Fundamental Interactions

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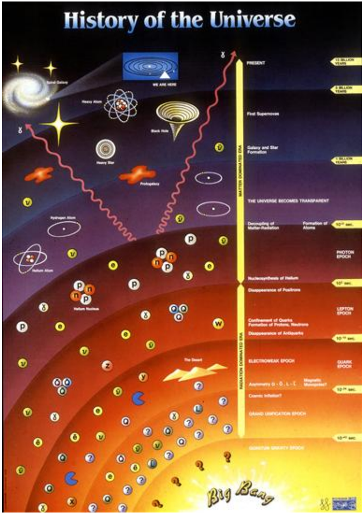
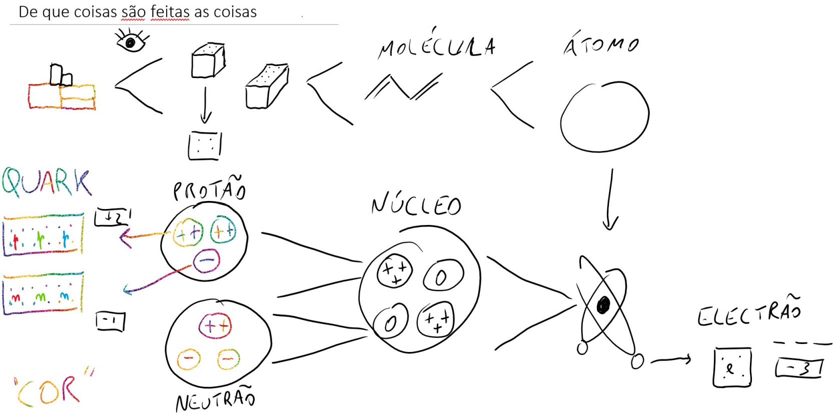
Scales


History of the Universe
PRESENT
First Supernovae
Galaxy and Star Formation
THE UNIVERSE BECOMES TRANSPARENT
PHOTON EPOCH
Nucleosynthesis of Helium
Disappearance of Positrons
LEPTON EPOCH
ELECTROWEAK EPOCH
QUARK EPOCH
GRAND UNIFICATION EPOCH
Big Bang

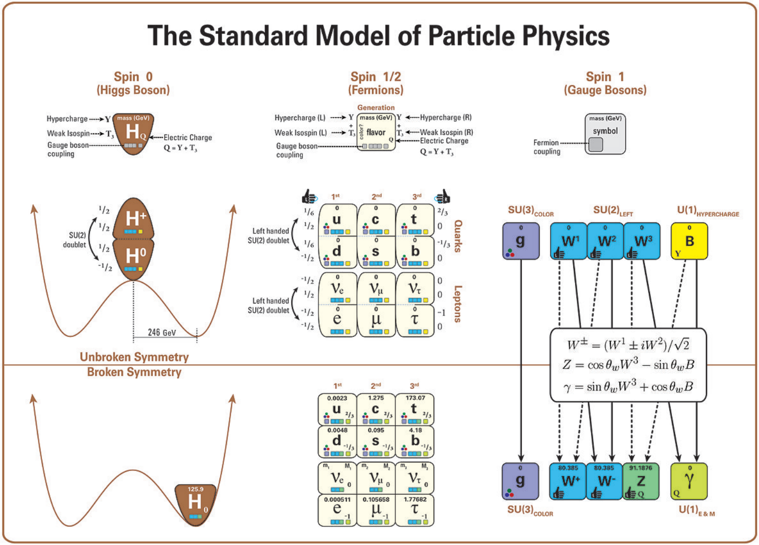
Things


De que coisas são feitas as coisas
MOLÉCULA
ÁTOMO
QUARK
PROTÃO
NÚCLEO
ELECTRÃO
NEUTRÃO
"COR"

More Things

	$SU(2)$		
	[Neutrino, Electron]$_{Left}$		(Electron)$_{Right}$
S	[Up, Down]$_{Left}$	(Up)$_{Right}$	(Down)$_{Right}$
U	[Up, Down]$_{Left}$	(Up)$_{Right}$	(Down)$_{Right}$
(3)	[Up, Down]$_{Left}$	(Up)$_{Right}$	(Down)$_{Right}$

