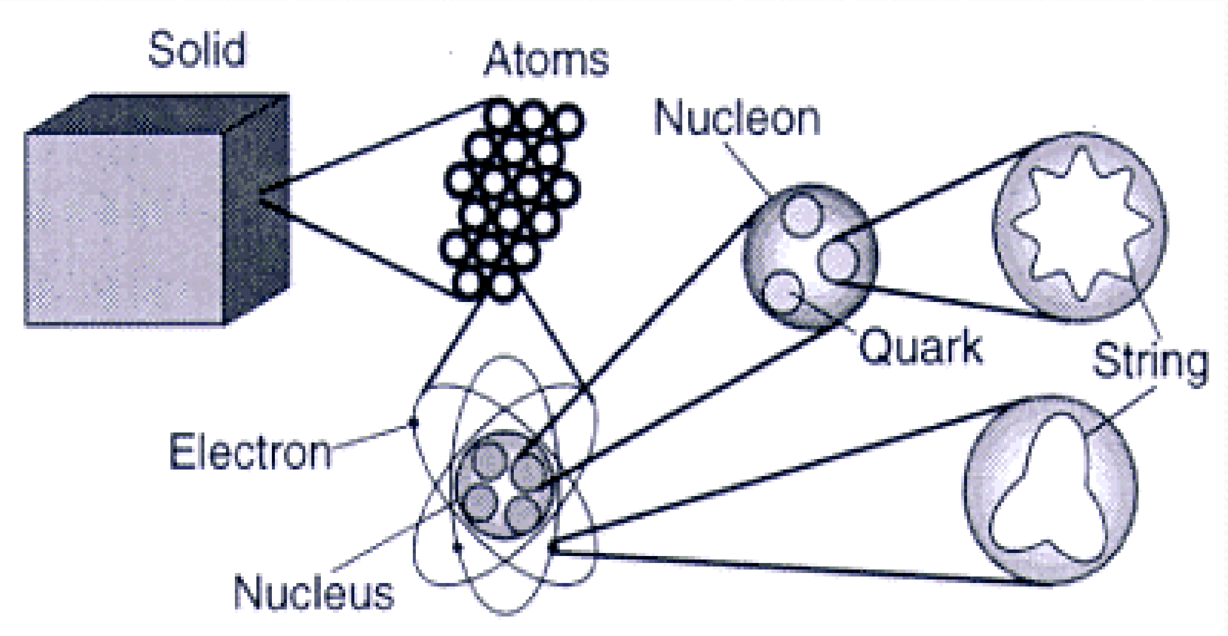
String theory and the AdS/CFT conjecture

1. Introduction to String theory

Robertus Potting, CENTRA/UAlg

10th School of Astrophysics and Gravitation, IST 2021

Strings: what are we talking about?



Why String theory?

- Four known elementary forces:
  - Electromagnetism
    - Electric & magnetic forces
    - electromagnetic waves
  - Gravity
  - Weak force
    - radioactive decay
  - Strong nuclear force

Why String theory?

Basic theoretical formalisms:

Quantum mechanics

- uncertainty principle
- probabilistic nature
- zero point energy
- wave function, Schroedinger equation

Quantum field theory

- theory of quantized fields
- particles = quanta of field fluctuations
- forces transmitted by virtual particles
- gauge symmetries
- renormalization

Electroweak (GWS) model

- describes E&M and weak forces
- EM force transmitted by photon
- weak force transmitted by W, Z bosons
- Higgs mechanism

Quantum Chromo Dynamics

- describes strong nuclear force
- strong force transmitted by gluons

Why String theory?

Classical mechanics

- definite values for position and velocity
- Equations of motion: Newton’s laws

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General Relativity

- based on equivalence principle
- curved space-time
- particles move along geodesics
- energy-momentum tensor sources curvature

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Gravitation

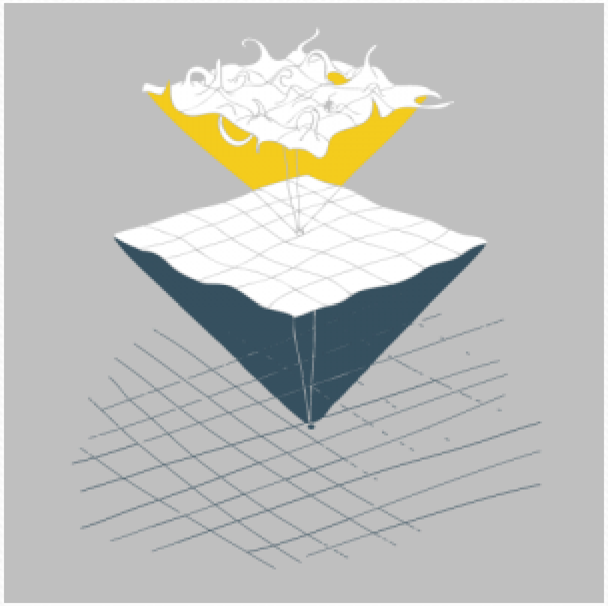
Why String theory?

- Classical footing:
  - Gravity
- Quantum footing:
  - Electroweak model
  - Quantum Chromodynamics

Difference between frameworks is unelegant and probably inconsistent.

Cure: develop quantum theory of gravity!

Unfortunately, quantizing GR as a QFT leads to inconsistencies.



Why String theory?

Superstring theory:

- Provides quantum completion of General Relativity
- UV finite at all orders of perturbation theory
- GR is recovered in limit of low energies
- Includes gauge interactions
- Able to yield 4-D theories with realistic matter (chiral fermions)
- Contains specifically new phenomena
- Amounts to a unified framework for “Theory of Everything”
- Potential to give insight into nature of space-time, black-hole information paradox, number of fermion families…

Strings: historical overview

- 1968 – 1972: String theory ("dual models") for describing strong interactions between quarks
    - String tension = strong interaction scale
- 1973 - : Strings as a theory of fundamental interactions
    - String tension = Planck scale
- 1983 : Green and Schwarz discover anomaly cancellation
- 1995 : Relevance of D-branes (Polchinski)
- 1998 : AdS-CFT correspondence (Maldacena): 2$^{\text{nd}}$ part by Miguel Zilhão
- 2003 - : Further developments:
    - String cosmology; the string theory "landscape"
    - Much interest from the pure mathematics community

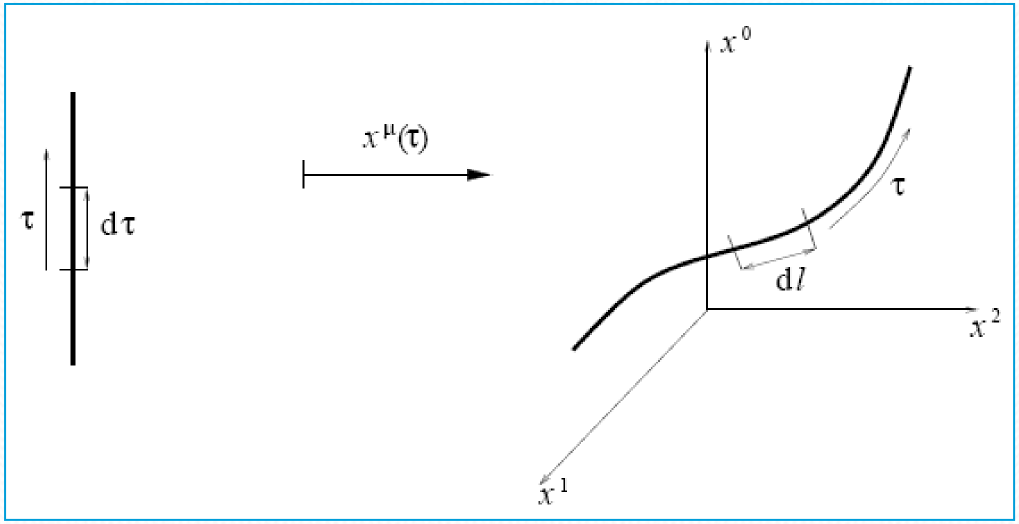
String dynamics

- p-brane: p-dim extended object moving in D space-time dims

- Spacetime coordinates:

