



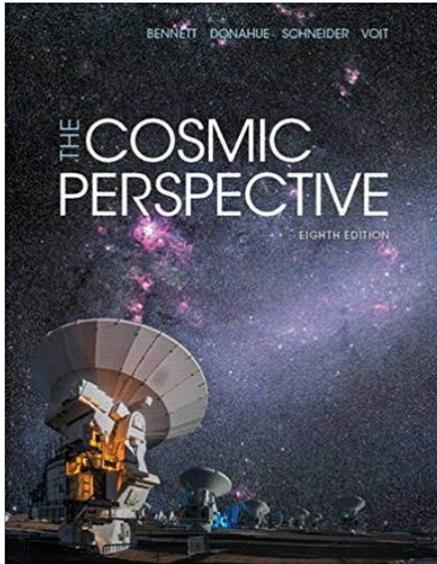
SAG10 - 10th School on Astrophysics and Gravitation

Astrophysics of Stars

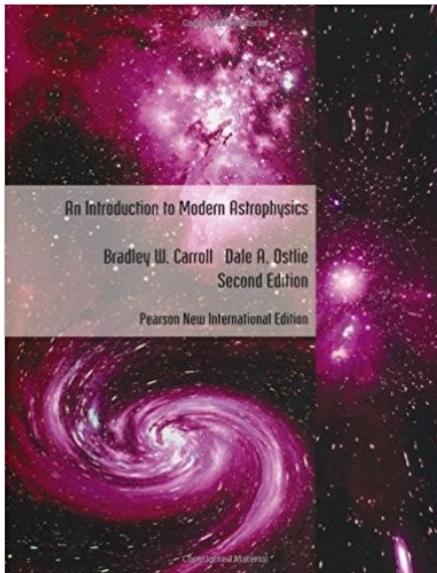
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Literature



- Bennett, Donahue, Schneider & Voit:
The Cosmic Perspective
(8th Edition, 2016)



- Carroll & Ostlie: **An Introduction to Modern Astrophysics**
(2nd Edition, 2013)

Course outline

- Basic properties of stars
- Stellar spectra and classification
- HR-diagram and the stellar main sequence
- Energy production
- Stellar evolution



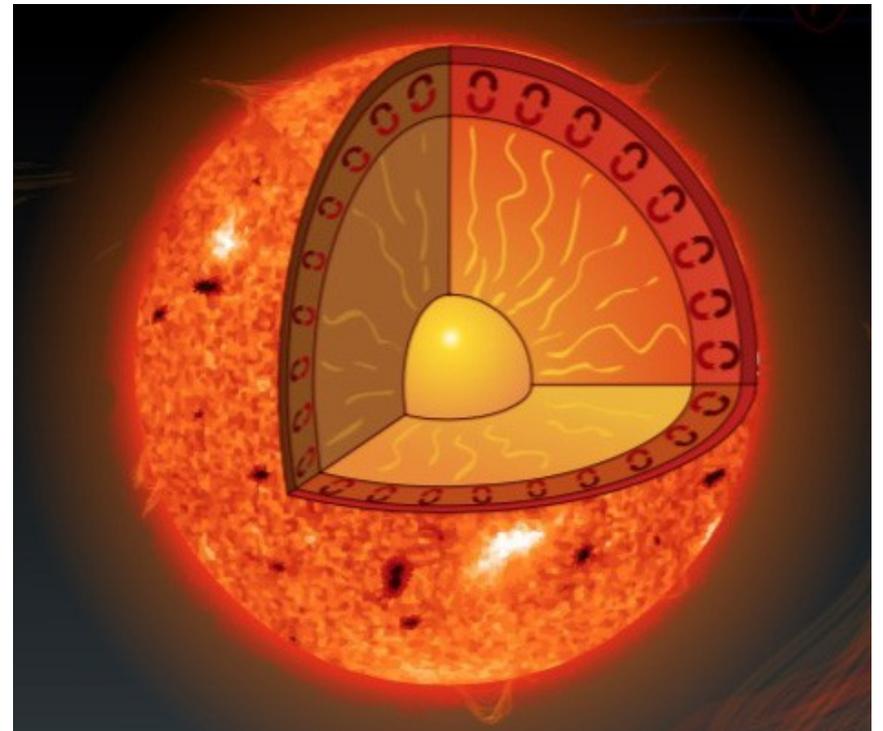
Course outline

- **Basic properties of stars**
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What is a star?

- Spherical object made predominantly of Hydrogen and Helium
- Held together by its own gravity
- Thermo-nuclear fusion of hydrogen in the core
(at present or in the past)



Luminosity vs. Apparent brightness

- **Luminosity** = total amount of energy emitted per unit time

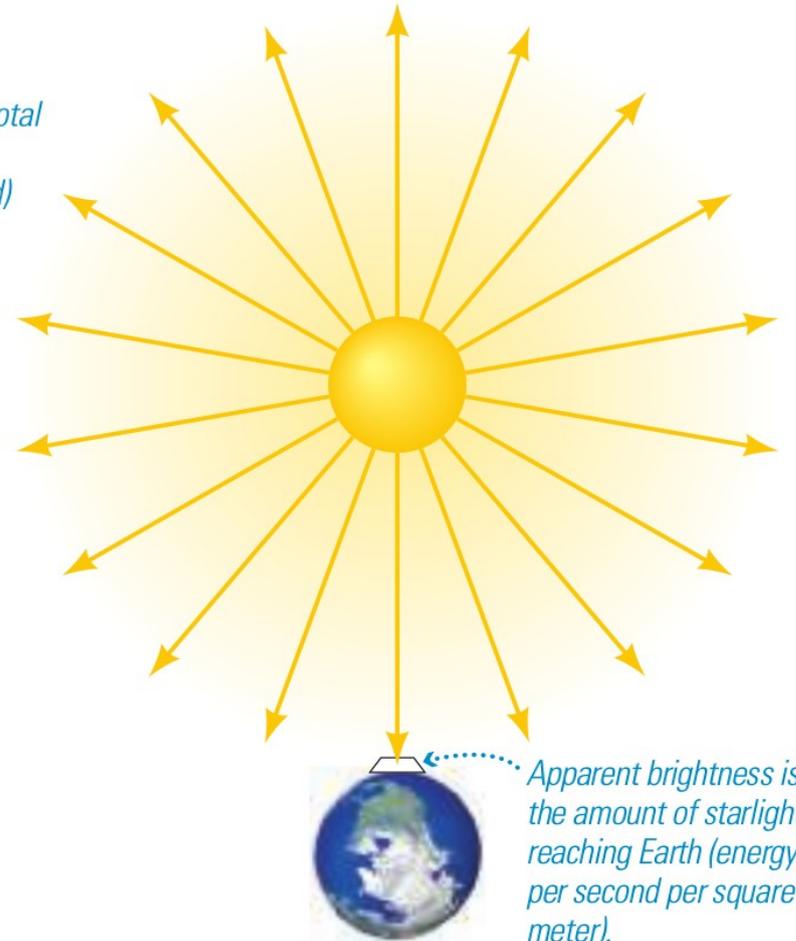
Physical unit: J/s or W

Intrinsic property

- **Apparent brightness**

= amount of power
(energy per sec) reaching
the observer per unit area
Physical unit: W/m^2

*Luminosity is the total
amount of power
(energy per second)
the star radiates
into space.*



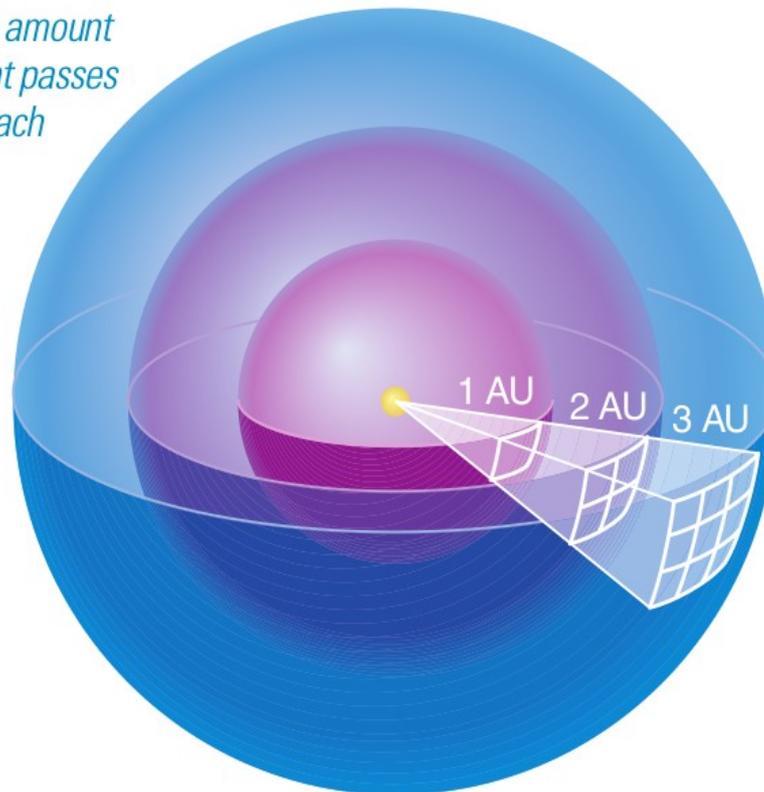
Not to scale!

Luminosity vs. Apparent brightness

- Inverse square law for light

$$\text{apparent brightness} = \frac{\text{luminosity}}{4 \pi \text{distance}^2}$$

The same amount of starlight passes through each sphere.



The surface area of a sphere depends on the square of its radius (distance from the star) . . .

. . . so the amount of light passing through each unit of area depends on the inverse square of distance from the star.

