# Black Holes

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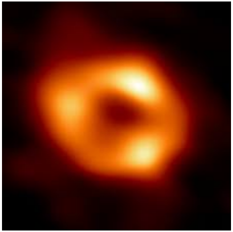
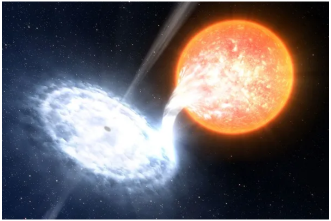
# Black holes are ubiquitous...

## In our Universe

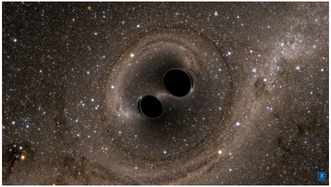


EHT Collab.

NASA



Illustration, ESA



Illustration, SXS

# Black holes are ubiquitous...

**In our Universe**

- Solar mass black holes - up to 100 $M_\odot$;
  Star collapse;
  Binary mergers;

- Intermediate black holes - 100 to $10^5$ $M_\odot$;
  Binary mergers;
  Accretion;

- Supermassive black holes - more than $10^5$ $M_\odot$;
  Specific seeds - $10^9$ $M_\odot$ clouds, direct collapse, multiple mergers?
  Very specific mechanisms - intermediate black hole with Super-Eddington accreation?

Largest observed by Hubble Telescope through gravitational lensing - $3 \times 10^9$ $M_\odot$.

# Black holes are ubiquitous...

## In physics

Astrophysics: Gravitational wave emission, gamma-ray burst engine, recoil, structure formation.

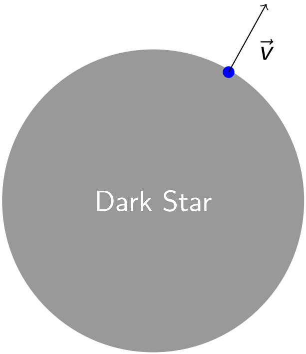
Particle Physics: dark matter, particle accelerator, etc.

Fundamental problems: cosmic censorship, physics at planck length.

Fluid analogue of gravity.

# What is a black hole?

**The Newtonian analogue - Dark star (The "Black")**




John Michell proposed the idea of a dark star in 1783.

For a particle to leave gravitational influence of a body, $v \geq v_{esc}$ (escape velocity).

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} \;\; ; \quad E \geq 0;$$

$$v_{esc} = \sqrt{\frac{2GM}{r}};$$

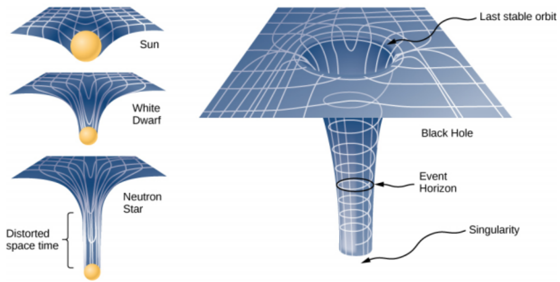
What if $v_{esc} = c$?
Light wouldn't escape (**Black**).
Happens if $\frac{2GM}{rc^2} = 1$. This carries on to GR!

# What is a black hole?

**In GR - The "hole"**

$$G_{ab} = g^{cd} R_{cadb} - \frac{1}{2} g_{ab} R_{cdef} g^{ce} g^{ef} = \frac{8\pi G}{c^4} T_{ab}$$

Interaction between curvature and matter.




| Object | Mass | $R_S$ |
|---|---|---|
| Atom | $10^{-26}$Kg | $10^{-51}$cm |
| Human being | 70 Kg | $10^{-23}$cm |
| Earth | $6 \times 10^{24}$Kg | 0.89 cm |
| Sun | $2.0 \times 10^{30}$Kg | 3.0 Km |
| Galaxy | $10^{11} M_\odot$ | $10^{-2}$ly |
| Universe (closed) | $10^{23} M_\odot$ | $10^{10}$ly |

$$R_S = \frac{2GM}{c^2}.$$

Event horizon: points of no return.

A singularity!