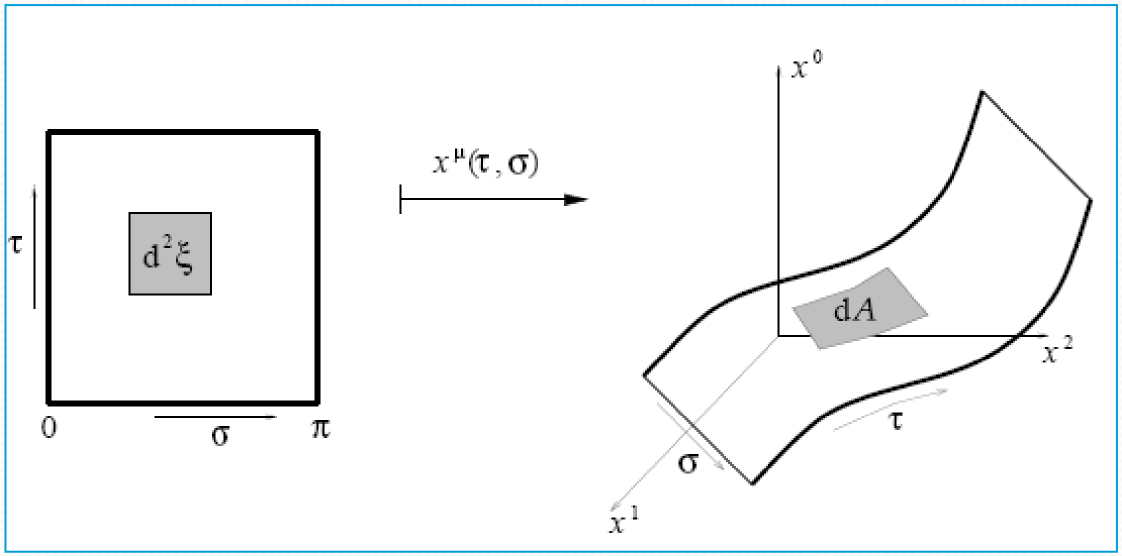
$$X^\mu(\xi_\alpha) \qquad \begin{cases} \alpha = 0, \ldots, p \\ \mu = 0, 1, \ldots, D-1 \end{cases}$$

Case p = 0:
(point particle)



String dynamics

Case p = 1:
(string)



String dynamics

- Nambu-Goto action: $$S = -T \int \sqrt{-\det h}\, d^{\mathrm{p}+1}\xi$$
  - measures worldsheet area
  - T: brane tension, dimension (mass)$^{\mathrm{p}+1}$
  - Induced brane metric: $$h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu \equiv \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

- *Equivalent description:* Polyakov action

$$S = -\frac{T}{2} \int d^{\mathrm{p}+1}\xi \sqrt{-\det h}\, \{h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu - (\mathrm{p}-1)\}$$

  - $h_{\alpha\beta}$: independent variable
  - Symmetries:
    - global Lorentz invariance
    - local reparametrization invariance
  - Quantization much easier than for Nambu-Goto action

String dynamics

- Case p=0 (particle)

Nambu-Goto action:

$$S = -m \int d\xi \sqrt{-\eta_{\mu\nu} \dot{X}^\mu \dot{X}^\nu}$$

$$P_\mu \equiv \frac{\partial L}{\partial \dot{X}^\mu} = -m \frac{\eta_{\mu\nu} \dot{X}^\nu}{\sqrt{-\eta_{\rho\sigma} \dot{X}^\rho \dot{X}^\sigma}}$$

It follows that $P_\mu P^\mu = -m^2$, representing a relativistic particle with mass *m*.

The equations of motion yield $\dot{P}_\mu = 0$, so the particle is free.

String dynamics

Polyakov action:

$$S = -\frac{m}{2}\int d\xi \sqrt{-h}\left(h^{-1}\eta_{\mu\nu}\dot{X}^{\mu}\dot{X}^{\nu} + 1\right)$$

Here h is taken to be a new, independent function of ξ .

Equations of motion: $h = \eta_{\mu\nu}\dot{X}^{\mu}\dot{X}^{\nu}$

Substituting back yields Nambu-Goto action.

S is invariant under reparametrizations of worldline $\xi \to \xi'(\xi)$

under which

$$h(\xi) = h'(\xi')\left(\frac{d\xi'}{d\xi}\right)^2$$

Can use this to fix $h(\xi)$ to a constant. Then the equations of motions for X^{μ} yield

$$\ddot{X}^{\mu} = 0 \qquad \Rightarrow \qquad X^{\mu}(\xi) = x_0^{\mu} + p^{\mu}\xi$$

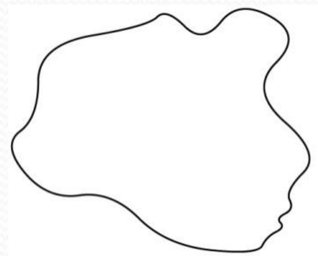
String dynamics

- Case p=1 (string)
  - String parameter $\xi^{\alpha} = (\tau, \sigma)$
  - Can fix to conformally flat string metric: $h_{\alpha\beta}(\sigma,\tau) = e^{\Phi(\sigma,\tau)}\eta_{\alpha\beta}$
    - Φ : scale factor
    - For p=1 action has additional symmetry: Weyl scaling of the 2-d metric. This allows to set Φ = 0
  - String equations of motion and constraints corresponding to reparametrization invariance: $\partial_\alpha \partial^\alpha X^\mu = 0$

$$T_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu - \frac{1}{2}\eta_{\alpha\beta}(\partial X)^2 = 0$$

  - Solution: introduce light-cone variables: $\tau, \sigma \rightarrow \sigma_\pm = \tau \pm \sigma$
    - $\partial_+ \partial_- X^\mu = 0$ $\quad X^\mu(\sigma_\pm) = X^\mu_L(\sigma_+) + X^\mu_R(\sigma_-)$
    - constraints: $(\partial_+ X_L)^2 = (\partial_- X_R)^2 = 0$

String dynamics



- Closed strings
  - Subject to periodicity condition: $X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+2\pi)$
  - General solution:

$$X^\mu = X_0^\mu + P^\mu\tau + \frac{i}{\sqrt{2}}\sum_{n\neq 0}\frac{1}{n}\{a_n^\mu e^{-in(\tau+\sigma)} + \tilde{a}_n^\mu e^{-in(\tau-\sigma)}\}$$

    - Units with $T = \frac{1}{2\pi}$
  - Reality condition for X^μ: $(a_n^\mu)^\dagger = a_{-n}^\mu$
  - Canonical quantization: $[X^\mu(\tau,\sigma), \dot{X}^\nu(\tau,\sigma')] = 2i\pi\delta(\sigma-\sigma')\eta^{\mu\nu}$

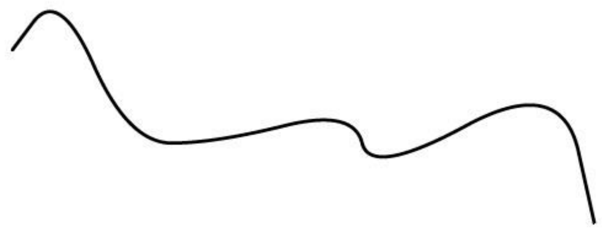
    implying: $[a_m^\mu, a_n^\nu] = [\tilde{a}_m^\mu, \tilde{a}_n^\nu] = m\delta_{m+n,0}\eta^{\mu\nu}$

    $[\tilde{a}_m^\mu, a_n^\nu] = 0 \quad , \quad [X_0^\mu, P^\nu] = i\eta^{\mu\nu}\,.$

    So we can identify $a_n^\mu, \tilde{a}_n^\mu$ $(a_{-n}^\mu, \tilde{a}_{-n}^\mu)$ for $n > 0$ with annihilation (creation) operators $\Longrightarrow$ harmonic oscillators!