Stellar luminosities

- Often expressed in solar luminosities (L_{\odot})
- $1L_{\odot}=3.828\times 10^{26}$ W
- Stars come in wide range of luminosities
roughly: $10^{-4} - 10^6 L_{\odot}$

Proxima Centauri

red dwarf

$d = 1.3$ pc

$m = 0.12 M_{\odot}$

$L = 0.0017 L_{\odot}$

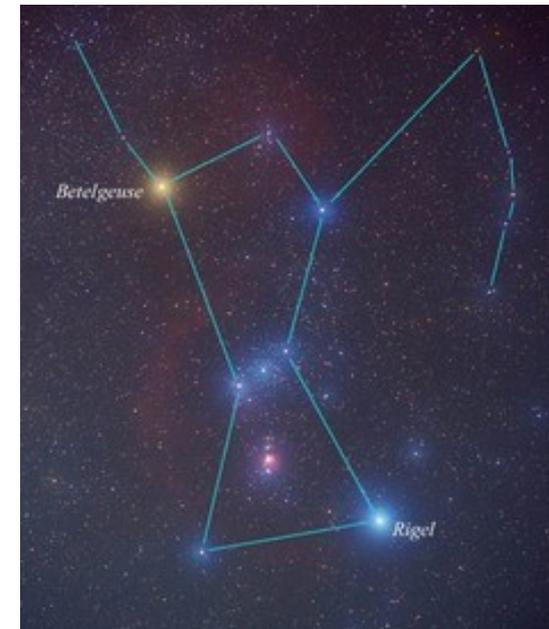
Betelgeuse (α Orionis)

red supergiant

$d = 220$ pc

$m \approx 12 M_{\odot}$

$L \approx 120,000 L_{\odot}$



Magnitude scale

- Often used by astronomers to express the stellar brightness

Apparent magnitude

- devised by Hipparchus (~135 BC) and Manilius (1st cent. AD)
- brightest star visible to the naked eye: $m=1$, faintest $m=6$

- Modern definition (introduced in the 19th century):

a difference of 5 magnitudes corresponds to a factor of 100 in brightness

$$\frac{F_1}{F_2} = 100^{(m_2 - m_1)/5} \quad \longrightarrow \quad m_1 - m_2 = -2.5 \log_{10} \left(\frac{F_1}{F_2} \right)$$

Magnitude scale

- Often used by astronomers to express the stellar brightness

Apparent magnitude

$$m_1 - m_2 = -2.5 \log_{10} \left(\frac{F_1}{F_2} \right)$$

- brighter the star, lower its magnitude (!)
- Magnitude scale is relative

Vega: $m=0$ (by definition)



Magnitude scale

Absolute magnitude

= apparent magnitude that a star would have if placed at a distance of 10 pc

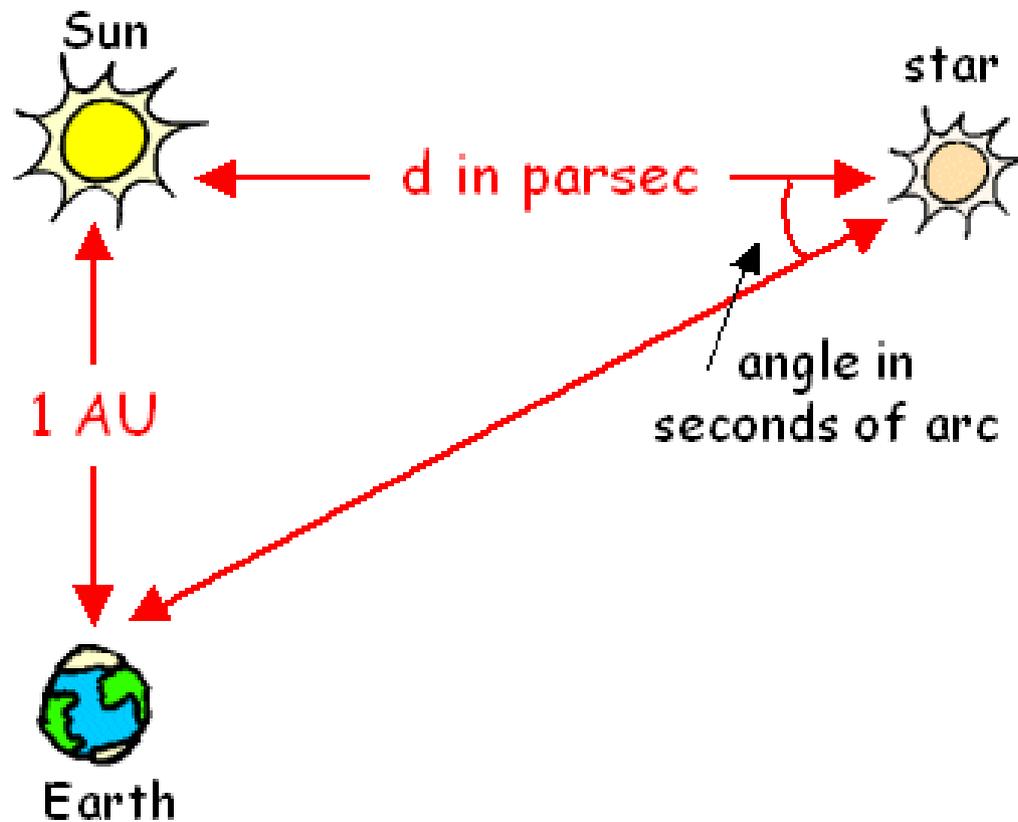
- way to express the luminosity (intrinsic)

Distance modulus $m - M = 5 \cdot \log_{10} \left(\frac{d}{10 \text{ pc}} \right)$

Star	m_v	M_v	Distance (pc)
Sun	-26.8	4.83	4.8e-6
Proxima Cen	11.1	15.6	1.3
Betelgeuse	0.5	-5.85	220
Sirius A	-1.46	1.42	2.6

Definition of parsec

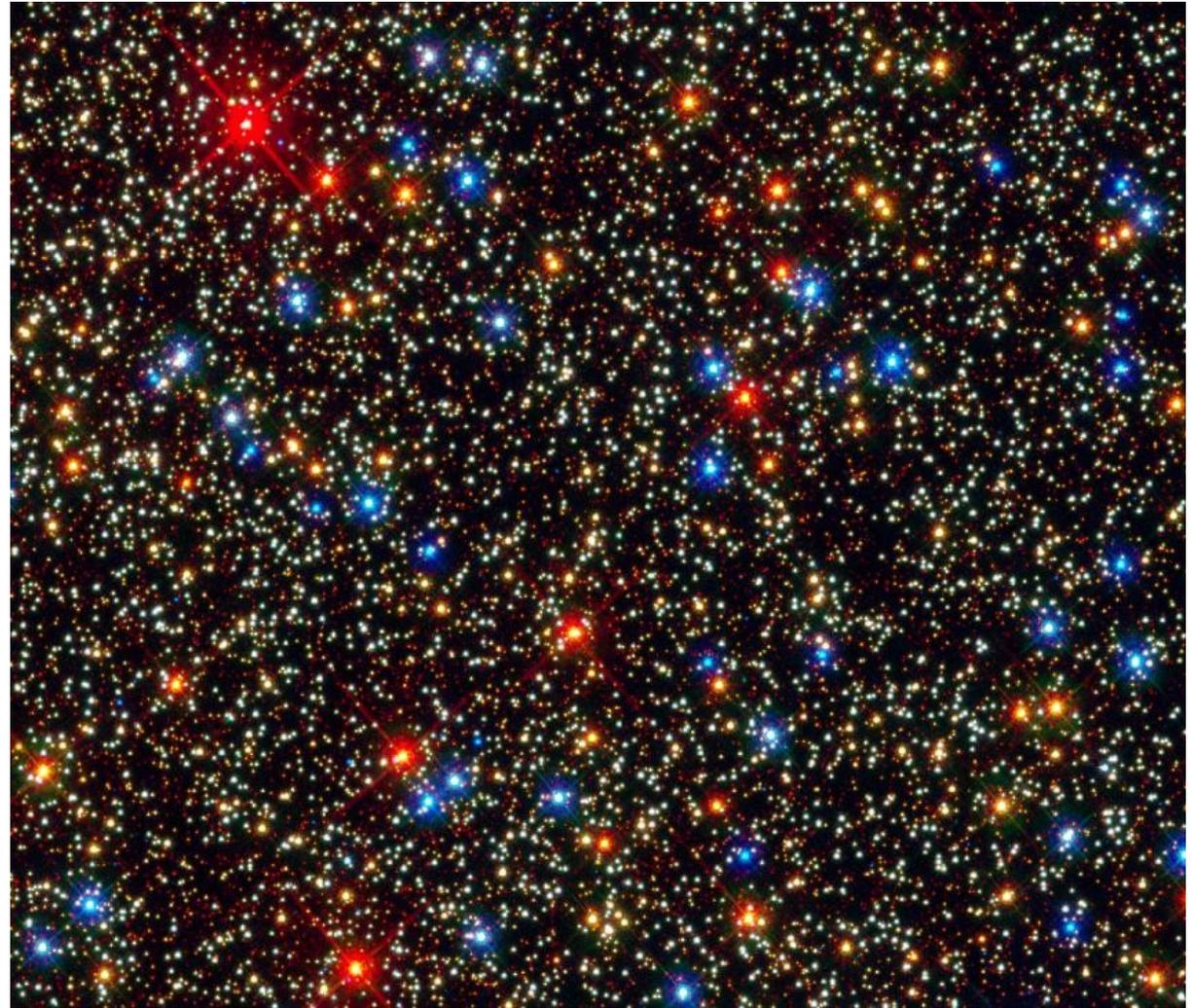
- A distance from which the 1 AU is seen at an angle of 1 arcsecond