# The Schwarzschild solution

**The metric of a spherically symmetric star and a black hole**

Karl Schwarzschild (1916) obtained the spherically symmetric solution for the metric $g_{ab}$ in GR:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2(\theta) d\phi^2)\,,$$

This describes the outer spacetime deformed by a spherically symmetric star - Birkhoff's theorem.

Also describes a black hole? Yes!
$M$ is the mass of the star or black hole.

Singularities at $r = 2GM/c^2$ and $r = 0$?
Use $G = 1$, $c = 1$.

**Remember**: Minkowski or flat spacetime has a metric:

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2(\theta) d\phi^2)\,.$$

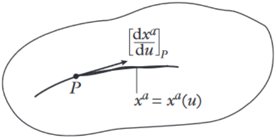
# The Schwarzschild solution

## Motion of particles

One of the ways to study a spacetime is to study its causal structure.

Curves in spacetime:
$x^a(u)$; $\dot{x}^a(u) = \frac{dx^a}{du}$ tangential vector;

Geodesics:
$\dot{x}^a \dot{x}^b g_{ab} = -s$,
if timelike $s = 1$, if null $s = 0$;
$\ddot{x}^a + \Gamma^a{}_{bc} \dot{x}^b \dot{x}^c = 0$;
$\Gamma^a{}_{bc} = \frac{1}{2} g^{ad} \left(\partial_b g_{dc} + \partial_c g_{db} - \partial_d g_{ab}\right)$;

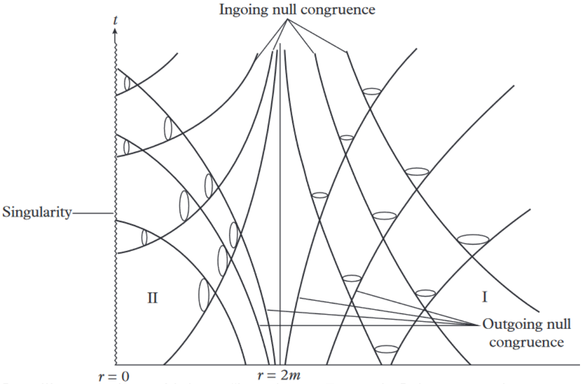



Ray d'Inverno and James Vickers, "Introducing Einstein's Relativity: a deeper understanding" (2021).

Schwarzschild:
$\dot{r}^2 + \left(1 - \frac{2M}{r}\right)\left(s + \frac{L^2}{r^2}\right) = E$
$\sqrt{E} = \left(1 - \frac{2M}{r}\right)\dot{t}$
$L = r^2 \dot{\phi}$

# The Schwarzschild solution

## Schwarzschild coordinates




Ray d'Inverno, James Vickers, "Introducing Einstein's Relativity: a deeper understanding" (2021).

Radial light rays:

$\dot{r} = \pm\sqrt{E};$

$\dot{t} = \left(1 - \frac{2M}{r}\right)^{-1}\sqrt{E};$

$$\frac{dr}{dt} = \pm\left(1 - \frac{2M}{r}\right);$$

Outgoing light rays:

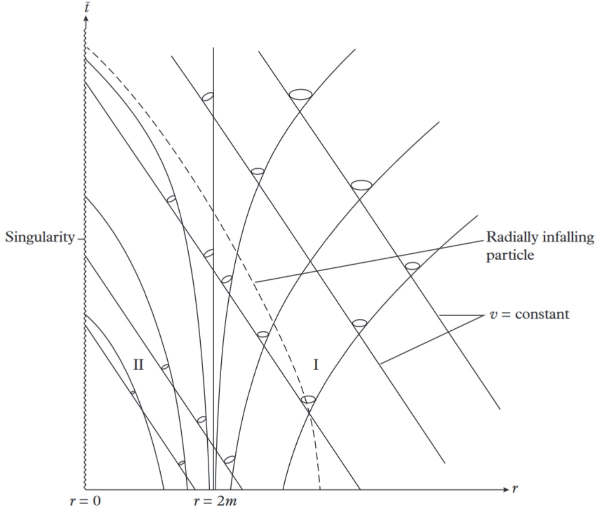
$t = r + 2M\log|r - 2M| + u;$

Ingoing light rays:

$t = -r - 2M\log|r - 2M| + v;$

# The Schwarzschild solution

## Advanced Eddington-Finkelstein coordinates - the black hole




Ray d'Inverno, James Vickers, "Introducing Einstein's Relativity: a deeper understanding" (2021).