String dynamics

- Open strings



- Lorentz-invariant boundary condition (Neumann): $\partial_\sigma X^\mu|_{\sigma=0,\pi} = 0$
- General solution:

$$X^\mu = X_0^\mu + P^\mu \tau + i\sqrt{2}\sum_{n\neq 0}\frac{1}{n}a_n^\mu e^{-in\tau}\cos(n\sigma)$$

  - Can be obtained by identifying closed string left- and right movers:

$$a \equiv \tilde{a}$$

String dynamics

- Free open string spectrum
  - Define vacuum of momentum p:

$$a_n^\mu |p>= 0 \qquad \tilde{a}_n^\mu |p>= 0 \qquad P^\mu |p>= p^\mu |p>$$

  - Physical states are obtained by acting with creation operators
  - Have to require that physical states $|\mathrm{phys}\rangle$ satisfy the constraints

$$L_m|\mathrm{phys}\rangle = 0 \qquad (m > 0)$$

$$(L_0 - a)|\mathrm{phys}\rangle = 0$$

where the Virasoro generators (string reparametrization group) are

$$L_m = \frac{1}{2\pi}\int_0^{2\pi} d\sigma\, e^{im\sigma}(\partial_+ X)^2 = \frac{1}{2}\sum_{m=-\infty}^{\infty} : a_{m-n}\cdot a_n :$$

$$L_0 = \frac{1}{2\pi}\int_0^{2\pi} d\sigma\, (\partial_+ X)^2 = \frac{1}{4}p^2 + \sum_{n>0} a_{-n}\cdot a_n = \frac{1}{4}p^2 + N$$

String dynamics

- *a* is a normal ordering constant
- Can prove that constraints eliminate negative-norm states from spectrum (example $||a^{\mu}_{-m}|p>||= m\eta^{\mu\mu} < 0$ if $\mu = 0$)
- Physical excitations essentially transverse
- Mass formula: $-\frac{1}{4}p^2 = N - a$
- First excited state $a^{T}_{-1}|p>$ with $N = 1$ is a spacetime vector with $D-2$ transverse independent components. Lorentz invariance implies it must be massless, thus $a = 1$
- Obtain string spectrum "tower":

$$N=0 \qquad |p> \qquad -\tfrac{1}{4}p^2 = -1$$ (tachyon)

$$N=1 \qquad a^{\mu}_{-1}|p> \qquad -\tfrac{1}{4}p^2 = 0$$ (massless)

$$N=2 \begin{cases} a^{\mu}_{-1}a^{\nu}_{-1}|p> \\ a^{\mu}_{-2}|p> \end{cases} \qquad -\tfrac{1}{4}p^2 = 1$$ (massive)

  - Regge trajectory: $$\frac{1}{4}M^2 = \frac{(J-1)}{\alpha'}$$

String dynamics

- Careful analysis of quantum algebra of Virasoro generators (conformal symmetry) reveals that one has to choose spacetime dimension D = 26!

- Closed string spectrum:
  - Direct product of right- and leftmovers
  - Constraints imply $M_L^2 = M_R^2$
  - *Example:* massless level: $a^{\mu}_{-1}\tilde{a}^{\nu}_{-1}|p>$
    - Contains spin-2 graviton (symmetric traceless) $G_{\mu\nu}$, scalar dilaton (trace) ϕ and Kalb-Ramond field (antisym. tensor) $B_{\mu\nu}$.
  - Thus gravity is unavoidable in string theory!

- Open string spectrum:
  - Can introduce additional quantum numbers associated to ends: "Chan-Paton factors". Massless vector becomes (non-abelian) gauge field.