Even More things



Interactions

$$dP/dt = F \tag{1}$$

$$dL/dt = r \times F \tag{2}$$

$$dE/dt = F.v \tag{3}$$

Invariances and Noether's Theorem

Translation

$$dP/dt = 0 \tag{4}$$

Rotation

$$dL/dt = 0 \tag{5}$$

Time translation

$$dE/dt = 0 \tag{6}$$

Conjugate variables

$$r \rightarrow P \tag{7}$$

$$\theta \rightarrow L \tag{8}$$

$$t \rightarrow E \tag{9}$$

$$(x, y, z, t) \rightarrow (P_x, P_y, P_z, E) \tag{10}$$

Charges

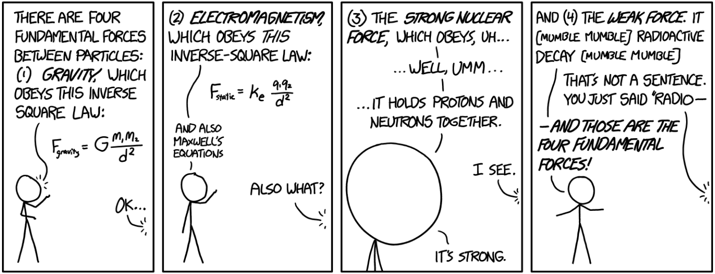
Gravity comparison with Electromagnetism

$$G \to \alpha \tag{11}$$

$$M \to Q \tag{12}$$

$$\frac{-GM}{r^2} \to \frac{-\alpha Q}{r^2} \tag{13}$$

Forces


THERE ARE FOUR FUNDAMENTAL FORCES BETWEEN PARTICLES: (1) GRAVITY, WHICH OBEYS THIS INVERSE SQUARE LAW:
$F_{gravity} = G\frac{m_1 m_2}{d^2}$
OK...
(2) ELECTROMAGNETISM, WHICH OBEYS THIS INVERSE-SQUARE LAW:
$F_{static} = k_e \frac{q_1 q_2}{d^2}$
AND ALSO MAXWELL'S EQUATIONS
ALSO WHAT?
(3) THE STRONG NUCLEAR FORCE, WHICH OBEYS, UH...
...WELL, UMM...
...IT HOLDS PROTONS AND NEUTRONS TOGETHER.
I SEE.
IT'S STRONG.
AND (4) THE WEAK FORCE. IT [MUMBLE MUMBLE] RADIOACTIVE DECAY [MUMBLE MUMBLE]
THAT'S NOT A SENTENCE. YOU JUST SAID "RADIO—
—AND THOSE ARE THE FOUR FUNDAMENTAL FORCES!

Gauss' theorem

$$\int_S E.dS = 4\pi\alpha Q_{in} \tag{14}$$

Point charge

$$\int \frac{\alpha Q}{r^2} dS = \frac{\alpha Q}{r^2} 4\pi r^2 = 4\pi\alpha Q \tag{15}$$

Generalisation for arbitrary dimension

$$S \to r^{D-1} \tag{16}$$

$$r^2 \to r^\beta \tag{17}$$

$$\tag{18}$$

Using Gauss we infer $\frac{r^{D-1}}{r^\beta}$ implying $\beta = D - 1$
For $D = 1$ there is no divergence

Dark Matter

Equilibrium means

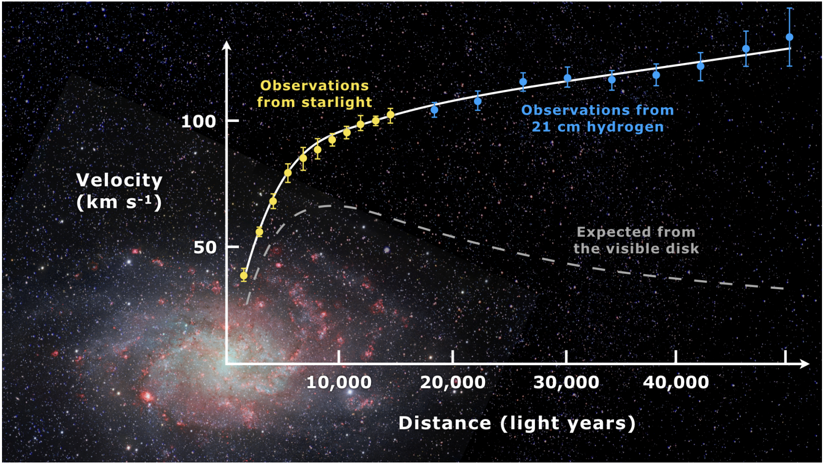
$$G\frac{Mm}{r^2} = \frac{mv^2}{r} \tag{19}$$