Change of coords:

$\bar{t} = t + 2M \log |r - 2M|$;

Outgoing light rays:

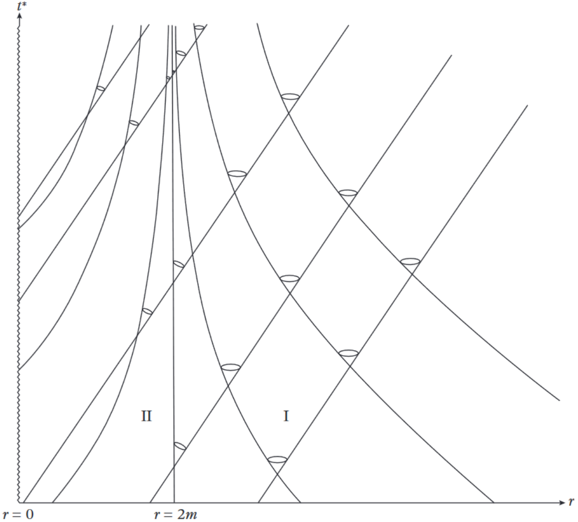
$\bar{t} = r + 4M \log |r - 2M| + u$;

Ingoing light rays:

$\bar{t} = -r + v$;

# The Schwarzschild solution

## Retarded Eddington-Finkelstein coordinates - the white hole




Ray d'Inverno, James Vickers, "Introducing Einstein's Relativity: a deeper understanding" (2021)

Change of coords:

$t^* = t - 2M \log|r - 2M|;$

Outgoing light rays:

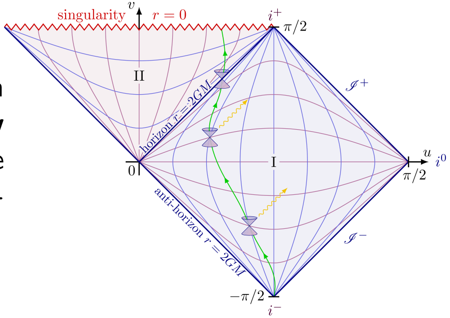
$t^* = r + u;$

Ingoing light rays:

$t^* = -r - 4M \log|r - 2M| + v;$

# The definition of a black hole

Black hole is a region of spacetime **causally disconnected** from future **null infinity, in asymptotically flat spacetimes.**

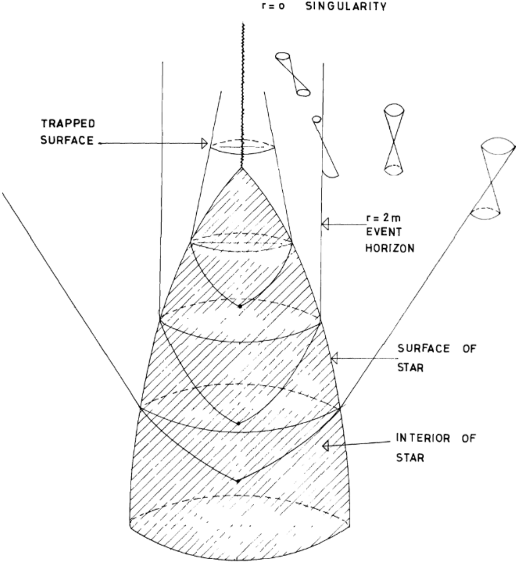



**Causally disconnected** - impossible to draw a light ray connecting the regions.

**Null infinity** - points at infinite distance reached by light rays.

Basically means: a region from which light or common matter can escape, in fact, they are fated to fall into a singularity (in spherical symmetry).

# Black holes from the collapse of matter




Hawking, "The Event horizon", Les Astres Occlus (1972).

**Oppenheimer and Snyder (1939)**
Spherical collapse of pressureless matter dust

**Penrose (1964)**
Collapse leads inevitably to a singularity.

# Weak cosmic censorship

But what about the singularity?

Weak Cosmic censorship (Penrose 1969): informally, a singularity is always enclosed by an event horizon (they are not naked), **for generic initial data**.

The conjecture relies on spacetimes that are physical! It is easy to find an exotic solution of GR with a naked singularity.

