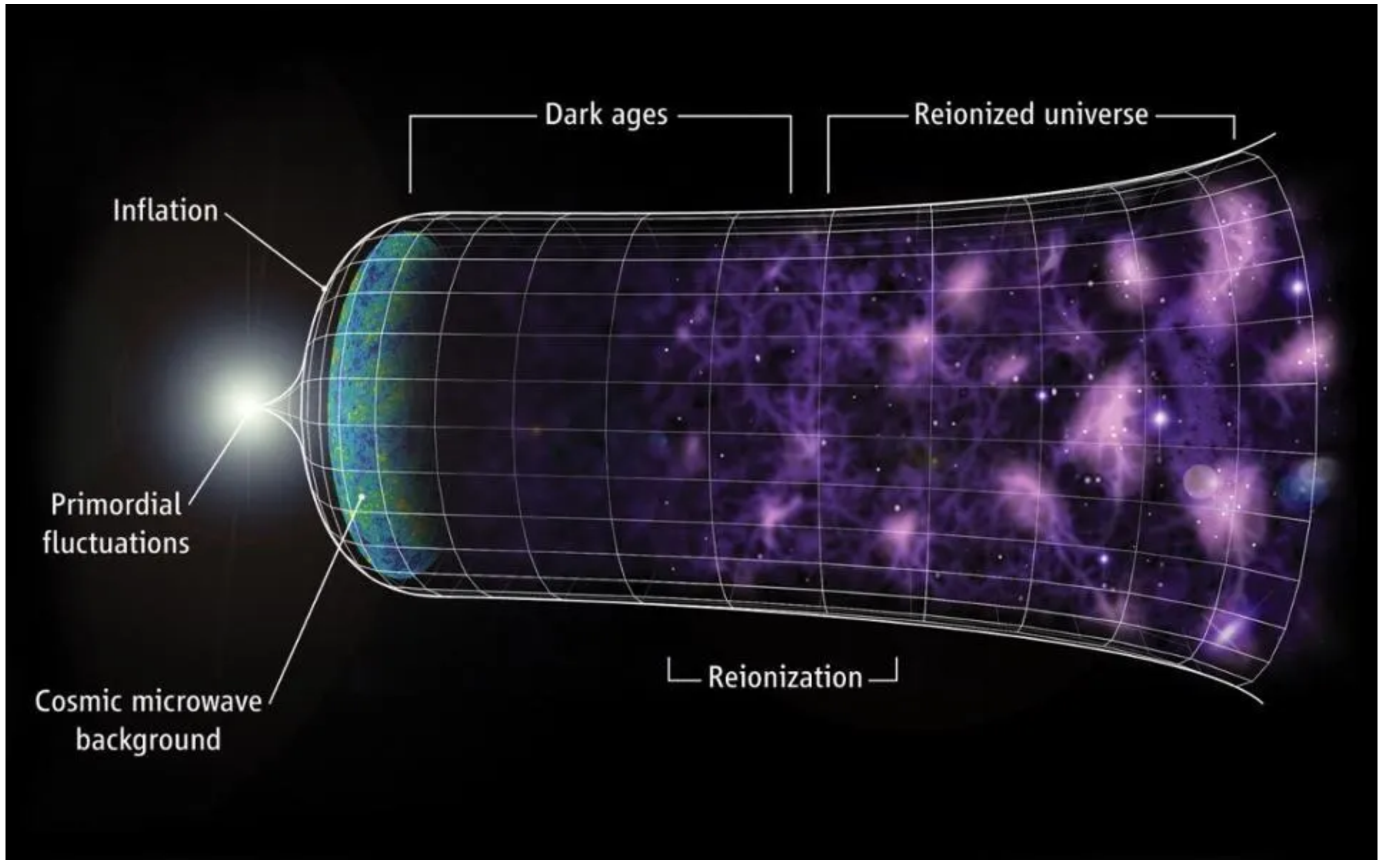
The inflationary paradigm

- So we want to postulate a period during which $\rho + 3p < 0$.

- This represents a violation of the strong energy condition (meaning that gravity has a net repulsive effect in such a case).

- From the 2nd Friedmann eq. we see this yields precisely accelerated expansion:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$



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The inflationary paradigm

- “[…] if there is a period of *accelerated* expansion then it is possible for the present Visible Universe to have evolved from a much smaller, and causally coherent primordial region. In this way, a region smaller than the horizon size at the grand unification epoch can have evolved into a region that is larger than the entire visible portion of the Universe today. The microscopic smoothness and graininess of that primordial region will be reflected in the present large-scale structure of the visible universe.”

John D. Barrow in “The Inflationary Universe: Modern Developments” (1988)

How much inflation is needed?

- The amount of inflation occurred is traditionally measured by the number of *e*-folds:

$$N = \int_{a_\mathrm{i}}^{a_\mathrm{f}} \mathrm{d}\ln a = \int_{t_\mathrm{i}}^{t_\mathrm{f}} H\,\mathrm{d}t.$$

- We need around 60 *e*-folds to successfully address the horizon problem.