Inside and outside a homogeneous ball of matter radius R (simplified galaxy)

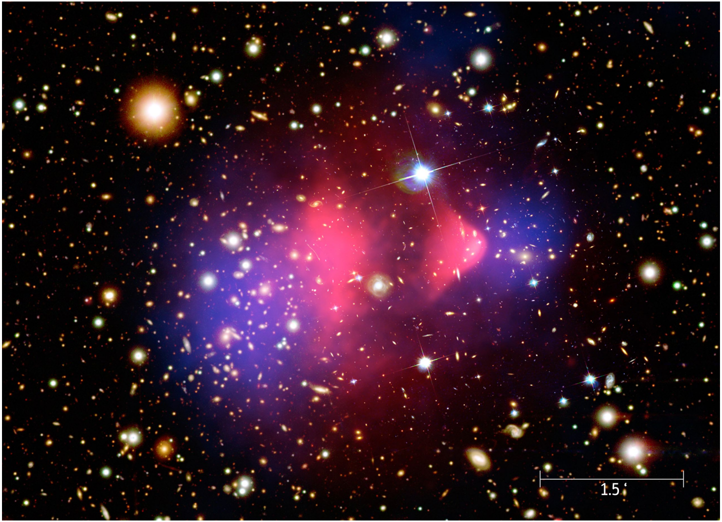
$M \sim r^3$, saturates at $M \sim R^3$

Expect
$v \sim r$ up to $r = R$
then
$v \sim r^{-1/2}$

Rotation Curve



Bullet Cluster, smoking gun



Field theories

Lagrangian and Action

$$L(x, \dot{x}) = \frac{1}{2} m \dot{x}^2 - V(x) \tag{20}$$

Euler-Lagrange

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0 \tag{21}$$

Lagrangian density and Action

Particle gets replaced by field $x \to \phi(x)$ and $L \to \mathcal{L}$

$$-\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2} m \phi^2 + \lambda \phi^3 + g \phi^4 \tag{22}$$

$$S[\phi(x)] = \int d^4 x \mathcal{L}[\phi(x), \partial_\mu \phi(x)] \tag{23}$$

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \tag{24}$$

QED

$$-\mathcal{L}_\psi = \bar{\psi}\gamma^\mu \partial_\mu \psi + m\bar{\psi}\psi \tag{25}$$

Global invariance

$$\psi \to \psi' = e^{iq\epsilon}\psi \tag{26}$$

Local invariance, if $\epsilon(x)$? No, because ∂_μ
Covariant derivative D_μ

$$D_\mu\psi(x) \to (D_\mu\psi(x))' = e^{iq\epsilon(x)}(D_\mu\psi) \tag{27}$$

$D_\mu \to \partial_\mu - iqA_\mu(x)$

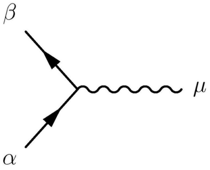
Gauge

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \epsilon(x) \tag{28}$$

$$-\mathcal{L}_{A+\psi} = \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \bar{\psi}\gamma^\mu \partial_\mu \psi + m\bar{\psi}\psi - iqA_\mu \bar{\psi}\gamma^\mu \psi \tag{29}$$

$$\rightarrow \quad \left(\frac{i}{\not{p} - m + i\varepsilon}\right)_{\beta\alpha}$$

$$\rightarrow \quad \frac{-i\eta_{\mu\nu}}{p^2 + i\varepsilon}$$



$$\rightarrow \quad -ie\gamma^{\mu}_{\beta\alpha}(2\pi)^4\delta^{(4)}(p_1 + p_2 + p_3).$$

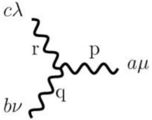
Yang-Mills

$$\psi_i(x) \to \psi_i(x)' = U_{ij}\psi_j(x) \tag{30}$$

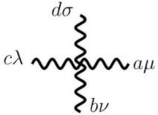
$U^\dagger = U^{-1}$
$SU(N)$: $N \otimes N = (N^2 - 1) \oplus 1$
N fundamental rep., spin $1/2$
$N^2 - 1$ generators, adjoint rep., spin 1.