Surface temperature and black body radiation



Orion constellation



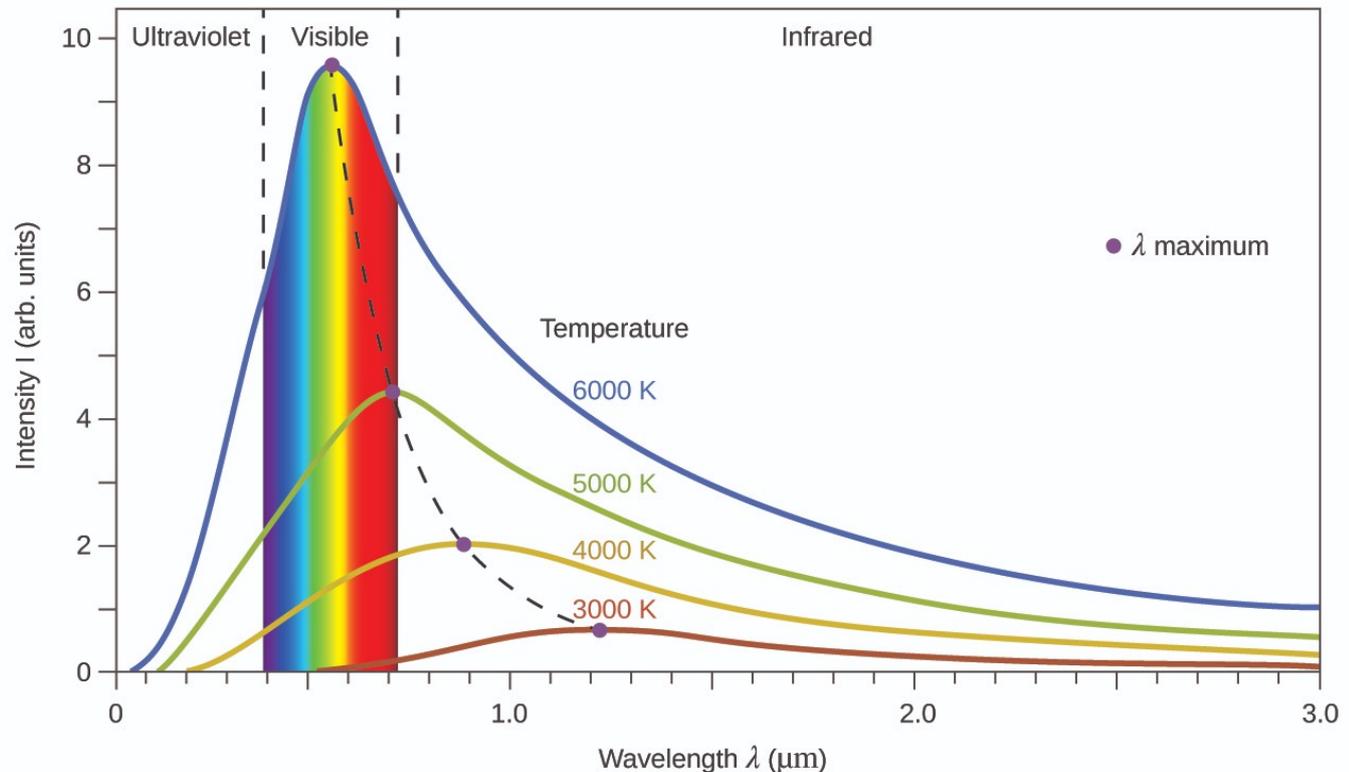
Central part of the Omega Cen globular cluster (HST)

Surface temperature

- Color is directly related to the surface temperature
- Cool stars are red, hot stars are blue
 - Betelgeuse: 3400 K
 - Rigel: 10100 K
- To a first approximation, stars emit as black bodies

Wien's displacement law:

$$\lambda_{\max} T = \text{const.}$$



Stefan – Boltzmann equation

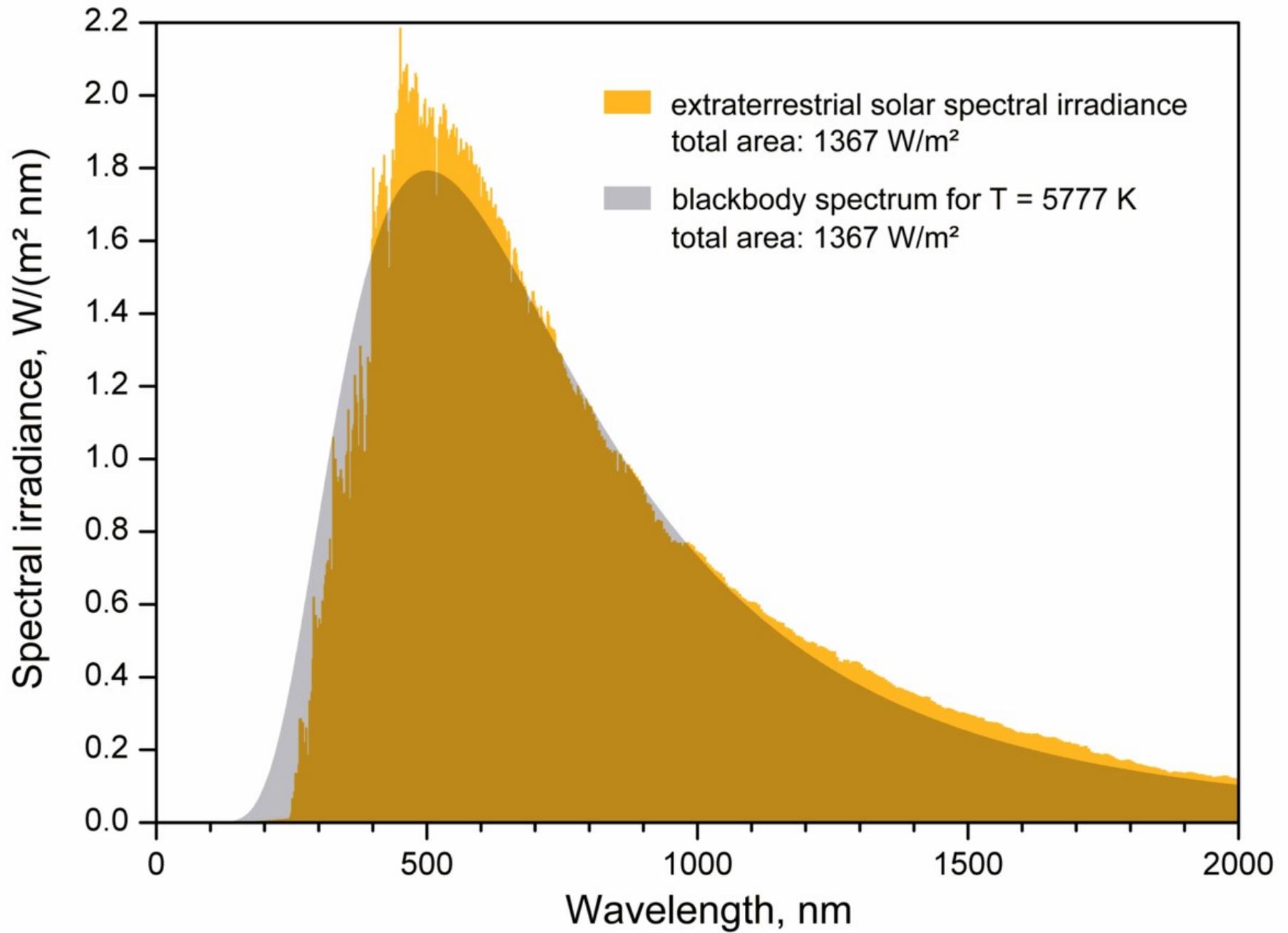
$$L = 4 \pi R^2 \sigma T_{\text{eff}}^4$$

luminosity radius effective temperature

$$\sigma = 5.6704 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \text{ S-B constant}$$

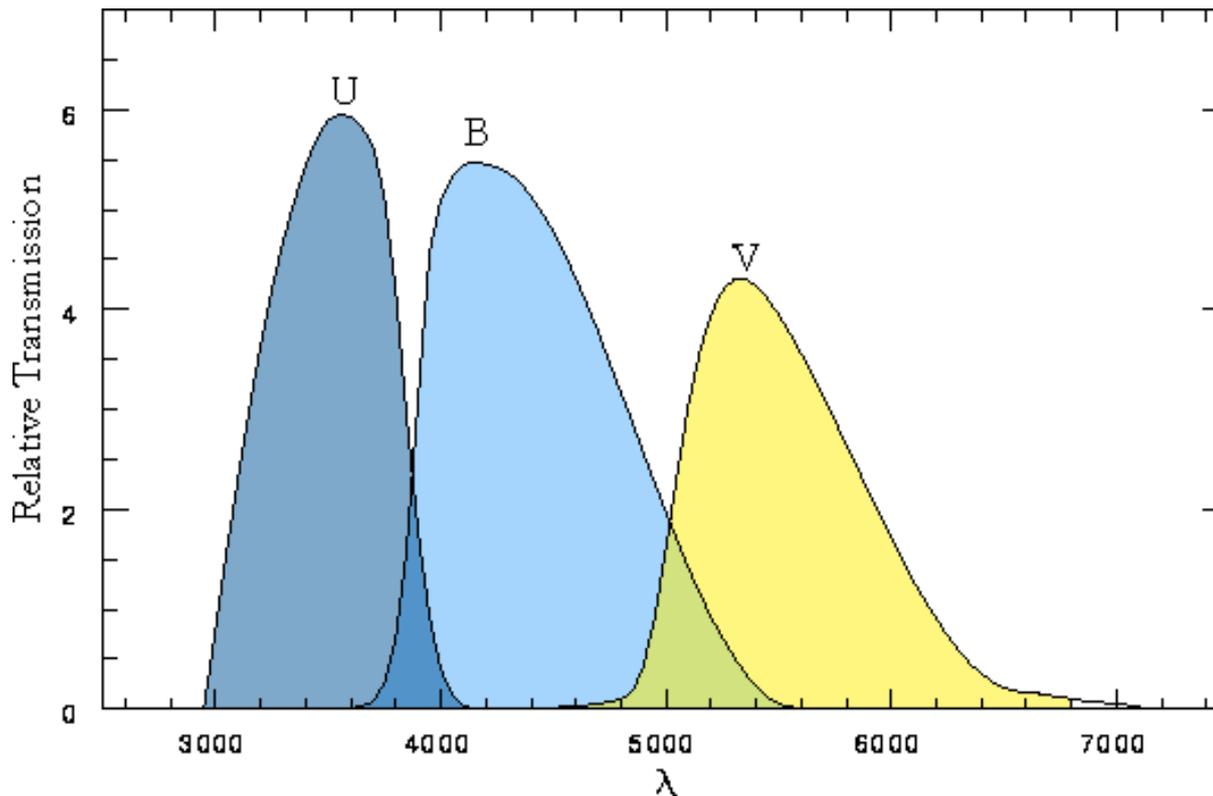
Concept of the **effective temperature**:

- Stars are not perfect black bodies
- T_{eff} = temperature of a blackbody with the same integrated surface flux as the star
- Sun: $T_{\text{eff}}=5777 \text{ K}$, $\lambda_{\text{max}}=501.6 \text{ nm}$



Concept of color

- Bolometric magnitude = measured over all wavelengths
- In practice, we observe in different wavelength passbands (filters)