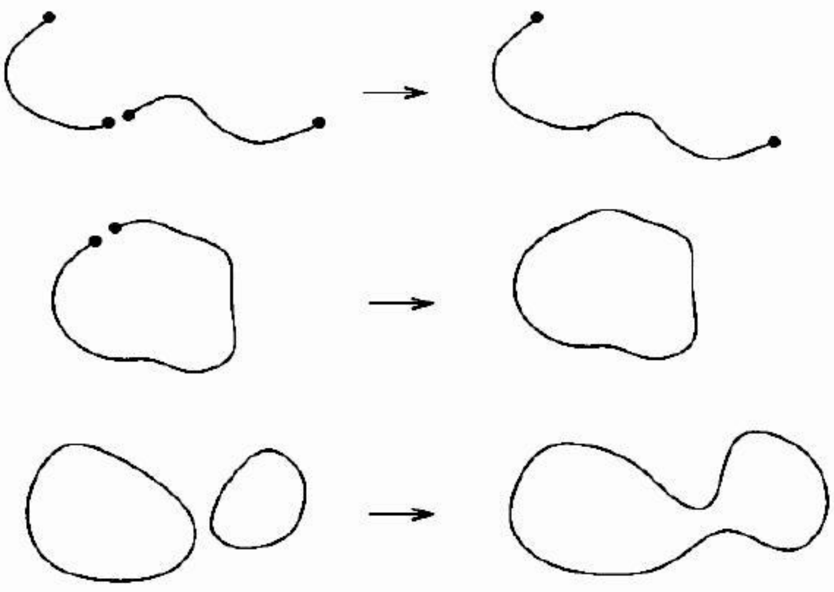
String dynamics

- String interactions

Open strings joining/splitting

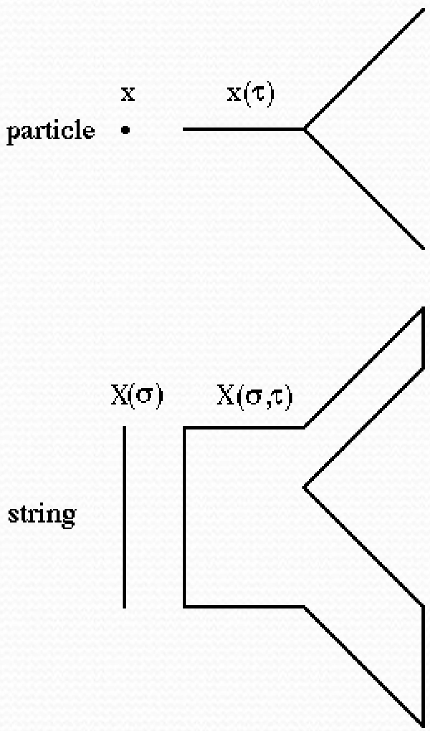
Open string to closed string

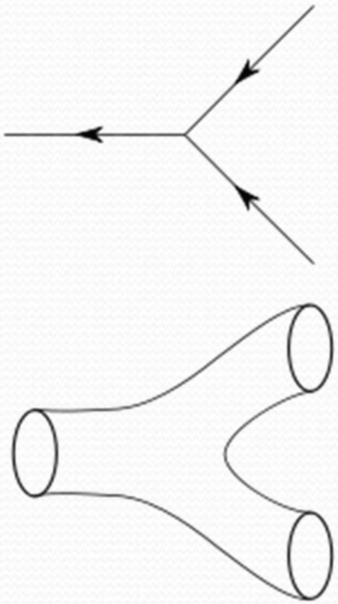
Closed strings joining /splitting



String dynamics

- String interactions: world sheet view

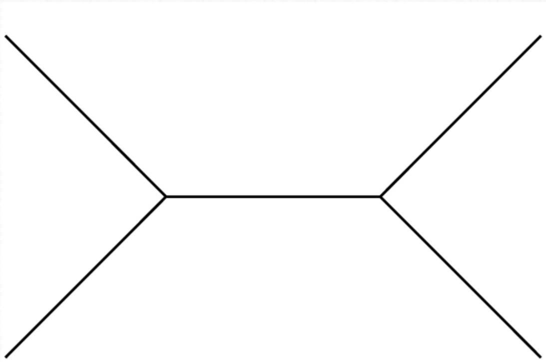


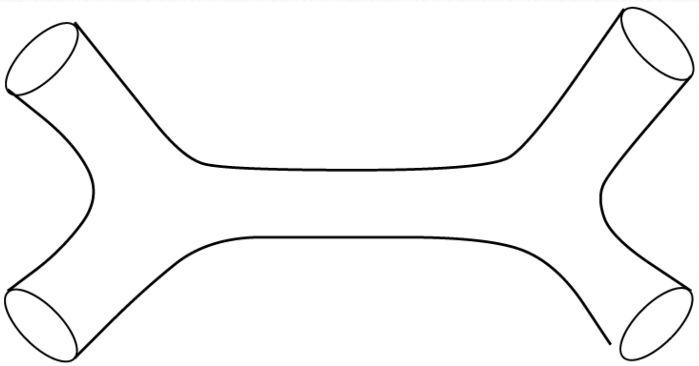


String dynamics

- String (Feynman-)diagrams

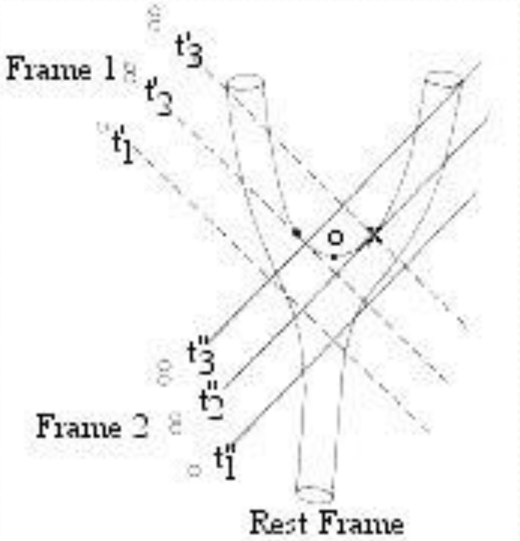
particle scattering: string scattering:





String dynamics

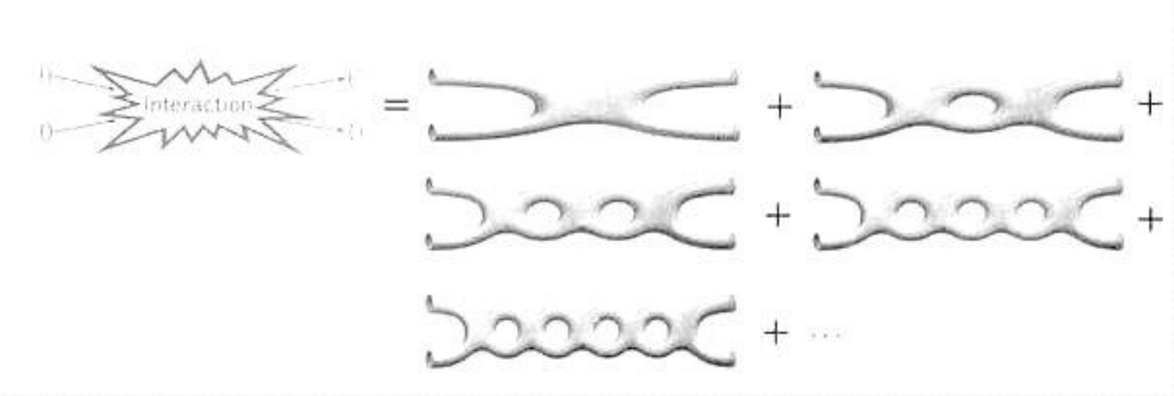
- String interaction not localized in unique singular point of world sheet!!



- String interaction uniquely defined by smooth world sheet (and topology of diagram)

String dynamics

- String amplitude sum over all possible diagrams with given external states and varying topologies



- Smooth interaction assures string perturbation theory is free of ultra-violet (short distance) divergences

Effective field theories

- Low energy effective actions
  - At energies much lower than string scale $\sqrt{T}$ can integrate out massive modes $\Longrightarrow$ effective action for massless modes
  - Most convenient approach: consider string propagation in presence of massless background fields. World-sheet action becomes a so-called nonlinear sigma model:

$$S = -\frac{1}{4\pi}\int d^2\xi \left[G_{\mu\nu}(X)\partial_\alpha X^\mu \partial^\alpha X^\nu + B_{\mu\nu}(X)\epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \right.$$
$$\left. - \phi(X)\mathcal{R}^{(2)} + A^a_\mu(X)J^\mu_\alpha + \ldots\right]$$

  - $G_{\mu\nu},\ B_{\mu\nu},\ \phi,\ A^a_\mu, \ldots$: backgrounds for massless fields

$G_{\mu\nu}$: metric; ϕ : dilaton A^a_μ : gauge fields

$B_{\mu\nu}$: Kalb-Ramond field (antisymmetric tensor)

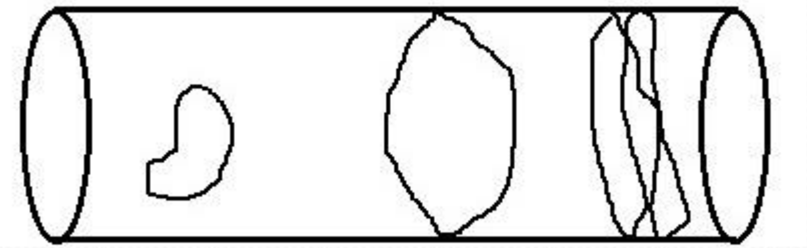
Effective field theories

- Imposing conformal invariance yields equations of motion for background fields, which can in turn be derived from effective action:

$$S^{26} \propto \int d^{26}X \sqrt{g} e^{-2\phi} \left[R - 4 D_\mu \phi D^\mu \phi + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right]$$

  - Here R is the Ricci scalar (Einstein-Hilbert action),
  - $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$ is the field strength for the Kalb-Ramond field

T-duality



- Excess of space-time dimensions: can apply compactification
- Simplest procedure: apply periodicity condition:

$$X \equiv X + 2\pi R$$

- Then free string solution generalizes to include winding modes:

$$X(\sigma,\tau) = X_L(\sigma_+) + X_R(\sigma_-), \qquad \sigma_\pm = \tau \pm \sigma$$

$$X_L(\sigma_+) = \frac{1}{2}x_0^L + \frac{1}{2}(p+w)\sigma_+ + \frac{i}{\sqrt{2}}\sum_{n\neq 0}\alpha_n e^{-in\sigma_+}$$

$$X_R(\sigma_-) = \frac{1}{2}x_0^R + \frac{1}{2}(p-w)\sigma_- + \frac{i}{\sqrt{2}}\sum_{n\neq 0}\tilde{\alpha}_n e^{-in\sigma_-}$$