Gauge bosons self-interaction

QED $U(1)$, Abelian $[a, b] = 0$
Yang-Mills $SU(N)$, non-Abelian $[a, b] \neq 0$



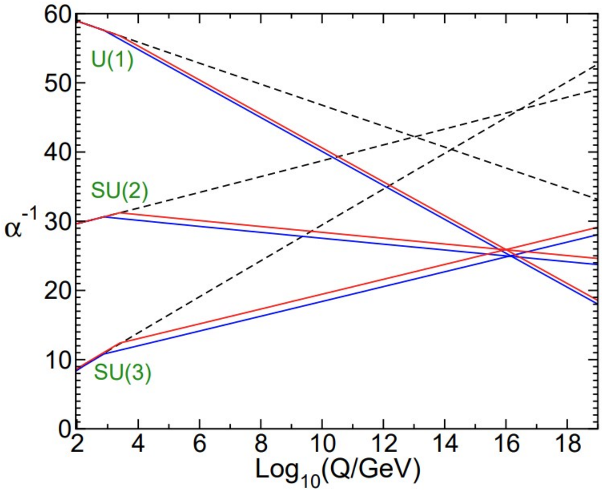
3–gluon vertex: $\Gamma^{abc}_{\mu\nu\lambda}(p, q, r) \;=\; -g f^{abc}[(p-q)_\lambda \eta_{\mu\nu} + (q-r)_\mu \eta_{\nu\lambda} + (r-p)_\nu \eta_{\mu\lambda}]$



4–gluon vertex:

$$\Gamma^{abcd}_{\mu\nu\lambda\sigma} = -ig^2 f^{abe} f^{cde}(\eta_{\mu\lambda}\eta_{\nu\sigma} - \eta_{\mu\sigma}\eta_{\nu\lambda}) -ig^2 f^{ace} f^{bde}(\eta_{\mu\nu}\eta_{\sigma\lambda} - \eta_{\mu\sigma}\eta_{\nu\lambda}) -ig^2 f^{ade} f^{bce}(\eta_{\mu\nu}\eta_{\sigma\lambda} - \eta_{\mu\lambda}\eta_{\nu\sigma})$$

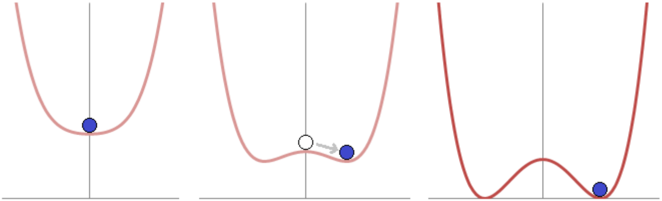
GUTs



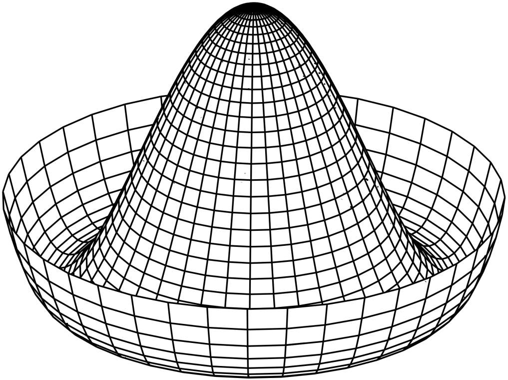
Spontaneous Symmetry Breaking

Landau theory (not from Particle Physics)

$$V(x) = -\frac{1}{2}\mu^2 x^2 + \frac{1}{4}\lambda x^4 = (-\frac{1}{2}\mu^2 + \frac{1}{4}\lambda x^2)x^2 \qquad (31)$$



Symmetry hidden at minimum



Complex scalar field

$$-\mathcal{L} = (\partial_\mu |\phi|)^2 - V(|\phi|) \tag{32}$$

Global invariance $\phi(x) \rightarrow \phi'(x) = e^{iq\epsilon}\phi(x)$
Expand around minimum...