Color index

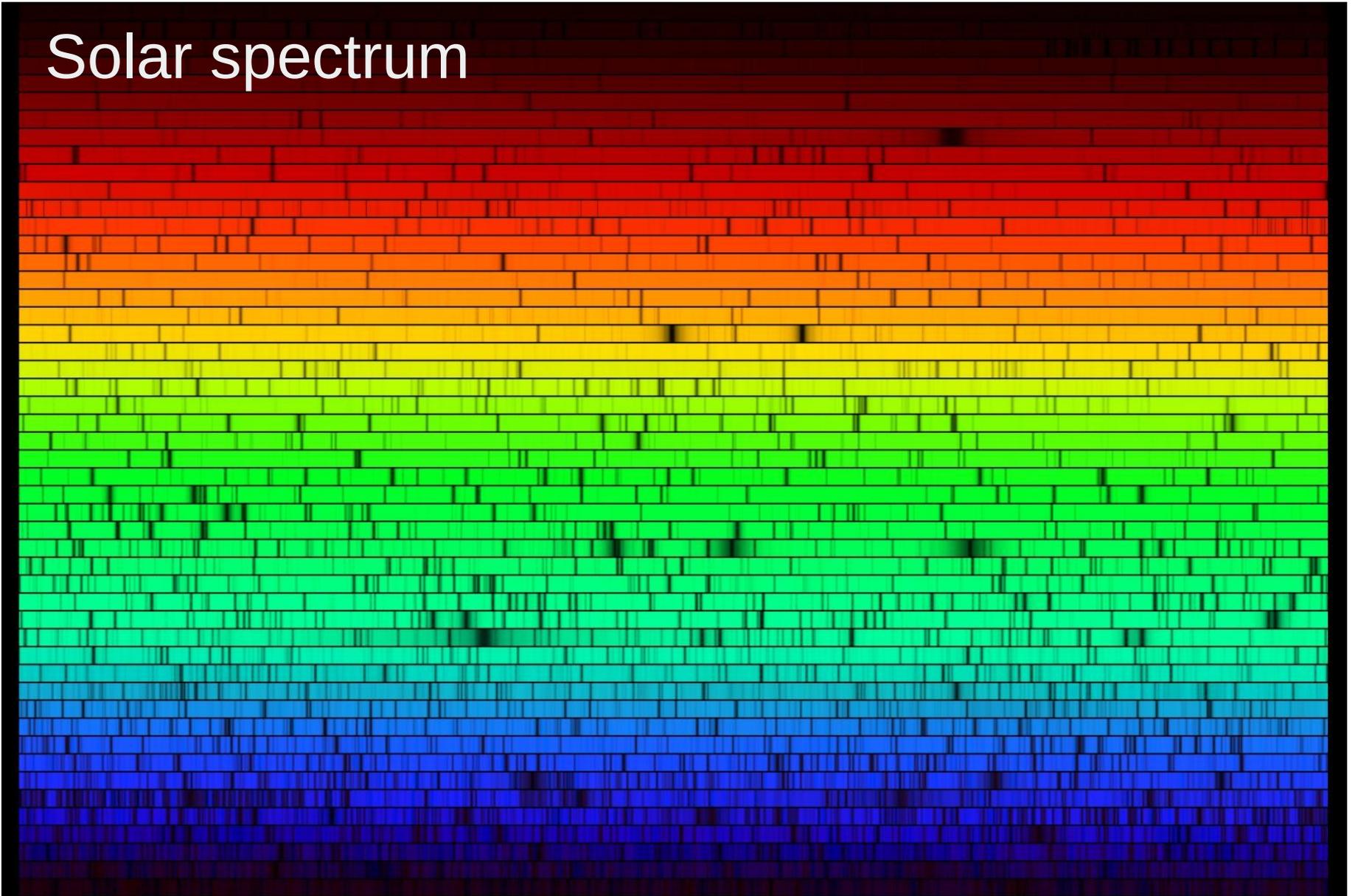
$$U-B=m_U-m_B$$

$$B-V=m_B-m_V$$

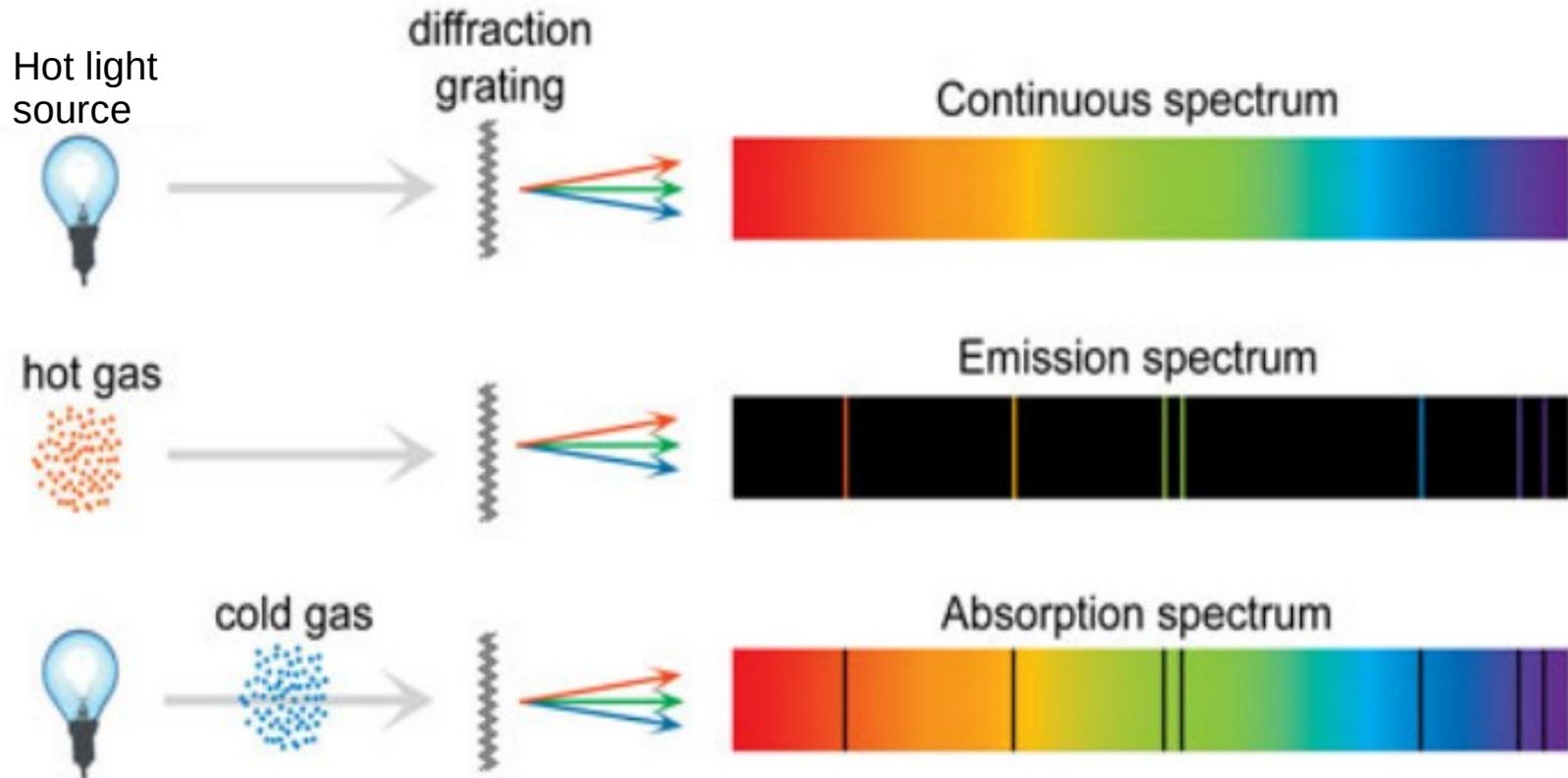
A star with a smaller (B-V) color index is bluer than the one with higher (B-V)

Absorption lines

Solar spectrum



Absorption lines



- Fraunhofer (1787–1826) – dark lines in Solar spectrum
- Kirchhoff (1824–1887) – laws of radiation

80

Fig. 6.

Joseph Fraunhofer

A a B c

D

E b

F

G



Sonnenspektrum
mit Fraunhofer-Linien

Joseph von Fraunhofer *Optiker und Physiker 1787-1826* Deutsche Bundespost

1987

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Spectral classification

- 1890: **Harvard classification scheme**
220,000 stellar spectra
classified by strength of Hydrogen lines (alphabetical)
- 1901: Annie Jump Cannon rearranged the scheme into the standard **O B A F G K M** sequence
- Only later it became evident that this scheme is a **surface temperature sequence**



Spectral classification

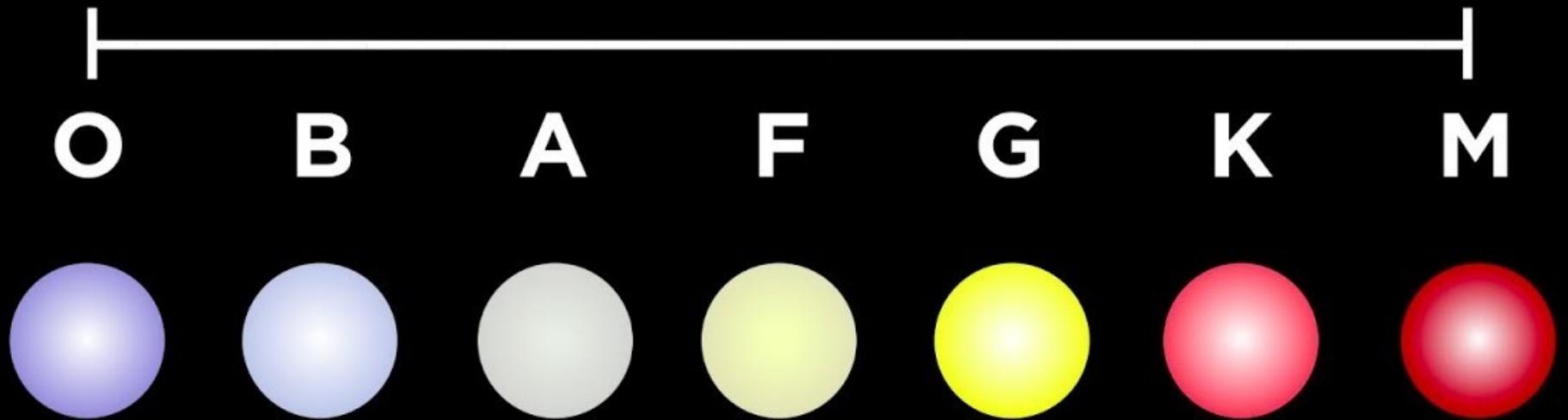
Key Absorption Line Features	Brightest Wavelength (Color)	Typical Spectrum
Lines of ionized helium, weak hydrogen lines	<89 nm (ultraviolet)	O
Lines of neutral helium, moderate hydrogen lines	89–290 nm (ultraviolet)*	B
Very strong hydrogen lines	290–390 nm (violet)*	A
Moderate hydrogen lines, moderate lines of ionized calcium	390–480 nm (blue)*	F
Weak hydrogen lines, strong lines of ionized calcium	480–560 nm (yellow)	G
Lines of neutral and singly ionized metals, some molecules	560–780 nm (red)	K
Strong molecular lines	>780 nm (infrared)	M

*All stars above 6000 K look more or less white to the human eye because they emit plenty of radiation at all visible wavelengths.