Question 3: Show this, knowing that the condition is that the largest scales observed today should be within the horizon at the beginning of inflation: $$\frac{1}{a_0 H_0} < \frac{1}{a_i H_i}$$

Use $a_i/a_f = \exp(-N)$

$T \sim 1/a$

$H_0 \simeq 10^{-42}$ GeV	$H_i \sim 10^{15}$ GeV
$T_0 \simeq 10^{-13}$ GeV	$T_f \sim 10^{8} - 10^{12}$ GeV

to show that

$$N > \ln\left(T_0/H_0\right) + \ln\left(H_i/T_f\right) \approx 67 + \ln\left(H_i/T_f\right)$$

Note: this is a crude estimate, assuming Hubble rate remains constant throughout inflation.

How to obtain inflation?

- Recall we want some matter (or something interpreted effectively as matter) with $\rho+3p < 0$.

- An obvious possibility is a cosmological constant ($p = -\rho$). But this is not viable because it is eternal …

- The most explored option is to consider a self-interacting scalar field, the *inflaton*, ϕ . This allows the possibility of adjusting its potential $V(\phi)$ at will …

- Another possibility: include higher order corrections in the gravitational theory. For instance, $\mathscr{L} = R - \alpha R^2$. [This can be recast as normal GR with a scalar…]

Slow-roll inflation

- In an isotropic and homogeneous universe the inflaton has:

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \qquad \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

so we want the potential energy to dominate over the kinetic energy for the inflationary period:

$$\rho + 3p = 2(\dot{\phi}^2 - V(\phi))$$

- This is precisely what slow-roll models accomplish.

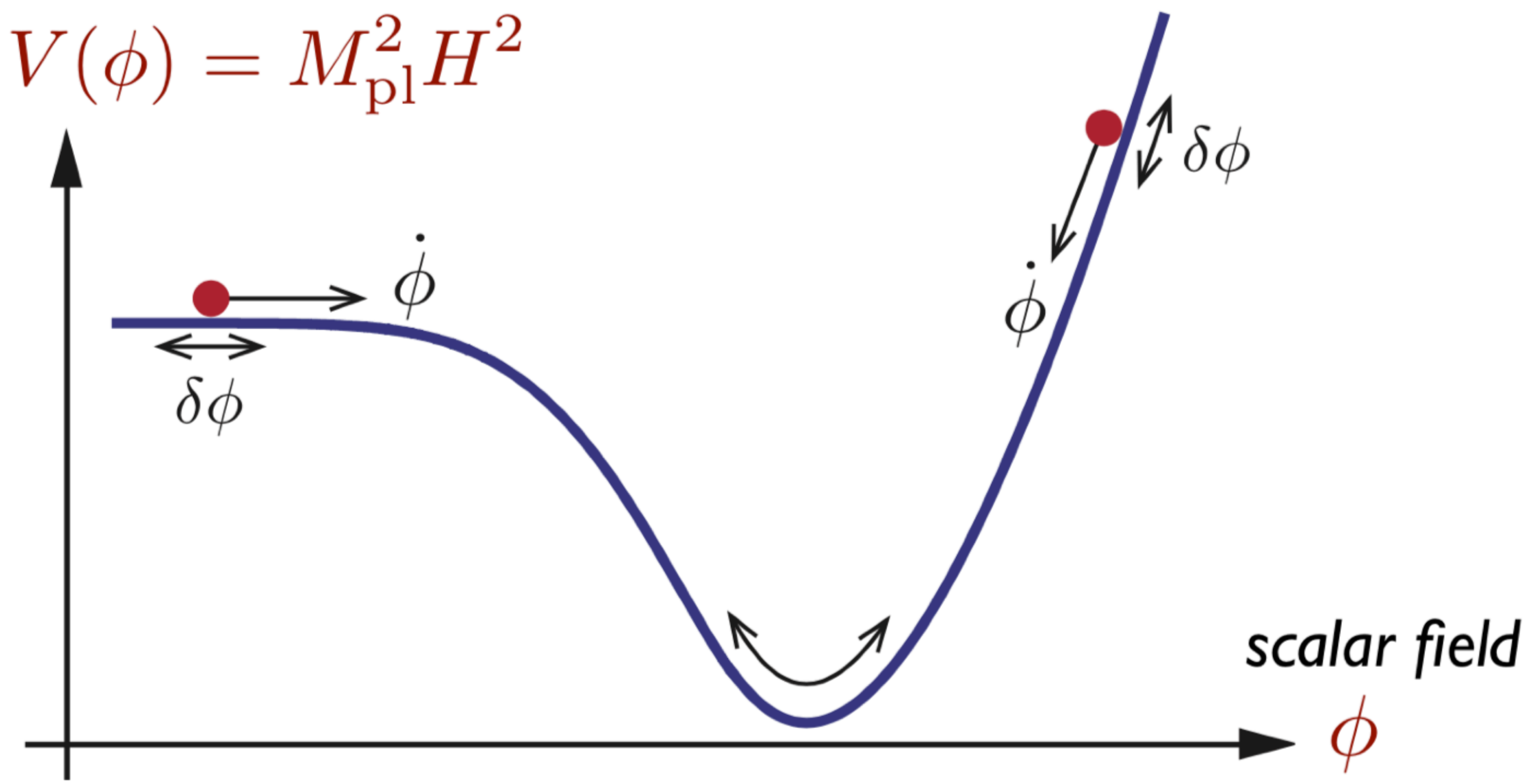
- Summing up:

Slow-roll parameters:

- first slow-roll condition: $(\phi')^2 < V \longleftrightarrow \varepsilon_V \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 < 1$
- second slow-roll condition: $\phi'' < 3H\phi' \longleftrightarrow \eta_V \equiv M_{\rm pl}^2 \left|\frac{V_{,\phi\phi}}{V}\right| < 1$

(first conditions must last)

Slow-roll models



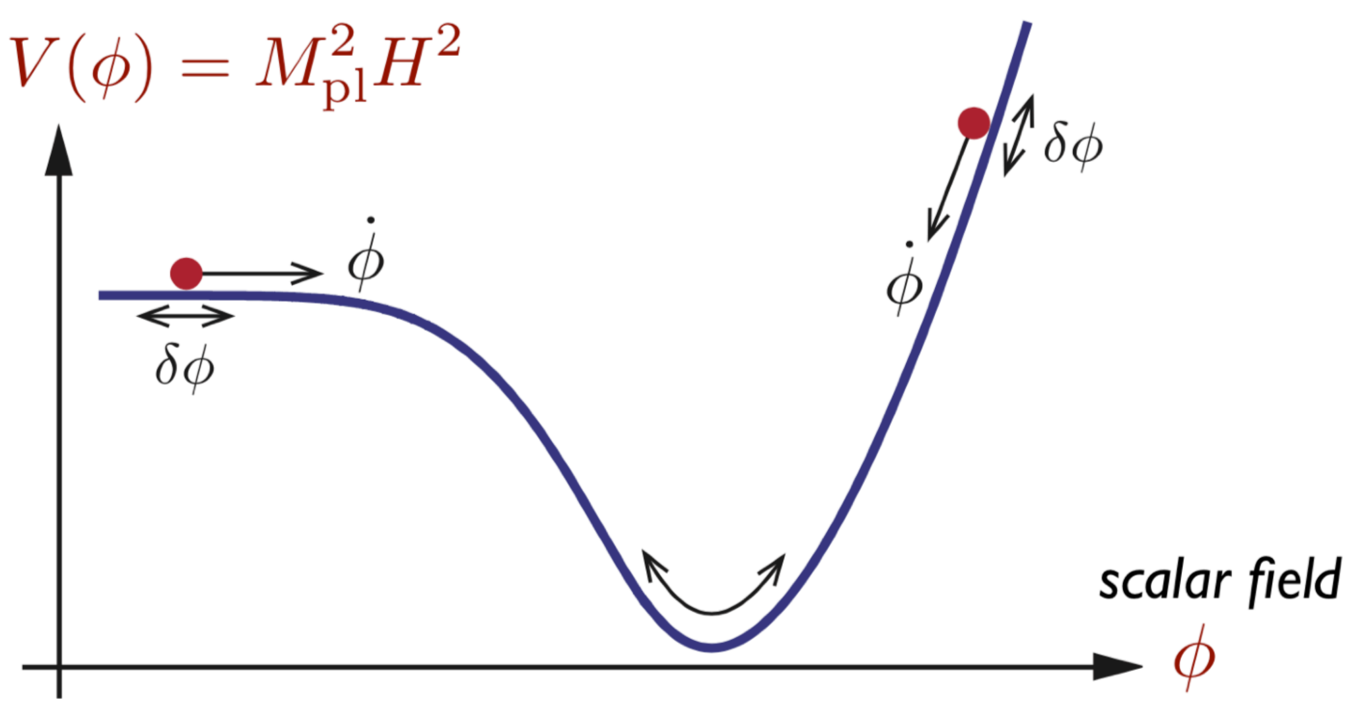
from Baumann (2011)

How to end inflation?

- To end inflation, the inflaton just needs to violate the slow-roll conditions.