  - Here *p* and *w* are quantized: $p = m/R$ $\qquad w = nR$

T-duality

- Closed strings
  - Define left- and right momenta for compact direction:

$$p_{L,R} = \frac{m}{R} \pm nR$$

  - Mass in 24+1 uncompactified dimensions becomes:

$$-\frac{1}{4}p^2 = \frac{1}{4}p_L^2 + N_L - 1 = \frac{1}{4}p_R^2 + N_R - 1$$

  - Spectrum is invariant under T-duality:

$$X_L \to X_L \ , \ X_R \to -X_R \quad \Rightarrow \quad R \to \frac{1}{R} \ , \ m \leftrightarrow n$$

  - T-duality exact symmetry to all orders in string perturbation theory
  - For special values of R there can be extra massless modes: enhanced gauge symmetry

T-duality

- Open strings
  - Imposing Neumann b.c. $\partial_\sigma X|_{\sigma=0,\pi} = 0$ implies n = 0 (so no windings)
  - T-duality effectively interchanges $\sigma \leftrightarrow \tau$, thus boundary condition becomes Dirichlet:

$$\partial_\tau X|_{\sigma=0,\pi} = 0$$

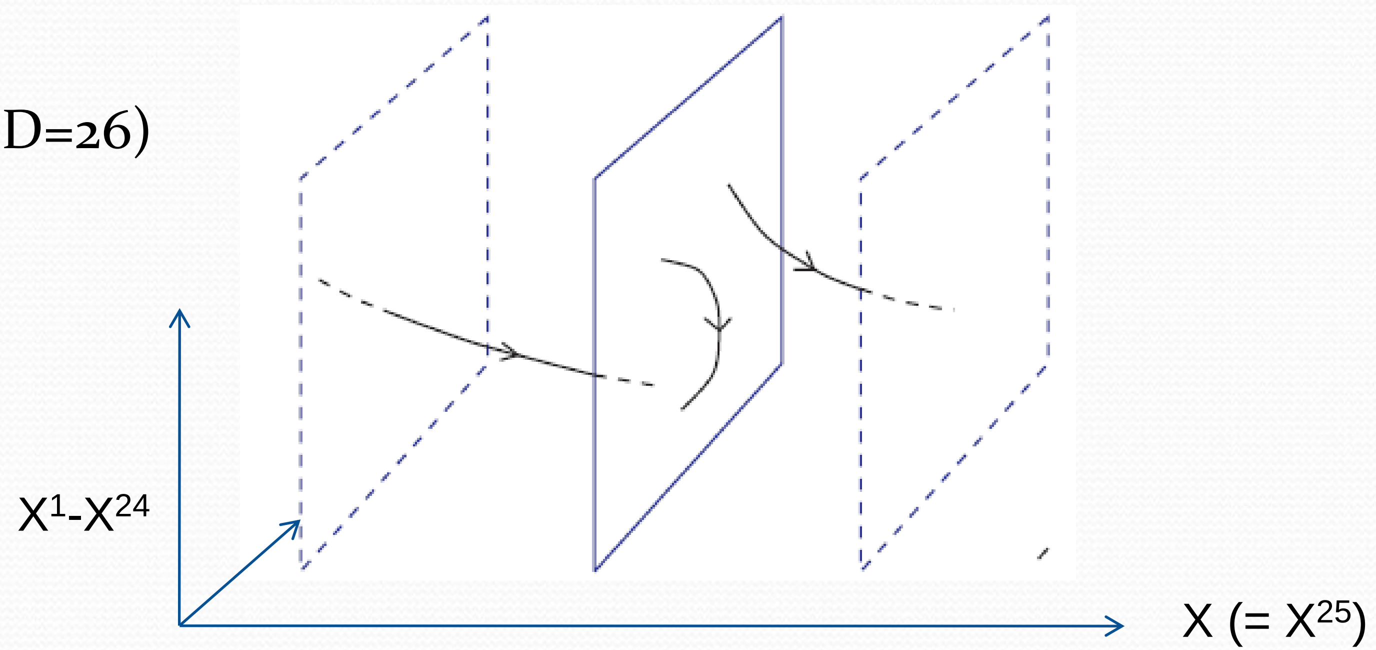
    - Ends of string fixed (at particular points of circle)
    - Coordinate expansion becomes:

$$\begin{aligned} X' &= X_R - X_L \\ &= X'_0 + 2\frac{n}{R}\sigma - \sqrt{2}\sum_{m\neq 0}\frac{1}{m}\alpha_m e^{-im\tau}\sin m\sigma \end{aligned}$$

  - Can in fact choose Neumann (N) or Dirichlet (D) b.c. independently for each string end: NN, ND, or DD

T-duality

(D=26)



- Result: open strings end on 24-dim hyperplane embedded in 25 space dimensions: D24-brane
  - Examples have winding nr. 0 and 1 (shaded planes are identified)

T-duality

- Procedure can be generalized to arbitrary subset of dimensions
- D-brane: surface on which open strings can end
  - Dp-brane:
    - Neumann b.c. for longitudinal directions $X^{\mu=0,..,p}$
    - Dirichlet b.c. for transverse directions $X^{I=p+1,..,25}$
  - Examples:
    - D0-brane: point
    - D1-brane: string
    - D2-brane: membrane
    - D25-brane: space filling ("canonical case" for open strings)
  - T-duality along transverse dir.: Dp-brane ⇨ D(p+1)-brane
  - T-duality along longitudinal dir.: Dp-brane ⇨ D(p-1)-brane

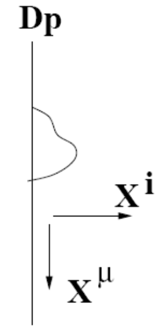
S-duality

- Duality symmetry implying the interchange:

  strong coupling $\longleftrightarrow$ weak coupling
- Oldest known example: Montonen-Olive duality of Maxwell theory
- Many examples in string theory:
  - Type IIB string theory w/ coupling constant g is equivalent to type IIB string theory w/ coupling constant $1/g$
  - Type I string theory w/ coupling constant g is equivalent to SO(32) heterotic string theory w/ coupling constant $1/g$
  - Type IIA and E8 heterotic string theories w/ coupling constant g are equivalent to M-theory with compact dimension of size g
- S-duality exchanges local charges with topological charges

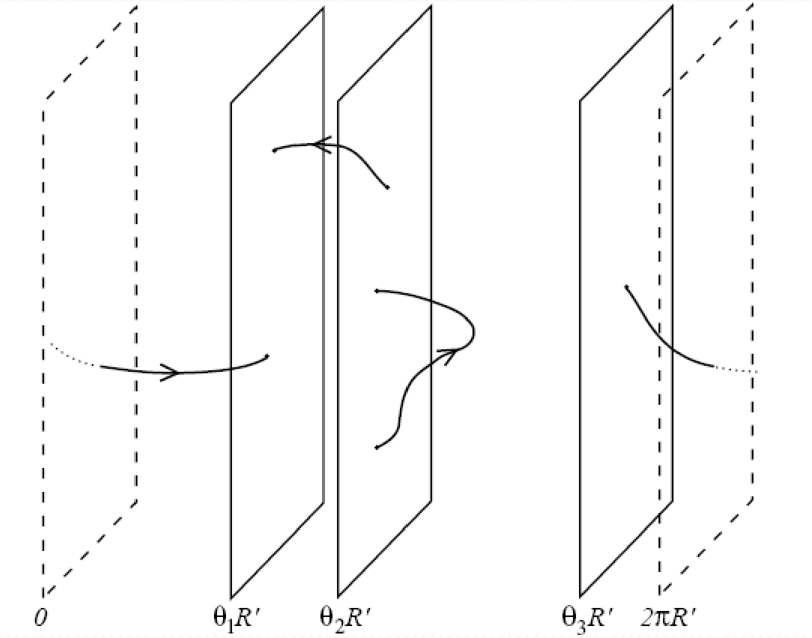
D-branes

- Open strings ending on Dp-brane:
  - Dirichlet b.c. for coordinates normal to Dp-brane
  - Neumann b.c. for coordinates tangential to Dp-brane
- Massless 26 dim. open string vector field $a^{\mu}_{-1}|p\rangle$ (beginning and ending on same Dp-brane) splits into:
  - massless p+1 dim. Maxwell field living on brane world-volume
  - massless scalar for each normal direction X^i
    - Correspond to fluctuations of position of Dp-brane. D-branes are dynamical objects!