$\mu^2 < 0$

$$V(|\phi|) = V(|\phi|)_{min} + \mu^2 |\phi|^2 + ... \quad (33)$$

$$\frac{\partial^2 V}{\partial |\phi|^2} = \mu^2 \quad (34)$$

Mass of field is μ^2

$\mu^2 > 0$ and Goldstone boson

$$\phi(x) = \frac{1}{\sqrt{2}}\rho(x)e^{i\theta(x)} \tag{35}$$

$$\partial_\mu \phi = \frac{1}{\sqrt{2}}e^{i\theta(x)}(\partial_\mu \rho(x) + i\rho\partial_\mu\theta(x)) \tag{36}$$

$$-\mathcal{L} = \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}\rho^2(\partial_\mu\theta)^2 + V(\frac{1}{\sqrt{2}}\rho) \tag{37}$$

$$\frac{\partial^2 V}{\partial \rho^2} \neq 0 \tag{38}$$

$$\frac{\partial^2 V}{\partial \theta^2} = 0 \tag{39}$$

Higgs mechanism

Gauge invariance $\phi(x) \to \phi'(x) = e^{iq\epsilon(x)}\phi(x)$
Gauge field $A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu \epsilon(x)$
Note $\theta(x) \to \theta'(x) + q\epsilon(x)$

$$-\mathcal{L} = \frac{1}{4} F^2_{\mu\nu}(A) + |D_\mu \phi|^2 + V(|\phi|) \tag{40}$$

$F_\mu \nu = \partial_\mu A_\nu - \partial_\nu A_\mu$
$D_\mu \phi = \partial_\mu \phi - iqA_\mu \phi$

Gauge Boson mass

$$D_\mu \phi = \frac{1}{\sqrt{2}} e^{i\theta} (\partial_\mu \rho - iq\rho(A_\mu - q^{-1}\partial_\mu \theta)) = \frac{1}{\sqrt{2}} e^{i\theta} (\partial_\mu \rho - iq\rho B_\mu) \tag{41}$$

Note B_μ is Gauge invariant

$$-\mathcal{L} = \frac{1}{4} F^2_{\mu\nu}(B) + \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2} q^2 \rho^2 B^2_\mu + V(|\phi|) \tag{42}$$

$M_B = |q\rho_{min}| = |qv|$

Higgs Boson mass

$$V(\rho) = -\frac{1}{2}\mu^2\rho^2 + \frac{1}{4}\lambda\rho^4 \tag{43}$$

$$-\mu^2 + \lambda\rho^2_{min} = 0 \to v = \sqrt{\frac{\mu^2}{\lambda}} \tag{44}$$

$$M_H^2 = \frac{\partial^2 V}{\partial\rho^2}|_{min} = -\mu^2 + 3\lambda\rho^2|_{min} = 2\mu^2 = 2\lambda v^2 \tag{45}$$

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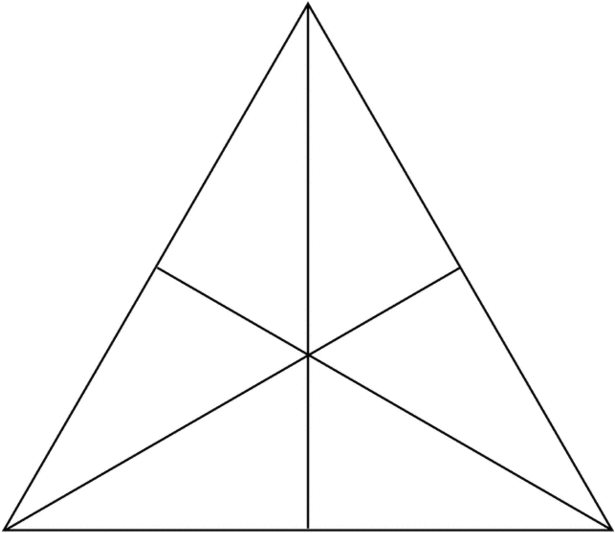
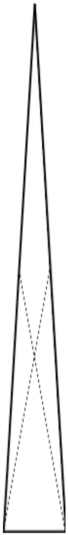
Equilateral action



Equilateral damage!