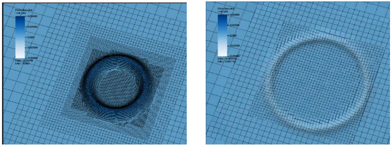
The resolution of the singularity may lie on a different (quantum) theory.

# Black holes from the collapse of fields

## The exception of weak cosmic conjecture

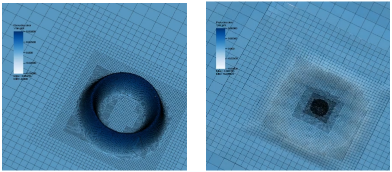
A bet by Preskill and Thorne against Hawking (Hawking lost but modified the bet).

Subcritical



Existence of naked singularity in scalar field collapse (Christodoulou, 1994).

Supercritical



Critical phenomena in black hole formation (Choptuik, 1993).

GRChombo sims from Choptuik's scalar collapse

(GRChombo youtube).

# Summary

- Black holes are ubiquitous on our Universe;
- Black holes are regions from which nothing can escape to very long distances (infinity);
- The event horizon separates the black hole from the outer spacetime;
- Collapse leads to a singularity, maybe they are all covered by an event horizon.

# The Kerr-Newmann family

## The metric

Kerr (1964) found an axisymmetric black hole solution and Newmann (1965) generalised it to a charged black hole:

$$ds^2 = -(dt - a^2 \sin^2\theta d\phi)^2 \frac{\Delta}{\Sigma} + \left((r^2 + a^2)d\phi - adt\right)^2 \frac{\sin^2\theta}{\Sigma}$$
$$\frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 \,,$$

$$\Delta = r^2 + a^2 - 2Mr + Q^2 \,,$$
$$\Sigma = r^2 + a^2 \cos^2\theta \,.$$

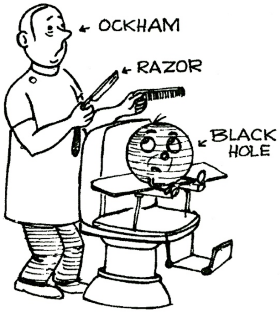
It describes a spacetime with an electrically charged, rotating black hole. Physical requirements: $M^2 \leq a^2 + Q^2$, otherwise naked singularity.

# The no-hair theorem

The Kerr-Newmann family describes a black hole using only three parameters: **mass** ($M$), **angular momentum** ($J = aM$) and **charge** ($Q$).

In General Relativity, these parameters suffice! Uniqueness theorem: Carter (1971), Robinson (1974), Mazur (1982), Bunting (1983).

"Black holes have no hair", Wheeler (1971).
But do they? Alternative theories, "soft hair", scalar and Proca hair.




Vishveshwara, Proc. Einstein Centenary Symposium (1979).

# The charge, astrophysically

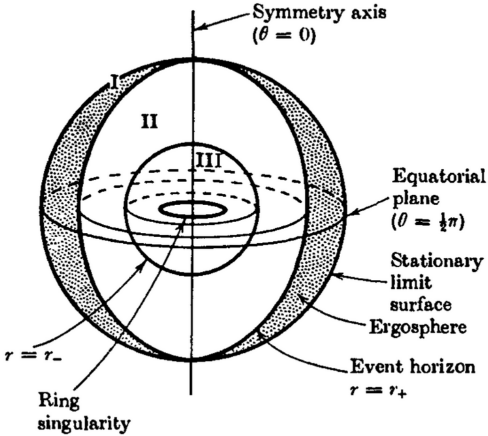
Astrophysically, there are bounds to the charge of the black hole.

From environment interaction (accretion of plasma, EM repels the same charges): $Q \leq 10^{-15} Q_{\text{max}}$.

From Schwinger effect (pair creation in vaccum): $Q \leq 10^{-5} \frac{M_\odot}{M} Q_{\text{max}}$.

It is assumed that most black holes in the universe are almost neutral (described by Kerr metric, $Q = 0$).