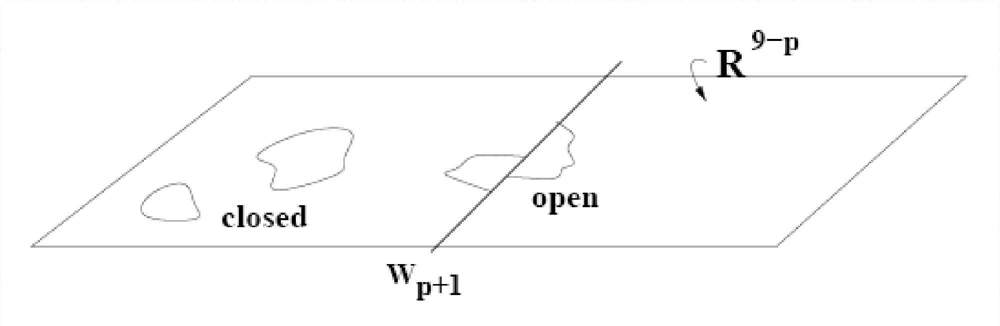
D-branes

- Open strings between parallel D-branes
  - String states carry quantum numbers i and j of D-branes
  - Extra contribution to mass² for nonzero distance between D-branes
  - For N overlapping D-branes get N^2 instead of N massless vectors: enhanced symmetry $U(1)^N \Rightarrow U(N)$ (Chan-Paton factor)



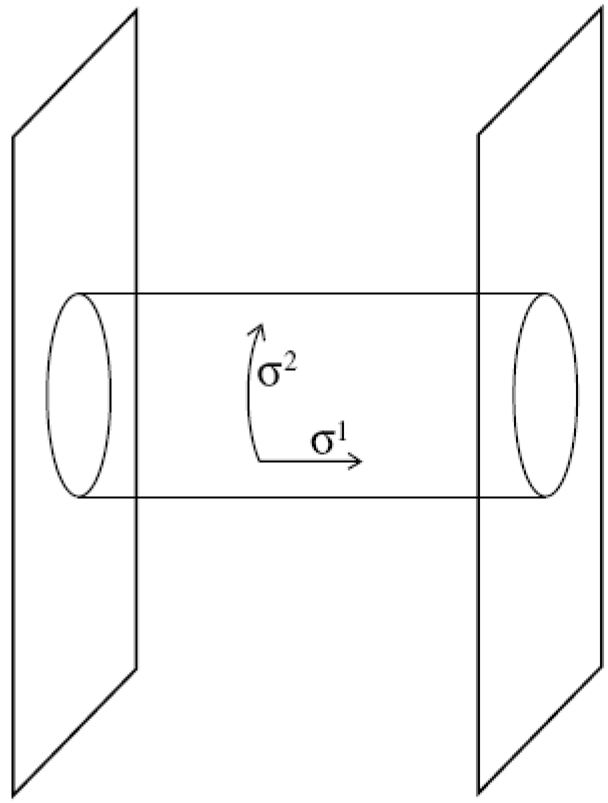
D-branes

- Closed strings
  - can move away from D-branes
  - Closed string modes describe fluctuations of bulk space-time around vacuum (gravitons, dilaton modes, etc.)



  - Closed strings do interact with D-branes: they can be exchanged between them!

D-branes



Exchange of closed strings between D-branes can be viewed as vacuum loop of open string (Casimir energy)

D-branes

- String charge
  - Maxwell charge naturally carried by points via Wilson line coupling to gauge field:

$$q\int A_\mu(X(\tau))\frac{dX^\mu(\tau)}{d\tau}d\tau \equiv q\int A_\mu(X)dX^\mu$$

  - Similarly, the string coordinates naturally couple to the Kalb-Ramond field through the coupling

$$-\int d\tau d\sigma \frac{\partial X^\mu}{\partial\tau}\frac{\partial X^\nu}{\partial\sigma}B_{\mu\nu}(X(\tau,\sigma))$$

    - Thus, the string carries "string charge"

D-branes

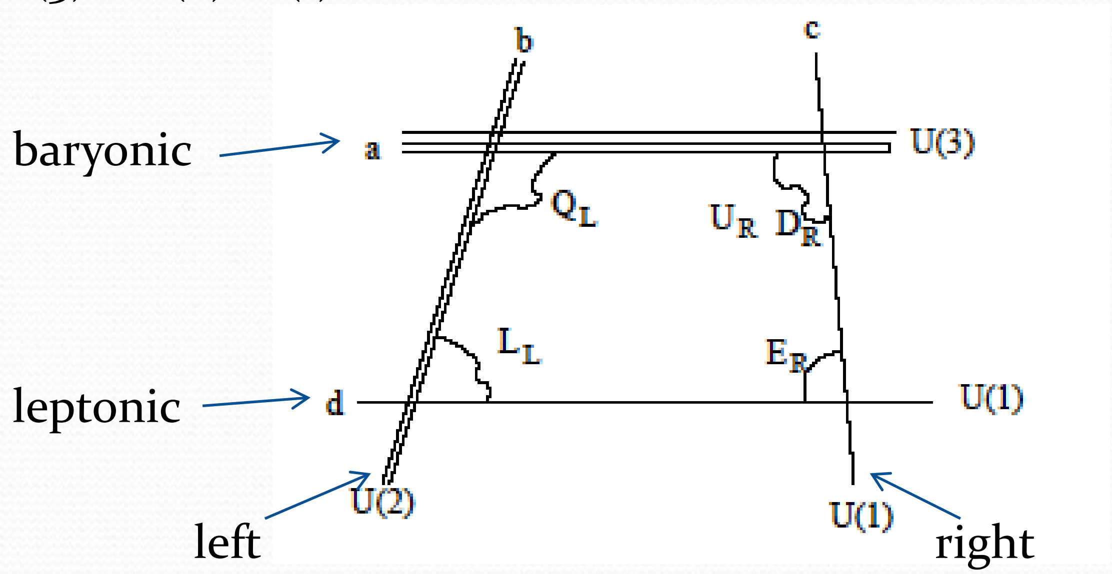
- D-brane charges
  - Dp-brane has (p+1)-dimensional world volume, couples naturally to massless antisymmetric tensor field with p+1 indices ((p+1)-form):

$$S_p = -\int d\tau d\sigma_1 \dots d\sigma_p \frac{\partial X^\mu}{\partial \tau} \frac{\partial X^{\nu_1}}{\partial \sigma^1} \dots \frac{\partial X^{\nu_p}}{\partial \sigma^p} A_{\mu\nu_1\dots\nu_p}(X(\tau, \sigma^1, \dots, \sigma^p))$$