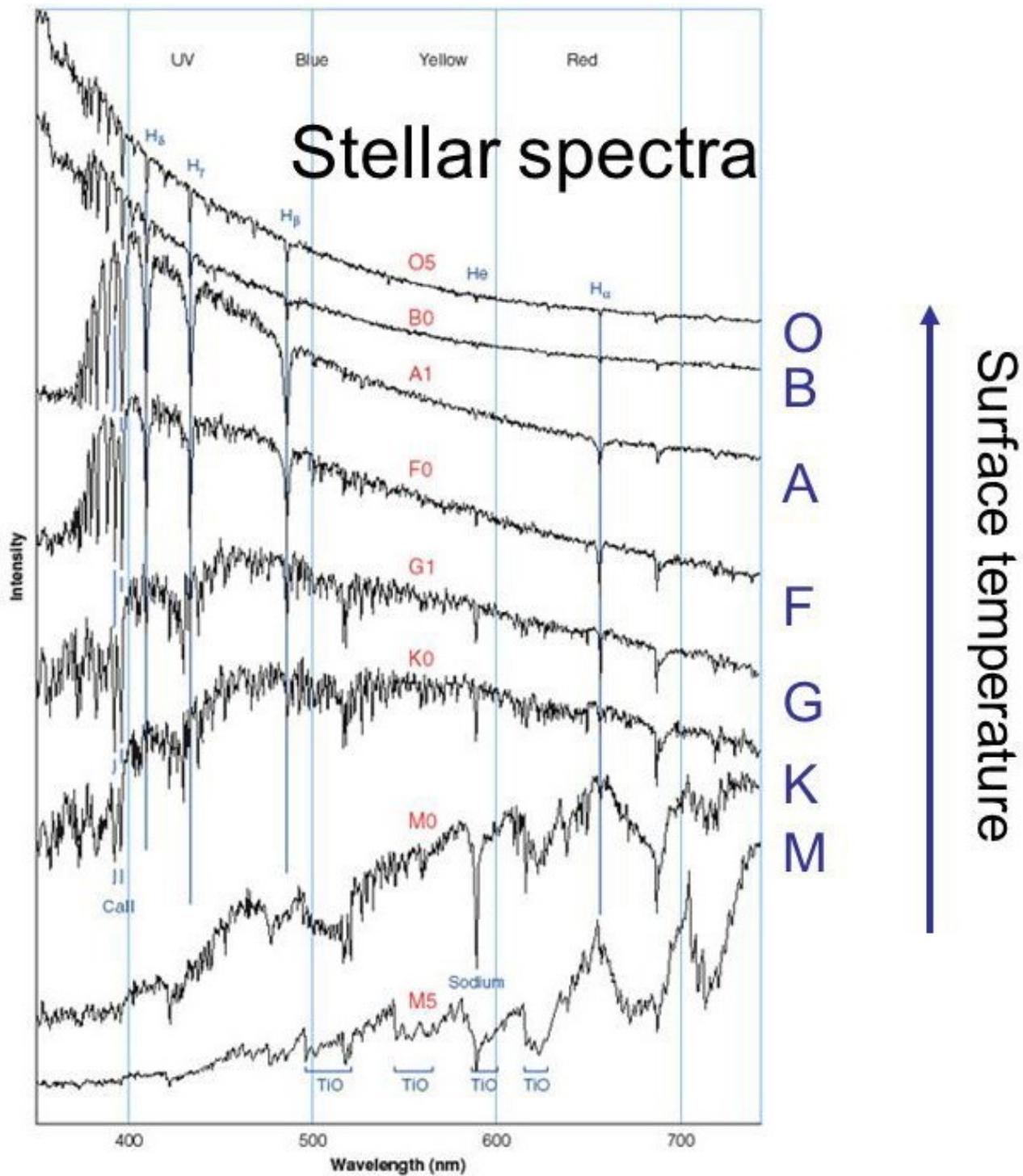
surface temperature

25,000 K

3,500 K

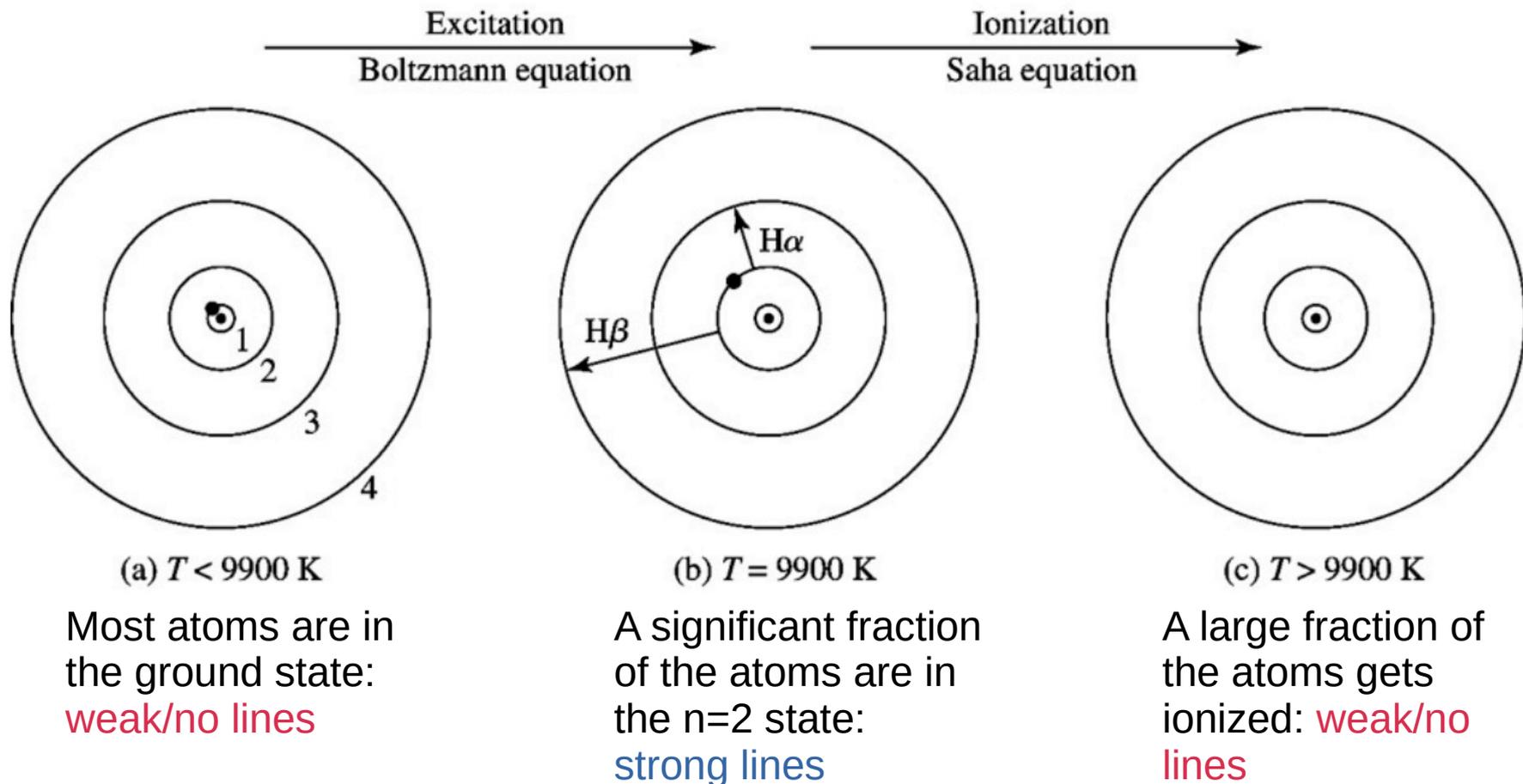


this correlates with the **color** of the star



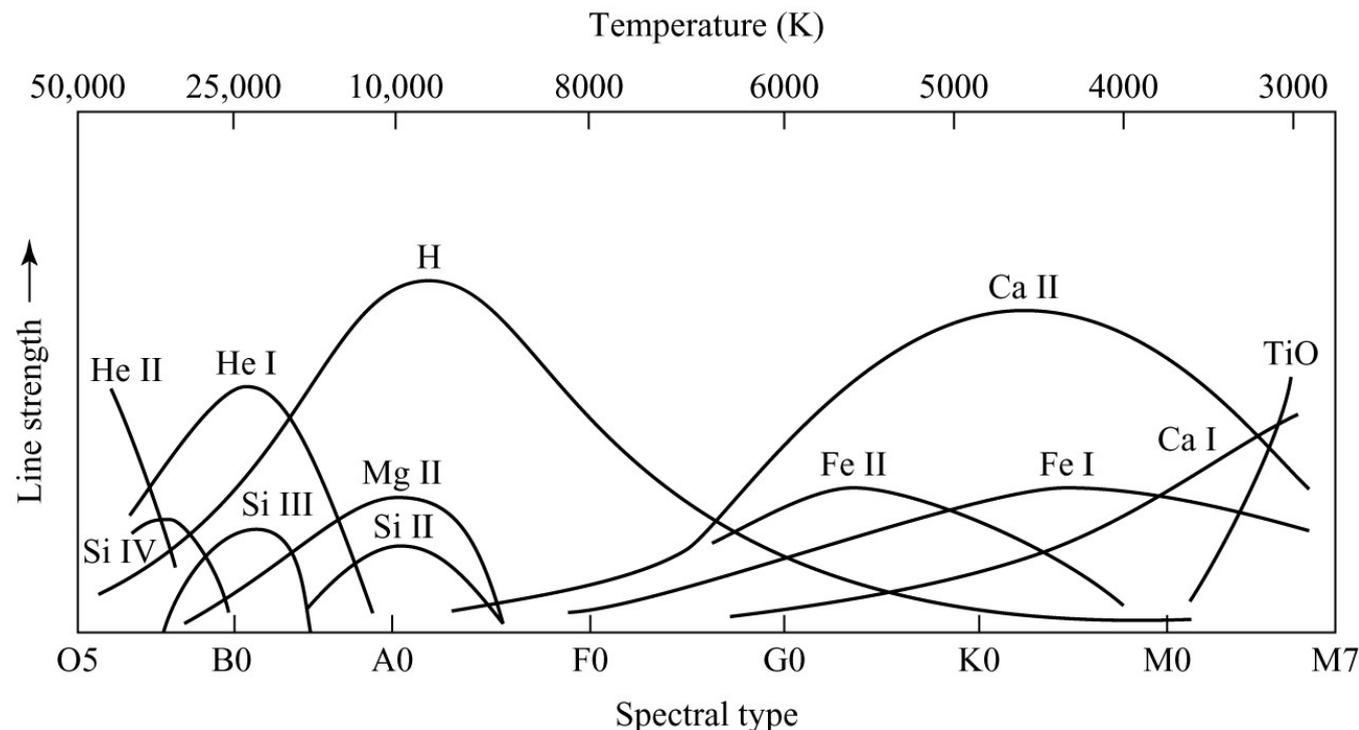
Temperature dependence – hydrogen atom

Balmer lines require a substantial fraction of the atoms to be in the first excited state ($n=2$)



Spectral classification

- The strength of different lines in stellar spectra is not due to elemental abundances, but reflects the **sensitive temperature dependence of the atomic states of excitation and ionization**
- **Temperature is the key to understanding the spectral classification sequence!**
 - The Sun has stronger line of Ca II than any of the H-lines, despite the fact that there is only 1 Ca atom per half a million H-atoms!



Spectral classification

- 1925: Cecilia Payne Gaposchkin, PhD thesis
 - (1) Stars are mainly composed of hydrogen
 - (2) Temperature, and not elemental abundances, behind the classification



Short intro

Intrinsic property

Observed property

Luminosity



Apparent brightness

$$\text{apparent brightness} = \frac{\text{luminosity}}{4 \pi \text{distance}^2}$$

Absolute magnitude (M)



Apparent magnitude (m)

$$m - M = 5 \cdot \log_{10} \left(\frac{d}{10 \text{ pc}} \right)$$

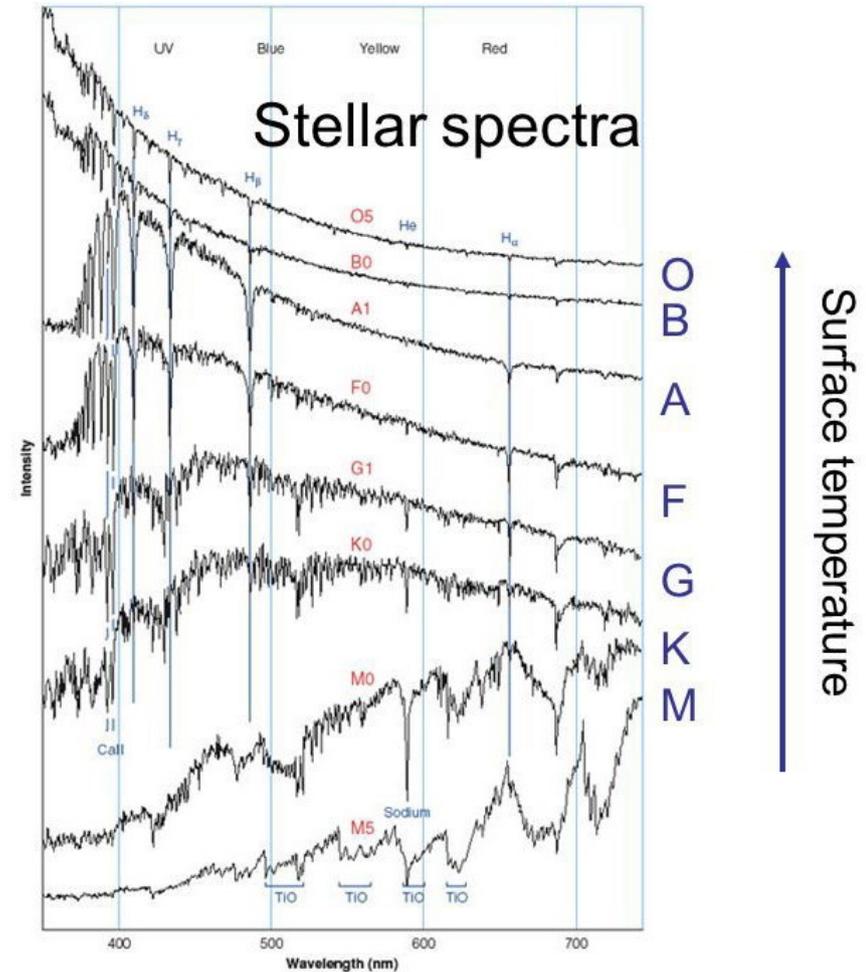
Short intro

Yesterday we learned that stars can be characterized by their

- **Luminosity** (total power emitted in a unit of time)
- **Surface (or effective) temperature**