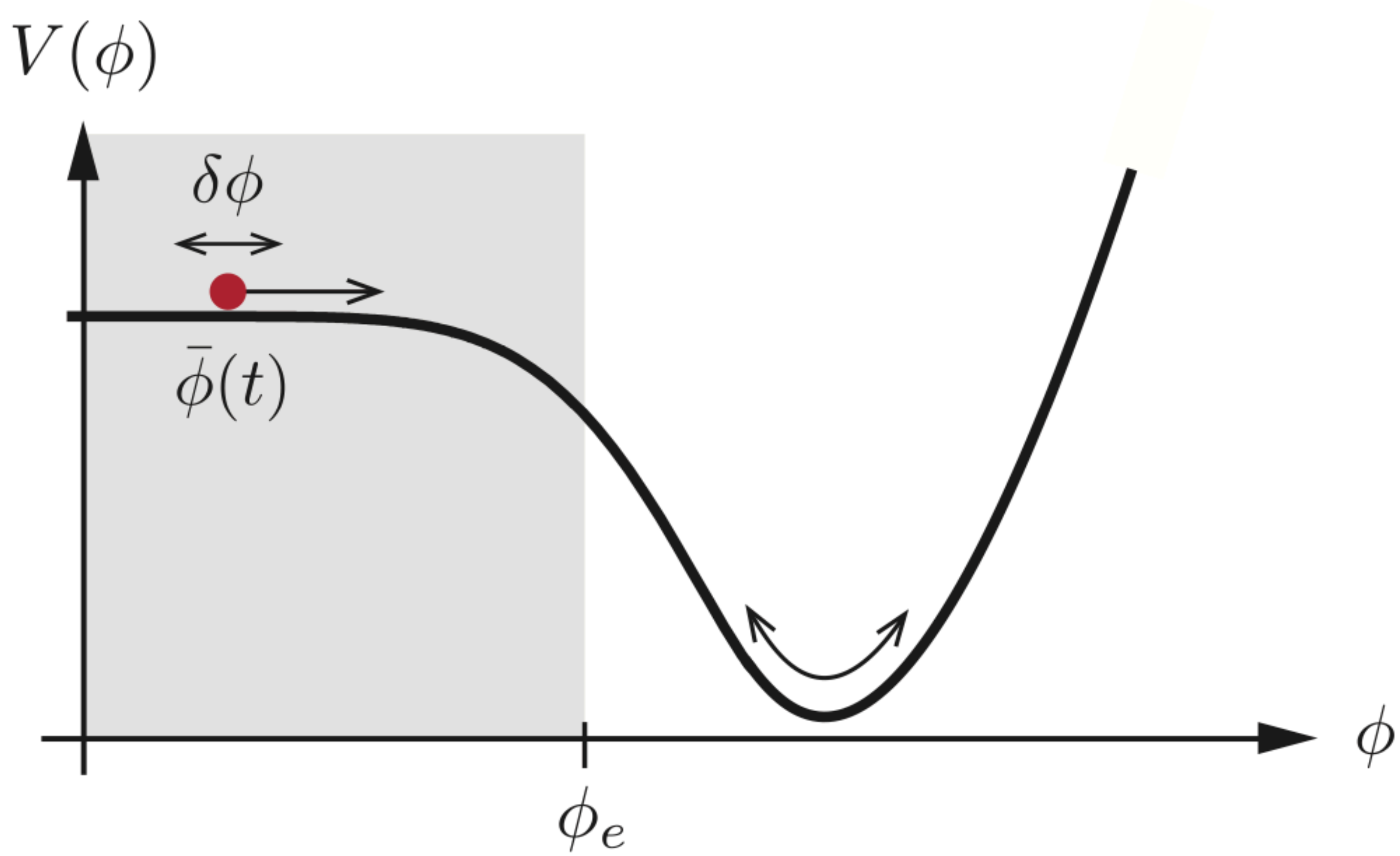
- However, this must be done in a way that is consistent with observations (to preserve the highly successful predictions of standard cosmology, like Big Bang Nucleosynthesis).

- This is the **graceful exit problem**.



from Baumann (2011)

Slow-roll models (and inflaton fluctuations)



from Baumann (2018)

- On top of the background evolution, $\overline{\phi}(t)$, one considers its quantum fluctuations, $\delta\phi(t, \mathbf{x})$.
- Regions acquiring negative fluctuations remain potential-dominated longer than regions with positive $\delta\phi$.
- Therefore, different parts of the universe undergo slightly different evolutions.
  After inflation, this induces density fluctuations $\delta\rho(t, x)$.

Observational evidence for inflation

- The study of the CMB has been the cornerstone of primordial feature searches.

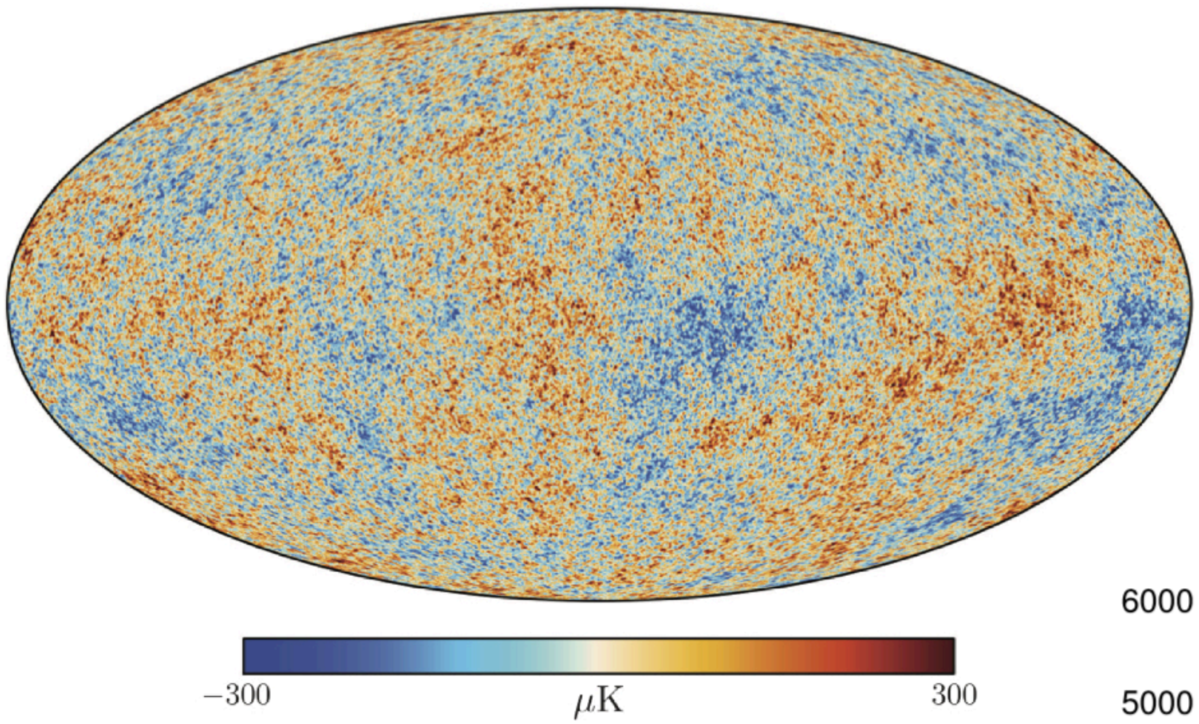
  [Large scale structure surveys and stochastic gravitational background provide complementary routes.]