  - Bosonic string: only massless antisymmetric tensor is $B_{\mu\nu}$
    - Bosonic string D-branes are uncharged
  - Type II superstrings have additional antisymmetric tensor fields in RR sector, leading to charged D-branes:
    - Type IIA: 1-form, 3-form $\Longrightarrow$ D0, D2
    - Type IIB: 0-form, 2-form, 4-form $\Longrightarrow$ D-1, D1, D3
  - Charged D-branes are stable, uncharged D-branes unstable

D-branes

- Model building using D-branes
  - Can use configuration of charged D-branes for model building in 4-dim. space-time
  - Example: use intersecting D6-branes wrapped on T^6 (leaving 10-6=4 large dimensions)
    - Fermions are open strings stretched between intersecting D-branes
    - Possible to obtain SU(3)*SU(2)*U(1) Standard Model



Outlook

- String theory is a large program aiming at unification of all interactions including gravity, at the most fundamental level
- Enormous progress has been made the past few decades in numerous different directions. At this point not clear what aspects will turn out to be just mathematics, what real physics
- Many breakthroughs as well as hope that theory will eventually be seriously confronted with experimental data has continued to drive researchers
- Many aspects of string theory have not been treated in this lecture:
  - Superstrings
  - Conformal field theory formulation
  - AdS /CFT correspondence → part 2 of this course, by Miguel Zilhão
  - String thermodynamics and black-hole information paradox
  - String field theory
  - Cosmological backgrounds
  - …….

Further Literature

Textbooks

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Further Literature

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Thanks for your attention!

The Superstring

- Problems with the bosonic string
  - Phenomenological: Absence of fermions
  - Stability: presence of tachyon
- Solution: superstring!
  - Add fermionic coordinates (Majorana-Weyl): $\psi^{\mu}_{L,R}(\sigma,\tau)$
  - Need worldsheet supersymmetry to avoid extra negative-norm states:

$$\delta X^{\mu}_{L} = -i\epsilon_R \psi^{\mu}_{L}$$
$$\delta \psi^{\mu}_{L} = \epsilon_R \partial_+ X^{\mu}_{L}$$

  - Absence of conformal anomaly implies $D = 10$
  - Two possible consistent periodicity conditions:

$$\psi^{\mu}(\tau,\sigma) = \pm\psi^{\mu}(\tau,\sigma+2\pi)$$

    - +: Ramond (R), -: Neveu-Schwarz (NS)

The Superstring

- General solution (left-movers):

$$\psi^\mu_{L,R} = \begin{cases} \sum_n d^\mu_n e^{-in(\tau\pm\sigma)} & \text{(R)} \\ \sum_r b^\mu_r e^{-ir(\tau\pm\sigma)} & \text{(NS)} \end{cases}$$

  - n: integer; r: half-integer

- Mass formula: $-\frac{1}{4}p^2 = N + N_\psi - a$

  - NS: $N_\psi = \frac{1}{2} \sum_{r\in Z+\frac{1}{2}} : b_{-r} \cdot b_r : , \qquad a = \frac{1}{2}$

  - R: $N_\psi = \frac{1}{2} \sum_{n\in Z} : d_{-n} \cdot d_n : , \qquad a = 0$

The Superstring

- Spectrum
  - NS sector
    - Vacuum: $|p>$ with $-(1/4)p^2 = -1/2$ *(tachyon)*
    - Massless vector: $b^{\mu}_{-1/2}|p>$
  - R sector
    - Fermionic coordinates have zero modes satisfying Clifford algebra (act as gamma matrices): $\{d_0^{\mu}, d_0^{\nu}\} = \eta^{\mu\nu}$
    - Thus vacuum corresponds to space-time spinor $|p, \alpha\rangle$ dimension $2^{d/2} = 32$. Zero modes act as:

      $$i\sqrt{2}d_0^{\mu}|p, \alpha\rangle = \gamma^{\mu}_{\alpha\beta}|p, \beta\rangle$$
    - World-sheet supersymmetry implies vacuum satisfy 10d massless Dirac equation, reducing dimensionality to 16 (Majorana fermion).

The Superstring

- Spectrum
  - Gliozzi-Scherk-Olive (GSO) projection
    - To resolve problem of tachyonic NS vacuum (instability), one applies the *GSO projection* (*F* = fermion number):

$$(-)^F = -1$$

    - NS sector: eliminates half-integer levels (including tachyon): bosons
    - R sector: imposes chirality at every level

$$(-)^F \equiv \gamma^{11}(-)^{\sum_{n=1}^{\infty} d_{-n}\cdot d_n}$$

    - At massless level: *NS*: massless vector (8 bosonic d.o.f.); *R:* Weyl Majorana fermion (8 fermionic d.o.f.)
    - Fermion-boson degeneracy holds at all levels: space-time supersymmetry

The Superstring

- Closed superstrings
  - Two theories: type IIA and type IIB, depending on relative chirality of left- and right-movers (opposite or same)
  - Massless spectrum: $(|\mu\rangle, |\alpha\rangle_L) \otimes (|\nu\rangle, |\beta\rangle_{L,R})$

bosons

NS-NS: $|\mu\rangle \otimes |\nu\rangle = 1 + 35_S + 28_A$ (dilaton, graviton, Kalb-Ramond field)

R-R: $|\alpha\rangle \otimes |\beta\rangle \overset{\text{IIA}}{=} 8_{1-\text{form}} + 56_{3-\text{form}}$

$\overset{\text{IIB}}{=} 1_{0-\text{form}} + 28_{2-\text{form}} + 35_{4-\text{form}}$

fermions

R-NS: $|\alpha\rangle \otimes |\nu\rangle = 8_L + 56_L$

NS-R: $|\mu\rangle \otimes |\beta\rangle \overset{\text{IIA}}{=} 8_R + 56_R$

$\overset{\text{IIB}}{=} 8_L + 56_L$

- Massless spectra coincide with N=2, D=10 supergravity

The Superstring

- Open superstrings (type I)
  - Can be defined by taking type IIB superstring and making the (Z_2 -) identification: $\sigma \rightarrow -\sigma \qquad (L \leftrightarrow R)$
  - Results in twisted and untwisted sector (Z_2 –eigenvalue 1 or -1)
    - *Untwisted sector (unoriented closed strings):* symmetrized NS-NS sector (dilaton and graviton) and antisymmetrized R-R sector (two-form)
    - *Twisted sector (unoriented open strings):* quantum consistency (absence of anomalies) requires Chan-Paton factors corresponding to SO(32) gauge group
- Heterotic strings
  - Tensor product of left-moving superstrings (D=10) and right-moving bosonic strings (D=26, 16 compactified).
    - Gauge symmetry $SO(32)$ or $E_8 \times E_8$