# Anatomy of a Kerr black hole




Hawking and Ellis, "Large structure of space-time" (1973).

$$r_+ = M + \sqrt{M^2 - a^2}$$

$$r_- = M - \sqrt{M^2 - a^2}$$

Outer ergosphere

$$M + \sqrt{M^2 - a^2 \cos^2(\theta)}.$$

# About cosmic censorship, again!

## The Cauchy horizon

Strong Cosmic censorship: GR is deterministic, it is possible to completely determine a physical spacetime up to the singularity.

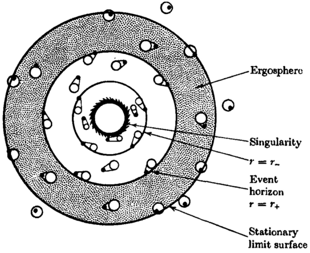
But GR evolution cannot be extended past the Cauchy horizon. Maybe the Cauchy horizon is unstable and create a singularity?

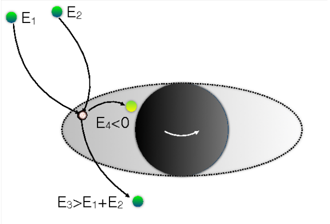
It seems to be of a different kind! (Dafermos and Luk, 2018).

# The extraction of energy

## The ergoregion

"Ergo" - work.




The energy of particles can be negative in the ergoregion.

In a collision, one can get a more energetic particle up to a limit (Penrose and Floyd, 1971).

Extraction with fields
**Superradiance**
(Bekenstein and Schiffer, 1998).

# The laws of black hole mechanics

## The first law

Imagine a particle falling into a black hole, the area of the event horizon changes as

$$\frac{\kappa}{8\pi} dA = dM - \Omega_H dJ - \Phi_H dQ \; .$$

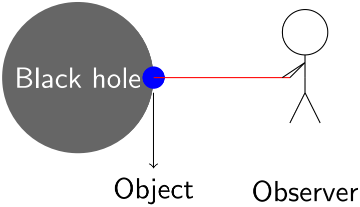
$M$ - black hole mass;
$J$ - black hole angular momentum;
$Q$ - black hole charge;
$A$ - event horizon area;
$\kappa$ - surface gravity;
$\Omega_H$ - angular velocity at event horizon;
$\Phi_H$ - electric potential at event horizon.

# The laws of black hole mechanics

## The surface gravity and zeroth law

$$\boxed{\frac{\kappa}{8\pi}} dA = dM - \Omega_H dJ - \Phi_H dQ \, .$$

The surface gravity is the acceleration felt by a stationary object at the horizon measured by an observer at infinity ($\kappa = \frac{1}{4M}$ for Schwarzschild).




**The zeroth law**: surface gravity is constant through the horizon.

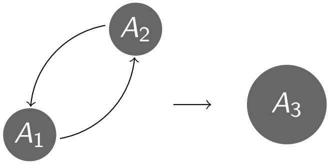
# The laws of black hole mechanics

## The second law

How much energy can we extract? Limited by second law.

$$\frac{\kappa}{8\pi} dA = dM - \Omega_H dJ - \Phi_H dQ \ .$$

The event horizon area must cannot decrease! $dA \geq 0$.




**Example**: Black hole binary

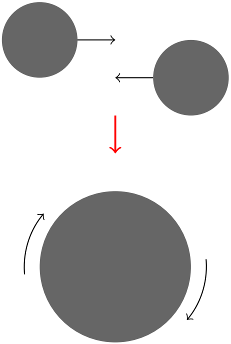
- Two black holes can coalesce into one;
- $A_3 \geq A_1 + A_2$;
- Black holes **cannot** bifurcate.

Area and entropy?

# The laws of black hole mechanics

## The third law

What about giving angular or charge energy to the black hole? Can it become **naked**? Remember the weak cosmic censorship.



$M < a$ Kerr with horizon
$M = a$ Extremal
$M > a$ naked singularity

Throwing objects or even collide with another black hole.
Numerically up to $a = 0.95M$ (Sperhake et al, 2009).

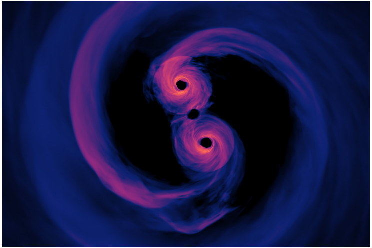
**Third law**: It takes infinite time (advanced time) for a subextremal black hole to become extremal $M = a$.