- It is strongly believed that initial conditions for the Big Bang have been created by **quantum fluctuations during inflation**.

- Inflation predicts specific statistics of the initial conditions, i.e. correlations between the CMB fluctuations in different directions in the sky.

$$\langle \delta T(\hat{\mathbf{n}})\, \delta T(\hat{\mathbf{n}}') \rangle = \sum_l \frac{2l+1}{4\pi} C_l\, P_l(\cos\theta)$$

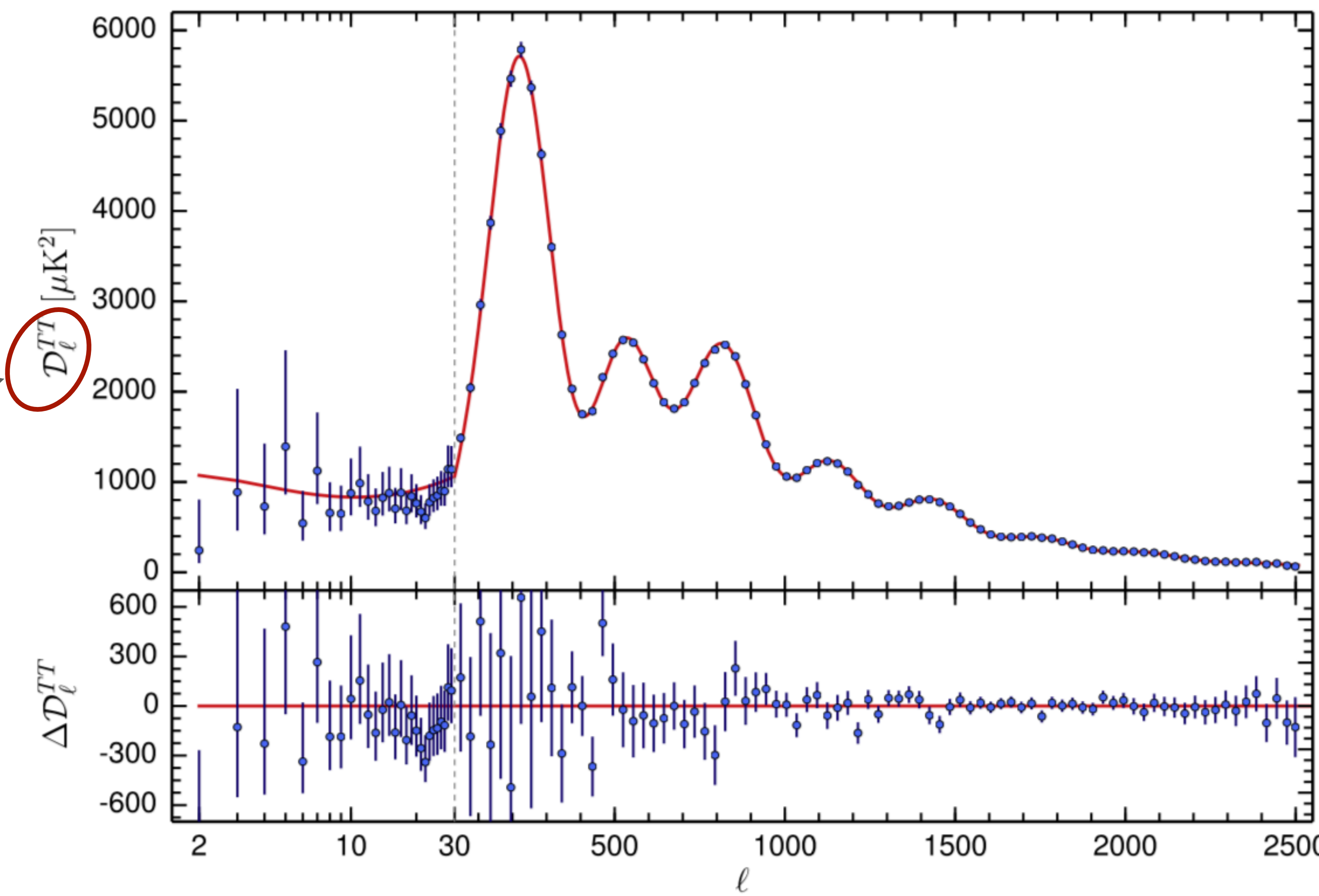
CMB temperature anisotropies



Angular power spectrum

$$\langle \delta T(\hat{\mathbf{n}})\,\delta T(\hat{\mathbf{n}}')\rangle = \sum_l \frac{2l+1}{4\pi} C_l P_l(\cos\theta)$$