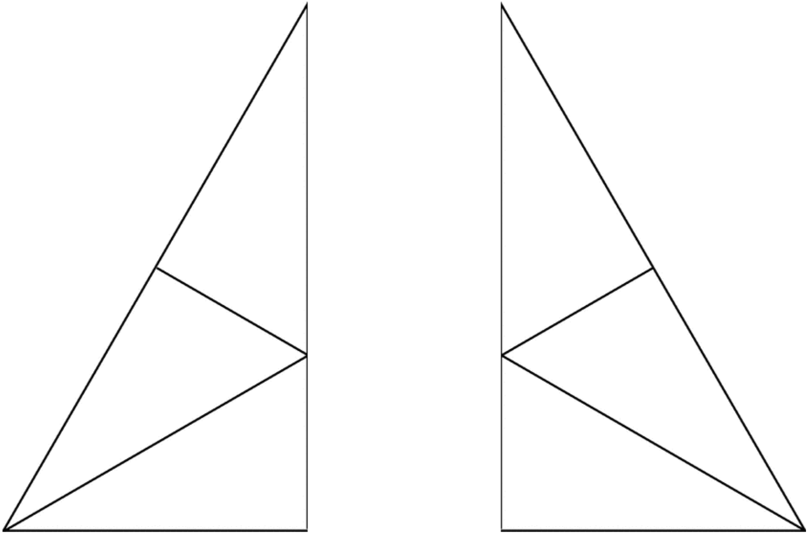
Symmetries

Symmetry if something unchanged
when I act on it with a specific transformation.

Polygon examples

Equilateral triangle (3 mirrors and 3 rotations (0, 120, 240))
Isosceles triangle (1 mirror and 1 rotation (0 degrees))

Building something invariant



Invariants under symmetries

Build something with symmetry (doesn't transform)
from parts that transform in specific ways

Join 2 square brackets [ ] to make a rectangle
Or use 2 minus signs - to build a plus sign
(with the symmetry of the square)

Important example: length of a vector is frame-invariant
(some properties of triangles are also invariant)

Building frame-invariants from vectors

Vectors transform in specific ways under frame transformations (e.g. rotation by an angle θ)
$(x, y) \rightarrow (x', y') = (x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta)$

But one can build, from vectors, quantities that are frame-independent:
Squared length of a vector (x, y), is $x^2 + y^2 = x'^2 + y'^2$.

More generally, scalar product of two vectors
$(x_1, y_1).(x_2, y_2) = x_1 x_2 + y_1 y_2 = (x'_1, y'_1).(x'_2, y'_2) = x'_1 x'_2 + y'_1 y'_2$

Invariants are very important as Physics should not depend on "arbitrary" choice of frame!

What about Particle Physics?

Particles have specific transformation properties
and interactions respect the invariant combinations

This restricts the “shape” of the theory, leading to predictions
(e.g. conservation of electric charge)

What about $U(1)$?

Particles with “charge” q under a $U(1)$ transformation (θ):
$\psi \to e^{iq\theta}\psi$
(i.e. get a phase proportional to their respective charge)

If that $U(1)$ is a symmetry of the theory, one must build quantities that remain invariant under any $U(1)$ transformation (for arbitrary θ).

(Scalar) QED mass term

Only have (scalar) electrons with charge $q = -1$, ψ:
$\psi \to e^{-i\theta}\psi$ and $\psi^* \to e^{+i\theta}\psi^*$

$U(1)$ invariant:
$m\psi^*\psi$

And that is basically the (scalar) QED mass term!

(Kind of) SM Yukawa interaction

"Electron doublet-vector" with hypercharge $q = -1$,
$\Psi = (\psi_1, \psi_2)$
"Boson doublet" with hypercharge $q = +1$ $\Phi = (\phi_1, \phi_2)$
the "scalar electron" ψ with hypercharge $q = -2$

$U(1)$ and frame-invariant:
$y(\Psi^*.\Phi)\psi$
which is kind of one of the interactions of the SM.