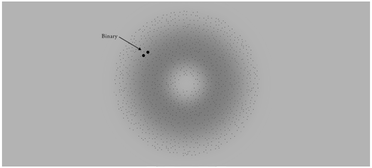
# Binaries as extreme events

Observing and studying binary mergers is a way to test GR in the strong curvature regime.

Can help on the study of their environment - dark matter.

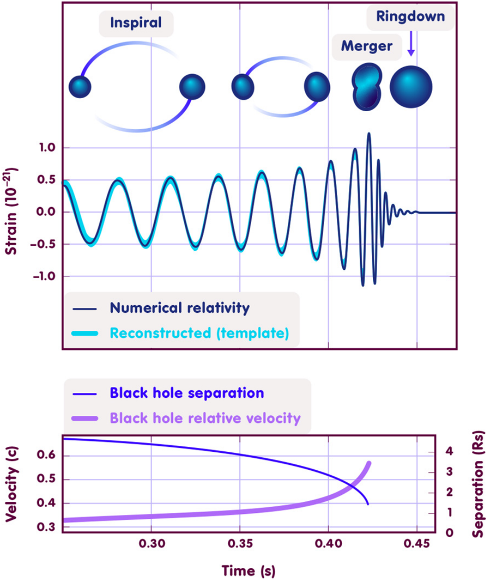
Two body problem cannot be solved analytically in GR - need for approximations and numerical simulations.





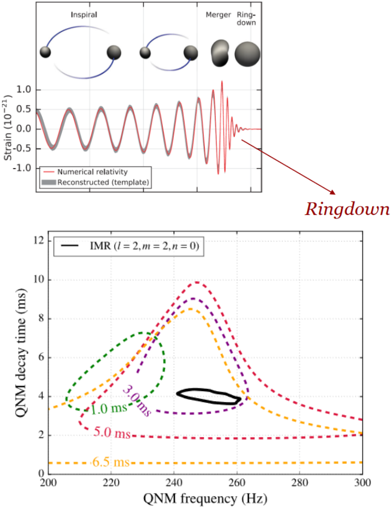
# How we detect them? Gravitational waves!

Virgo from LIGO/Virgo collab.



# Test of Kerr geometry

Are real black holes described by Kerr geometry? Use "Black hole spectroscopy"

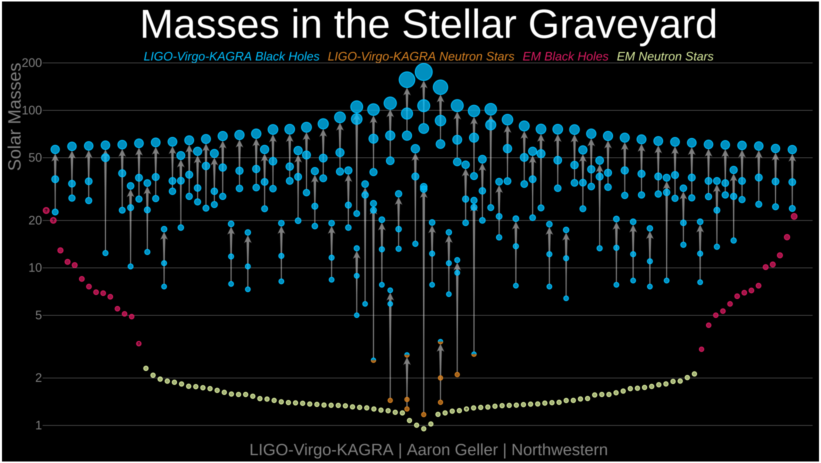



The final black hole "vibrates" at specific damped frequencies - Quasinormal modes.

Quasinormal modes depend only on the mass and spin of Kerr black holes.

Needed at least two modes to test no-hair theorem. Only the least damped can be "detected" by LIGO (Abbott et al, 2016).

# The catalog (2021)




The launch of LISA will give us many more events and a better grasp into the nature of black holes.

# Summary

- Black holes are described by mass, area and charge;
- Black holes can do work, extraction of energy is possible;
- Black holes always increase their event horizon area;
- The measurement of gravitational waves and the use of multimessenger astronomy can be used to test the nature of black holes.

Thank You!