Angular power spectrum
(normalized differently)



from Planck 2015

Observational evidence for inflation (curvature perturbations)

- Small fluctuations in the CMB **temperature** reflect spatial variations in the **density** of the primordial plasma.

  (Also related with perturbations of the spacetime geometry,

  $$ds^2 = a^2(\tau)\left[-d\tau^2 + e^{2\zeta(\tau,\mathbf{x})}\,\delta_{ij}\,dx^i dx^j\right],$$

  where ζ is the so-called **curvature perturbation**.)

- It is common to define the dimensionless power spectrum $\mathcal{P}_\zeta(k)$ as

  $$\langle\zeta(\mathbf{k})\zeta^*(\mathbf{k}')\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_\zeta(k)\,\delta_D(\mathbf{k}-\mathbf{k}')$$

- Note: Since these fluctuations are tiny, we can use linear perturbation theory.

Observational evidence for inflation (curvature perturbations)

- For **curvature (scalar) perturbations** one finds:

Scalar power spectrum: $$\mathcal{P}_\zeta(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$$

Scalar spectral index

- The amplitude A_s and the spectral index n_s depend on the slow-roll parameters in the following way:

$$A_s = \frac{1}{24\pi^2} \frac{1}{\varepsilon_V} \frac{V}{M_{\rm pl}^4} \,,$$

$$n_s = 1 - 6\varepsilon_V + 2\eta_V$$

- Note: k_* is some fiducial scale, conventionally taken to be $k_* = 0.05\ \mathrm{Mpc}^{-1}$.

Observational evidence for inflation (tensor perturbations)

- For **tensor perturbations (gravitational waves)**,

$$\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + (\delta_{ij} + 2\gamma_{ij})\mathrm{d}x^i\mathrm{d}x^j\right]$$

  the story is similar, and one finds:

Power spectrum of tensor fluctuations: $\mathcal{P}_\gamma(k) = A_\mathrm{t}\left(\frac{k}{k_*}\right)^{n_\mathrm{t}}$

Tensor spectral index

- In this case only the spectral index n_t depends on (only one of) the slow-roll parameters:

$$A_\mathrm{t} \equiv \frac{2}{\pi^2}\frac{H_*^2}{M_\mathrm{pl}^2},$$

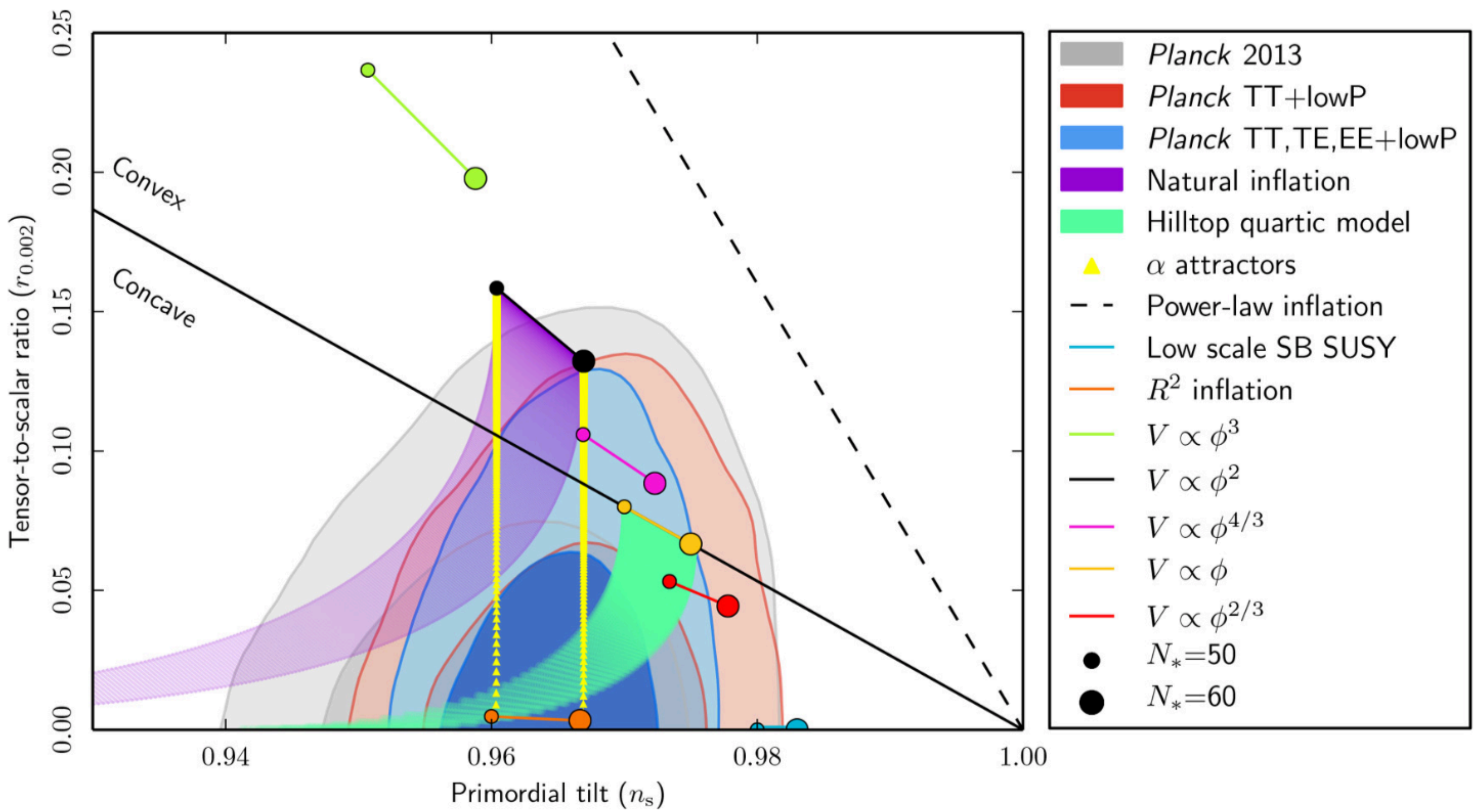
$$n_\mathrm{t} \equiv -2\varepsilon_* .$$