 - Physical explanation behind the stellar classification system

Luminosity can be expressed a function of T_{eff} and the star's radius

$$L = 4\pi R^2 \sigma T_{eff}^4$$



Modern spectral classification

M-K (Morgan-Keenan) system

- each class divided into **subclasses**, e.g. B0...B9
- New spectral classes **L, T, Y** added in 1990s to 2010 (brown dwarfs)
- Luminosity classes

Ia = Bright Supergiants

Ib = Supergiants

II = Bright Giants

III = Giants

IV = Subgiants

V = Dwarfs

Examples

Sun: G2V

Betelgeuse: M2Ib

Proxima Cen: M5.5V

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Hertzsprung-Russell (H-R) diagram

- Ejnar **Hertzsprung** (1873–1967)

Henry Norris **Russell** (1877 - 1957)

analyzed stars whose absolute magnitudes and spectral types had been accurately determined

Two important observations:

- **SpT < G** → abs. magnitude and SpT correlated
 - **SpT ≥ G** → same SpT, large span of magnitudes
- Hertzsprung called the brighter stars GIANTS

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$$R = \frac{1}{T^2} \sqrt{\frac{L}{4\pi\sigma}}$$

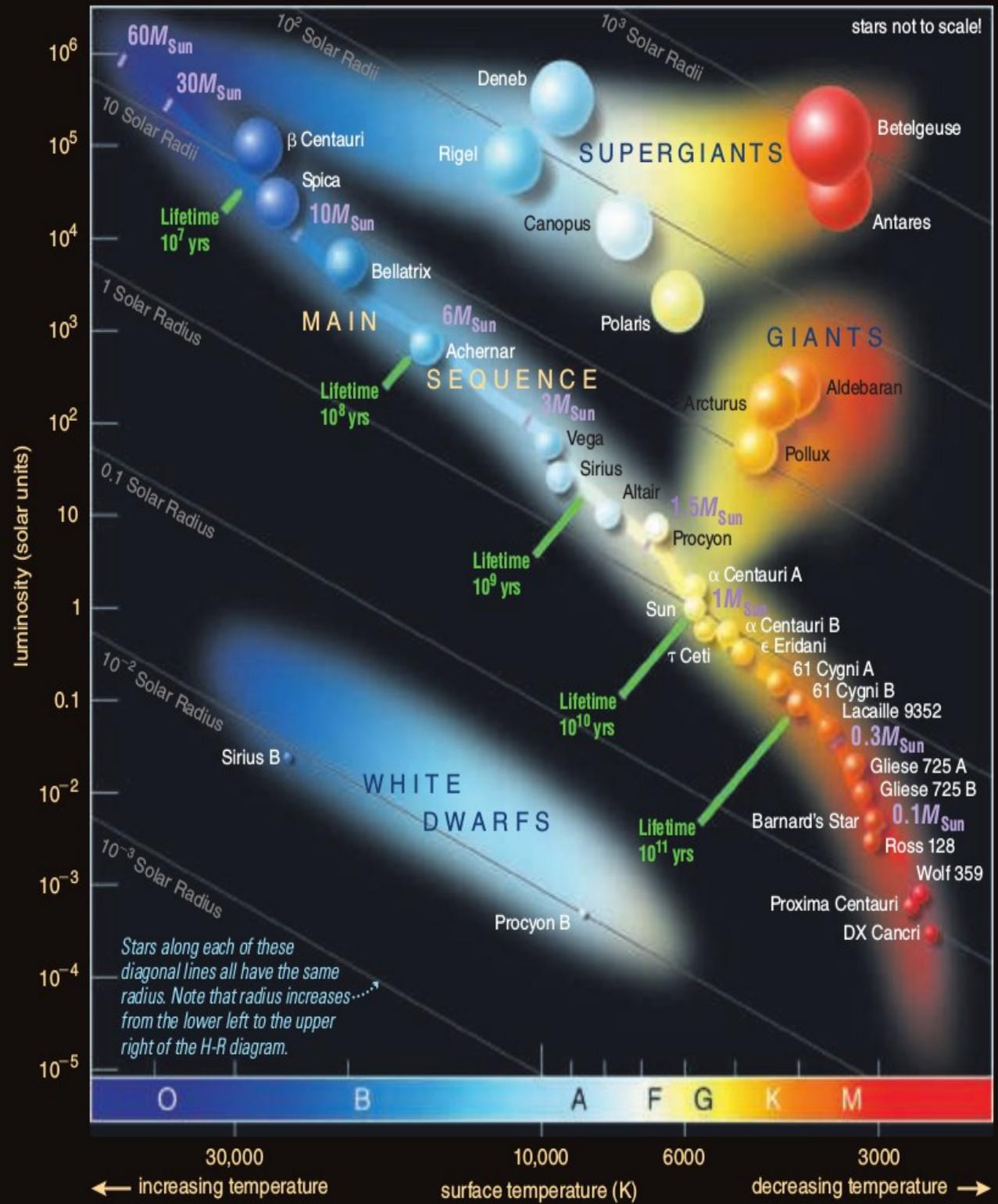
- From Stefan-Boltzmann law:

- Two stars with same T_{eff} and different L , must have a different size

H-R diagram

- Observational diagram
- Relates **surface temperature** of a star to its **luminosity** or **absolute magnitude**

Important: placing a star on the y-axis requires the knowledge of the distance to it (far from trivial!)