Note the invariants you can build depend on:
the symmetries of the theory
the particle content of the theory

(Kind of) SM fermion mass term

$U(1)$ and frame-invariant:
$y(\Psi^*.\Phi)\psi$

Φ gets a non-zero “vaccuum expectation value”
a specific value everywhere, in a specific direction
(this breaks both symmetries)
$\langle\Phi\rangle = (0, v)$

Then
$y(\Psi^*.\Phi)\psi = y(\Psi_1^*, \Psi_2^*).(\Phi_1, \Phi_2)\psi \to y(\Psi_2^* v)\psi$
Which looks like $m\psi^*\psi$ for $m = yv$

The Standard Model

Gauge group: $SU(3)_C \times SU(2)_L \times U(1)_Y$

Chiral spin $1/2$ fermions (left and right)
Quarks: colour triplets of $SU(3)_C$
Left fermions are doublets of $SU(2)_L$
Spin 0 scalar, doublet of $SU(2)_L$

The Standard Model is very successful but...

- Neutrinos have masses (νSM)
- Dark matter (no viable explanation)
- Matter / antimatter asymmetry (no viable explanation)
- Hierarchy problem (fine-tuning between parameters)
- Strong CP problem (fine-tuning between parameters)
- Gauge couplings (additional free parameters) - GUT?
- Flavour problem (many additional free parameters) - FS?

BSM solutions involve additional fields and symmetries

The Standard Model (1 generation)

Gauge group: $SU(3)_C \times SU(2)_L \times U(1)_Y$

Quarks (Q, u_R, d_R): colour triplets of $SU(3)_C$
LH fields (Q and L): doublets of $SU(2)_L$
e_R just $U(1)_Y$
(νSM: add ν_R, complete singlet)

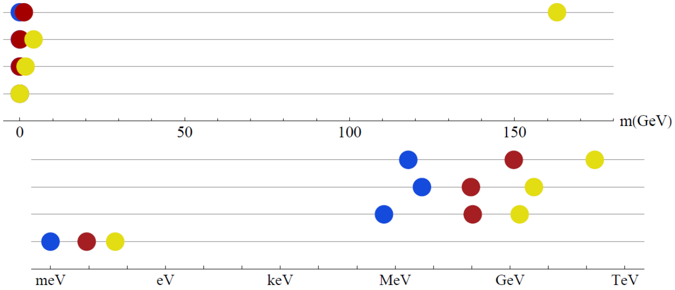
Scalar H also doublet of $SU(2)_L$
$\langle H \rangle$ breaks $SU(2)_L \times U(1)_Y \to U(1)_{em}$

Mass terms: $m_f F_\alpha f_R$ not invariant under $SU(2)_L$

But $y_f(\epsilon^{\alpha\beta} H_\alpha F_\beta) f_R$ is...
$y_f \langle H \rangle F f_R \to m_f F f_R$ with $m_f = y_f \langle H \rangle$

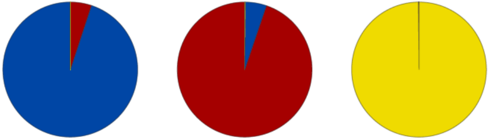
The Standard Model flavour problem: masses

3 fermion generations? Masses span orders of magnitude?



The Standard Model flavour problem: mixing

3 generations of quarks, small mixing



3 generations of leptons, large and peculiar mixing

(mixing between weak and mass eigenstates)

Summary of data: quark mixing

Wolfenstein parametrisation

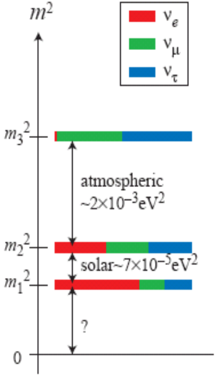
$$V_{CKM} \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$

$\lambda \simeq 0.23$ (Sine of the Cabibbo angle)

Summary of data: lepton mixing

Tri-bi-maximal (TBM) mixing

$$V_{PMNS} \simeq \begin{pmatrix} -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$



Beyond the Standard Model with family symmetries

Without $y_f H F f_R$, $\mathcal{L}_{\nu SM}$ has accidental symmetry $SU(3)^6$

FS: upgrade subgroup of $SU(3)^6$ to actual symmetry of $\mathcal{L}$

1. Generations charged differently under FS
2. Yukawa couplings no longer invariant
3. FS must be broken somehow...