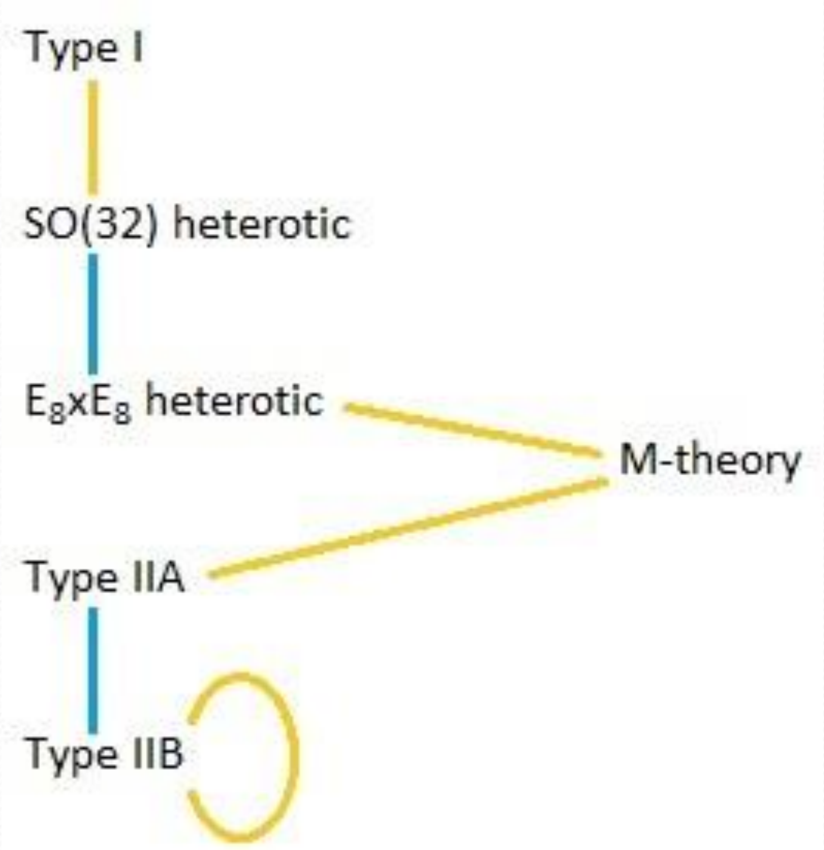
The Superstring

Summary of the 5 theories:

	Type IIA	Type IIB	$E_8 \times E_8$ Heterotic	SO(32) Heterotic	Type I SO(32)
String Type	Closed	Closed	Closed	Closed	Open (+ closed)
Oriented?	Yes	Yes	Yes	Yes	No
$D = 10$ Supersymmetry	$N = 2$ (non-chiral)	$N = 2$ (chiral)	$N = 1$	$N = 1$	$N = 1$
$D = 10$ Gauge groups	none	none	$E_8 \times E_8$	SO(32)	SO(32)
R-R sector	$C^{(1)}C^{(3)}$	$C^{(0)}C^{(2)}C^{(4+)}$	none	none	$C^{(2)}$

The Superstring

Web of duality symmetries between the 5 theories:



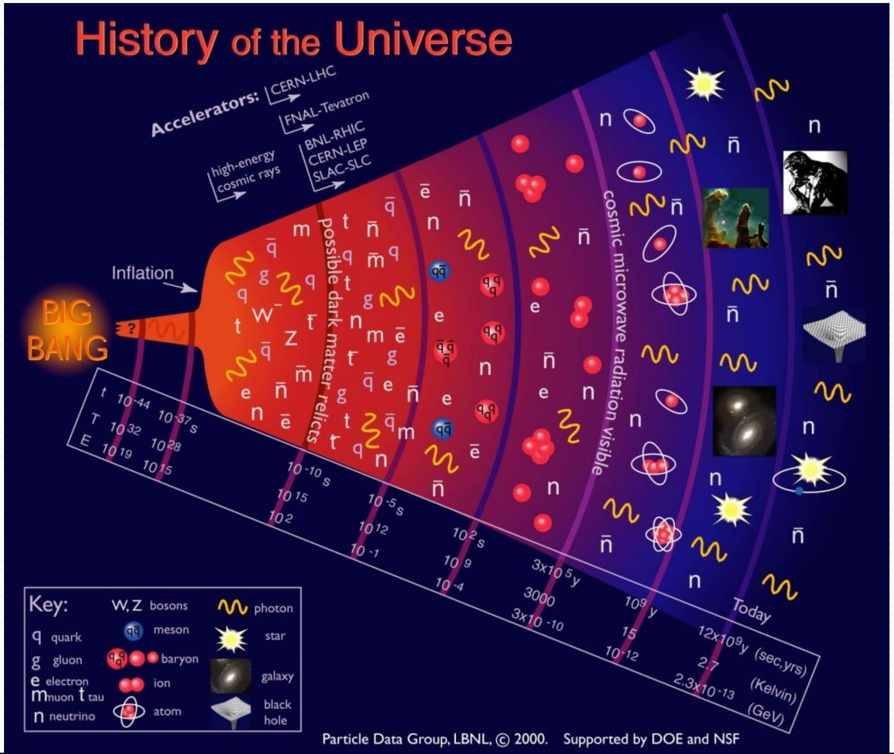
Blue lines: T duality
Yellow lines: S duality

Why String theory?

- Current status of Elementary Particle Physics
  - Described by quantum gauge field theory
    - Gauge group $SU(3)_c * SU(2)_w * U(1)_Y$
    - $SU(2)_w * U(1)_Y$ symmetry spontaneously broken to U(1) of Maxwell theory at ~ 100 GeV by Higgs field
    - Experimentally tested up to about 1 TeV (or, down to ~ 10^{-19} m)
  - Chiral matter (Weyl fermions)
    - Mass introduced by Yukawa coupling to Higgs
    - Three families with (almost) massless neutrinos
- Current status of Gravitational interactions
  - Described by classical theory: General Relativity
    - Interactions encoded in space-time metric via principle of equivalence, yielding Einstein equations (Einstein-Hilbert action)
    - Typical mass scale $M_p \sim 10^{19}$ GeV
    - Experimentally tested down to ~ 10^{-7} m

Why String theory?

- Combination of Standard Models successfully describes many phenomena in Astrophysics and Cosmology:
  - Star birth and evolution
  - Big bang nucleosynthesis
  - Large scale structure formation based on inflation and cold dark matter



Why String theory?

- Many questions remain:
  - Dual description insatisfactory: would like gravitational interactions consistent at quantum level. How?
  - Can all interactions be described in unified setup?
  - Why are there two widely different mass scales?
    - Are there other scales between M_W and M_p?
  - Why gauge sector as it is? Why 3 families?
  - Why is the proton stable? (Is it?)
  - Why 4 dimensions?
  - Why is the cosmological constant so small?
  - Why ……, ………, ………?

Why String theory?

Some proposals for Physics beyond the Standard Model:

1. Grand Unified Theories (GUT's)
   - Standard model gauge group remnant of larger simple group
   - Broken at GUT scale $\sim 10^{16}$ GeV (if no new physics above M_W)
   - Partially explains fermion quantum numbers
   - Proton decay usually possible
2. Supersymmetry (SUSY)
   - Global symmetry relating bosonic and fermionic degrees of freedom
   - Boson/fermion partners have equal mass
   - Mass of chiral fermion is forced to be zero
     - Explanation of low-mass fermions
   - So far SUSY partners of known fermions not observed
     - SUSY must be broken at ~1 TeV (problematic)

Why String theory?

3. Supergravity
   - Supersymmetry realized as local gauge symmetry, together with diffeomorphism invariance (General Relativity)
   - Includes gravitino and SUSY matter
   - SUSY breaking can be accomplished spontaneously at ~ 1 TeV
   - Does not solve quantum gravity problem (UV divergent)
4. Extra dimensions
   i. Kaluza-Klein theories: 4-D gauge bosons as components of metric tensor in higher D
      - Problematic for phenomenology (S-M gauge group, chiral fermions)

   ii. Brane-world models: Standard Model lives on brane in D > 4 space-time, while gravity propagates in 'bulk'
      - Present bound: extra dimensions < 0.1 mm (big!):
      - Allows for large 4-dim M_P from much smaller D-dim M_P (possible explanation of hierarchy problem!)
      - Treats gravity classically