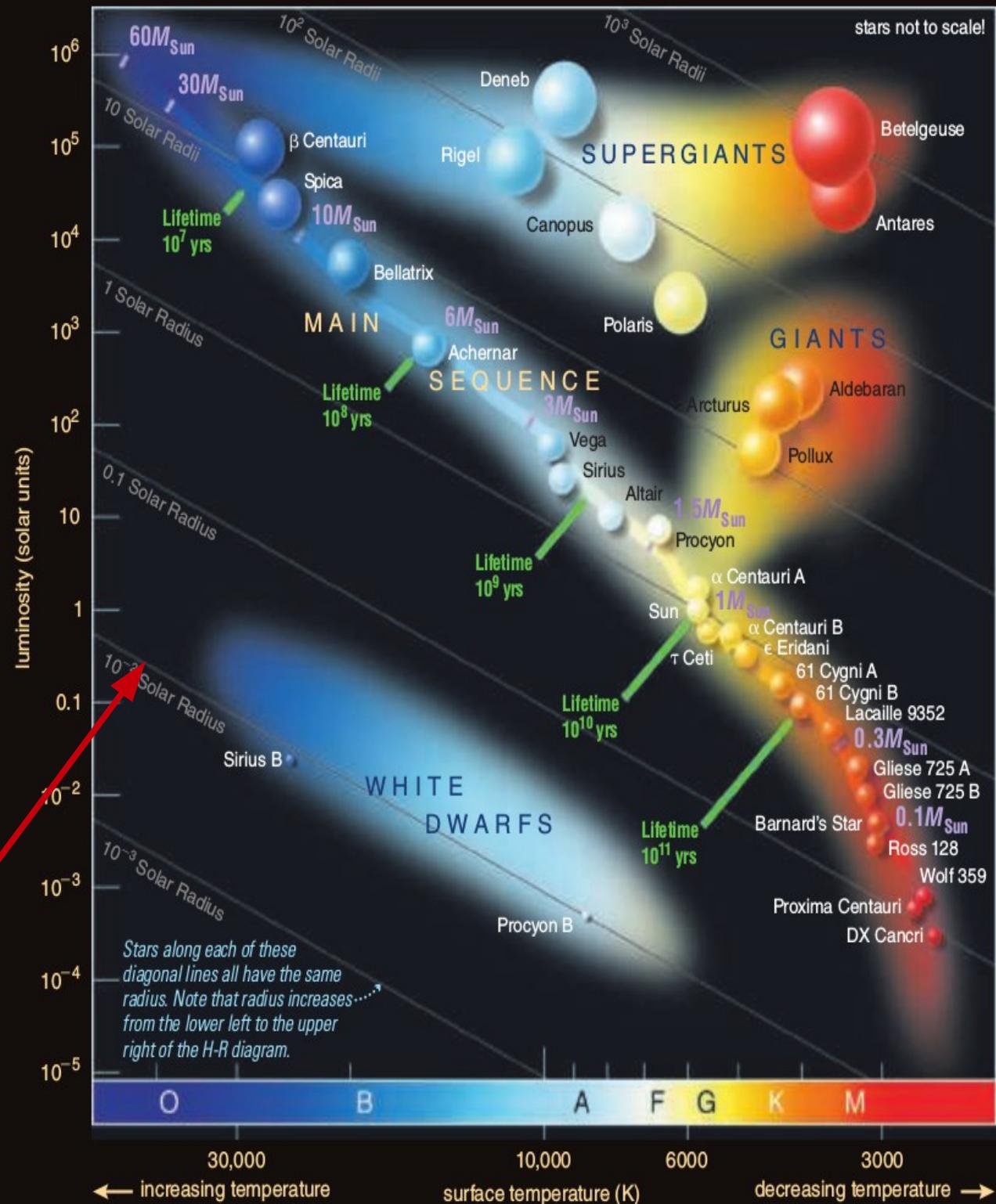
H-R diagram

Stellar radii:

$$10^{-2} - 10^3 R_{\odot}$$

$$R_{\odot} = 6.95508 \times 10^8 \text{ m}$$

Lines of constant radius



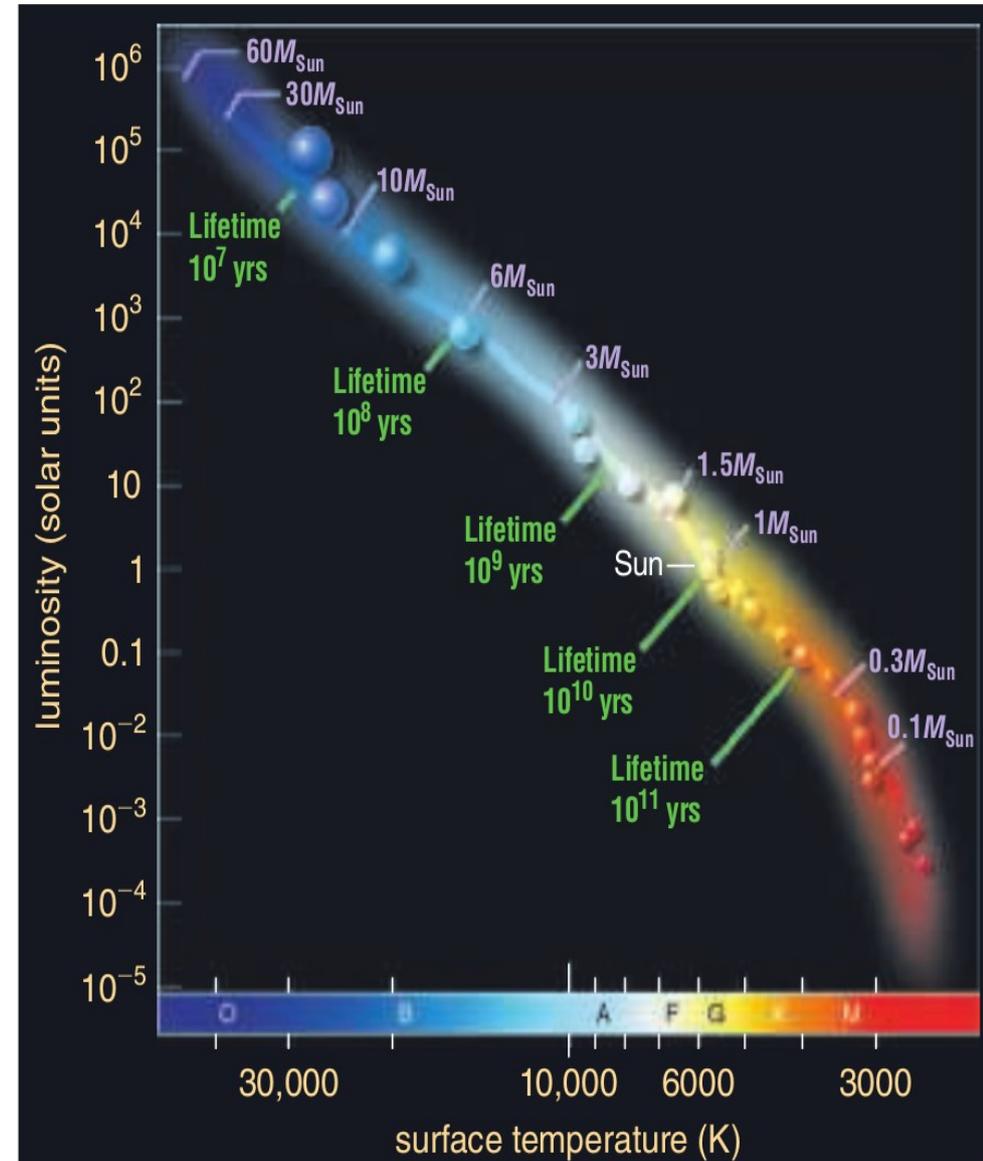
Main sequence (MS)

- Most stars located on the MS (>80%)
- MS stars fuse H into He
- Star's position along the MS related to its mass

Stellar masses:

0.08 to $\sim 100 M_{\odot}$

$M_{\odot} = 1.989 \times 10^{30}$ kg



Main sequence (MS)