- The (small) tensor-to-scalar ratio, $r \equiv \frac{A_\mathrm{t}}{A_\mathrm{s}} = 16\varepsilon_*$,

  is another prediction of slow-roll models.

Observational evidence for inflation

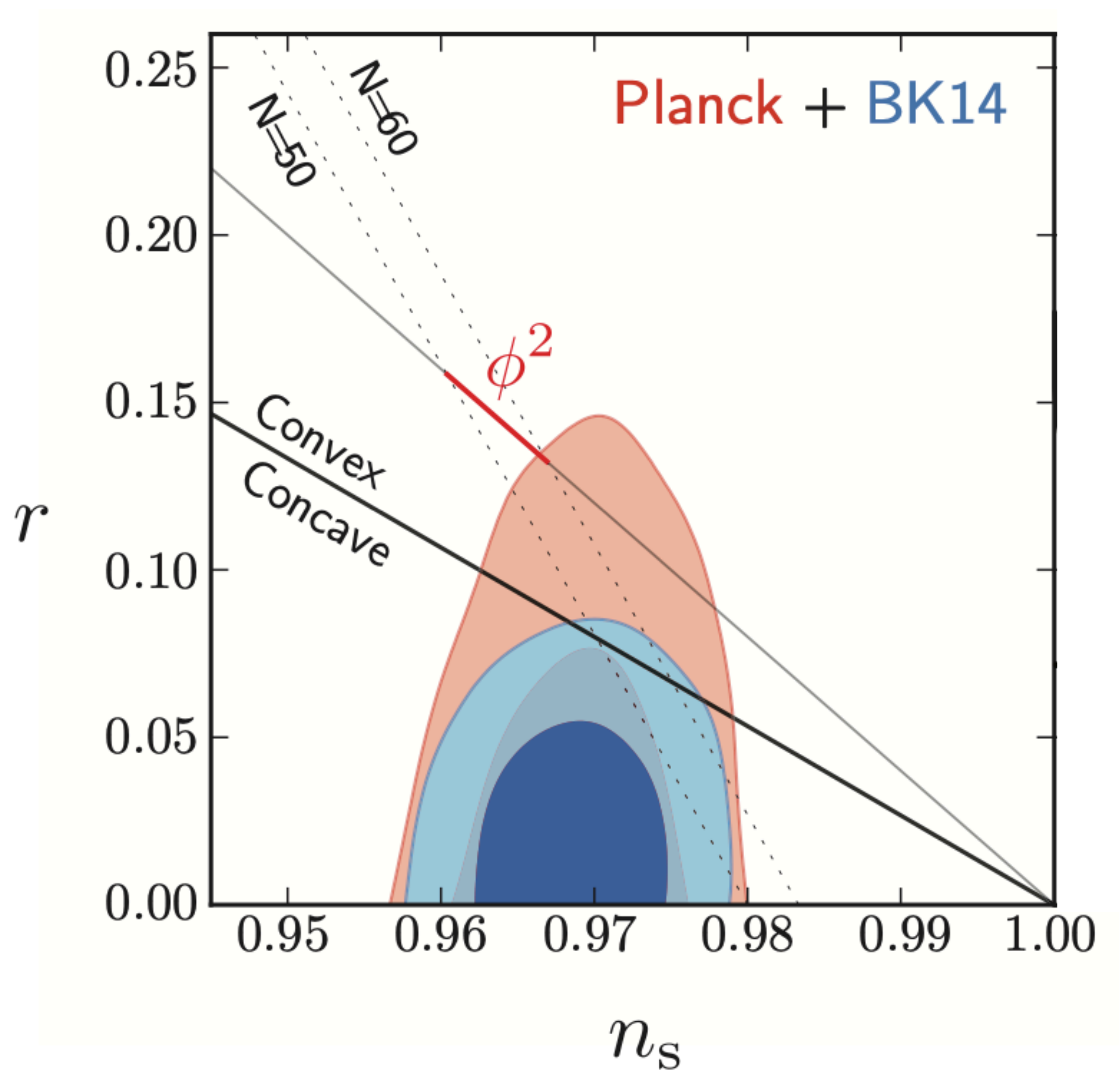
- From Planck data alone:



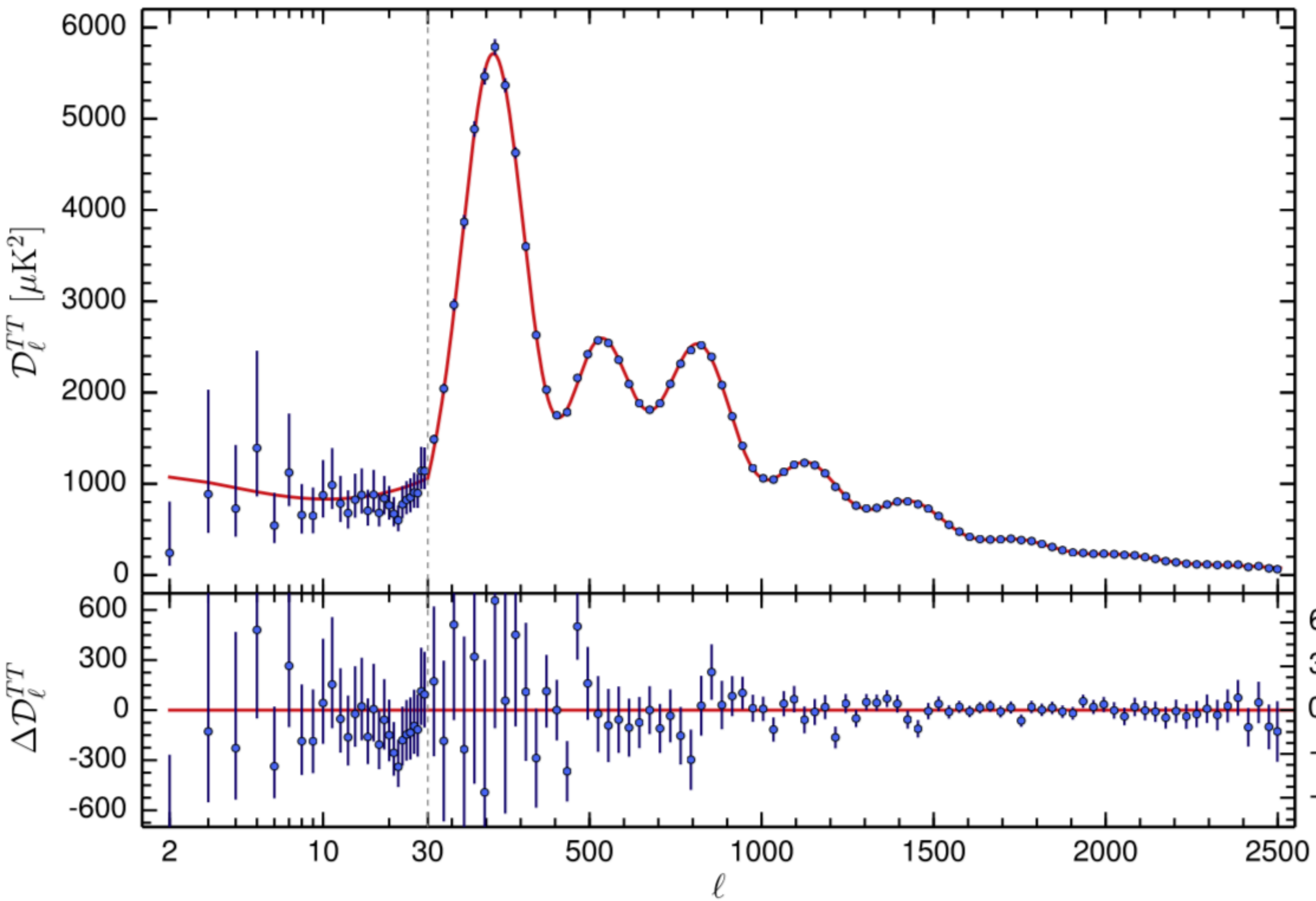
from Bouchet (2015)

Observational evidence for inflation

- From Planck + BICEP2 (2015):



Observational evidence for inflation



from Planck 2015

Conclusion

- Observations of CMB anisotropies (and polarization) are in excellent agreement with slow-roll inflation.

- Important evidence in favor of inflation:

  - nearly scale-invariant spectrum
  - near-Gaussian fluctuations
  - absence of vector perturbations

Open problems

- How did inflation start? Singularity?
- Tensor modes (primordial gravitational waves)?
- Non-Gaussianities (beyond free fields)?
- Single field vs. multi-field inflation?

The end
(of the inflationary universe)