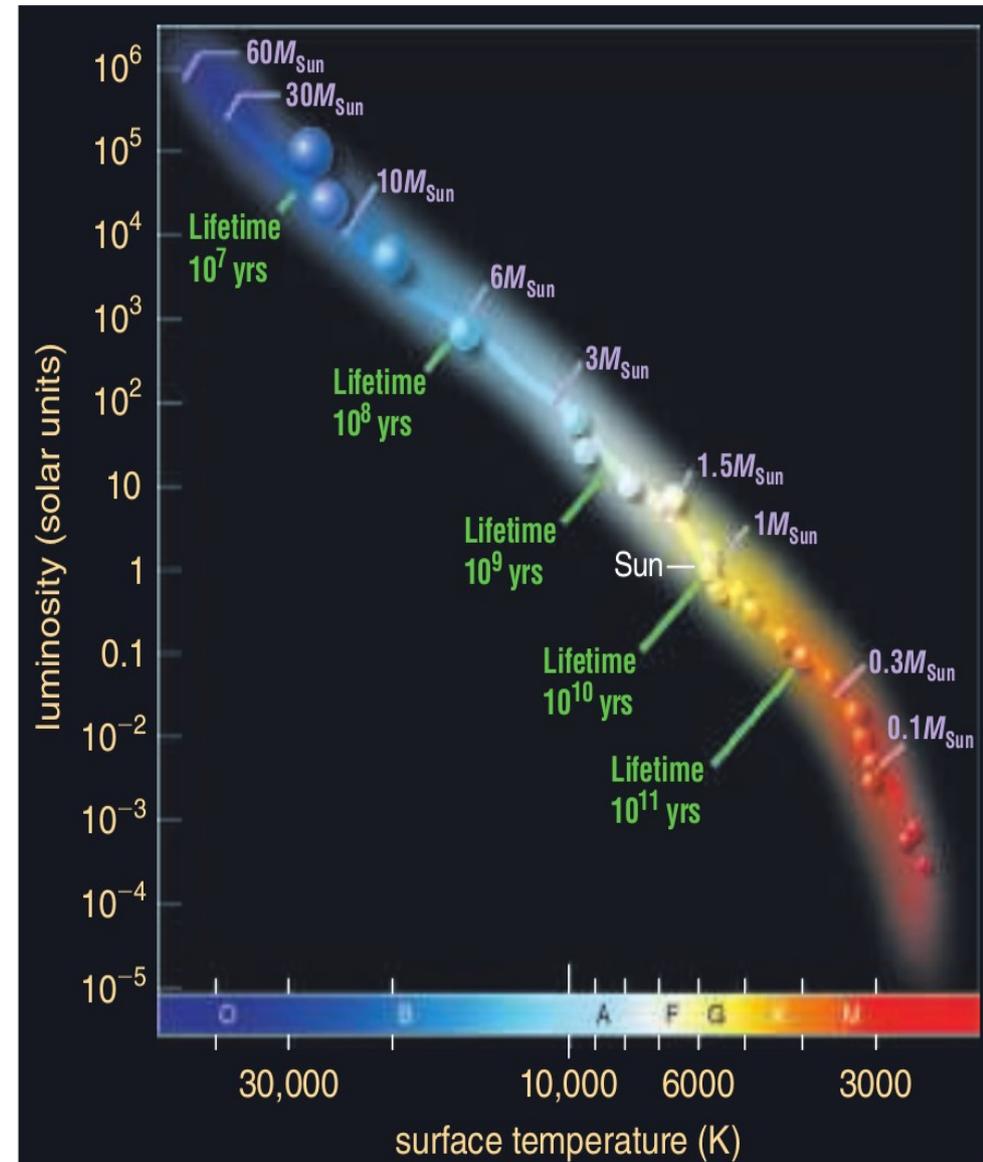
- Stars spend most of their lifetime on the MS
- Lower mass stars last longer

MS lifetime depends on

(a) **mass** - how much H is available in the core

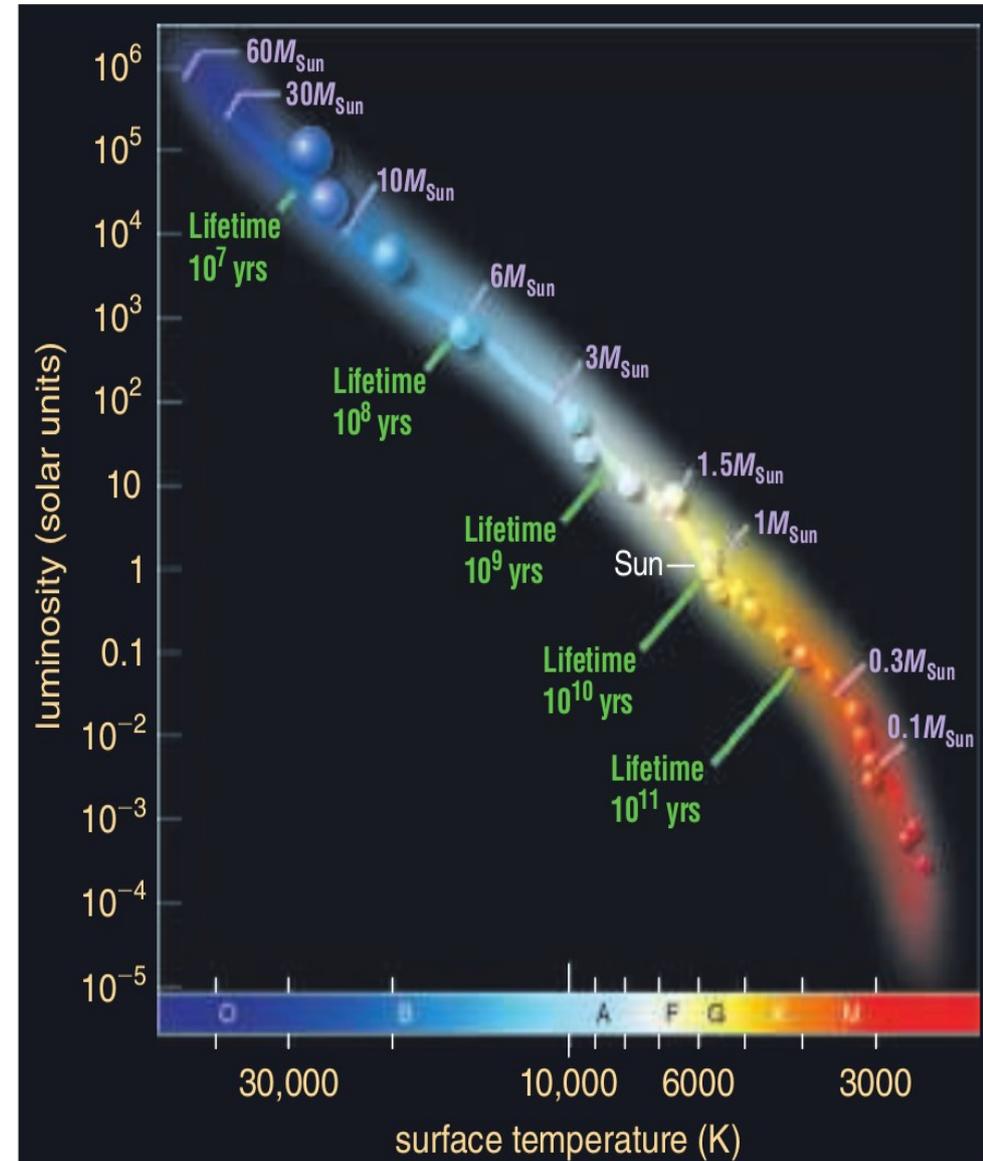
(b) **luminosity** - how fast the stars uses up the fuel ($L \sim M^{3.5}$)

$$\text{lifetime} \propto M^{-2.5}$$



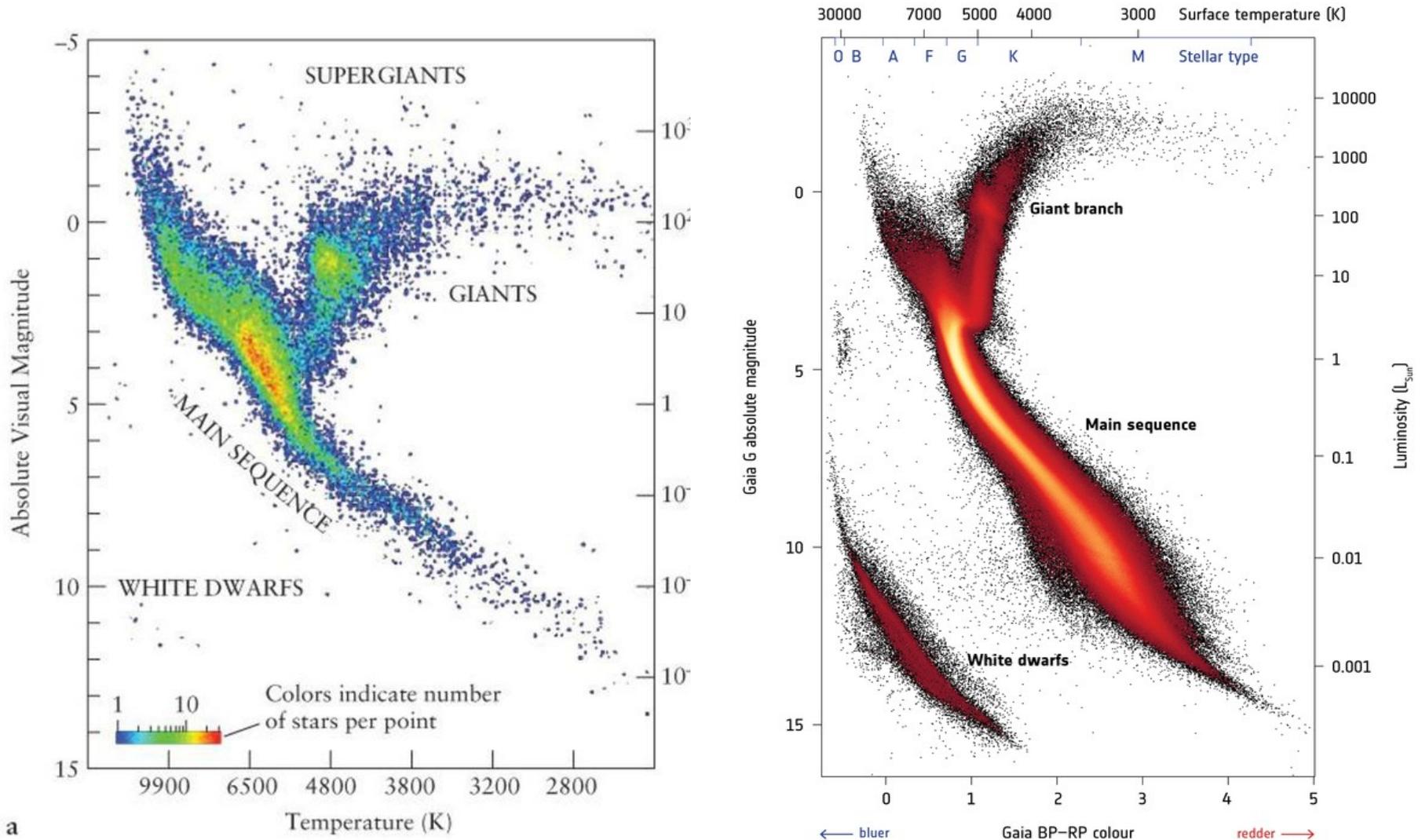
Main sequence (MS)

- A very **massive star** has lots of fuel, but it burns it fast
~ a few Myr
- A **low-mass stars** $0.3M_{\odot}$
~ 300 Gyrs
longer than the age of the Universe!



H-R diagram is an observational diagram

HRD from Hipparcos (~40 000 stars) and from Gaia Satellites (4 million stars)



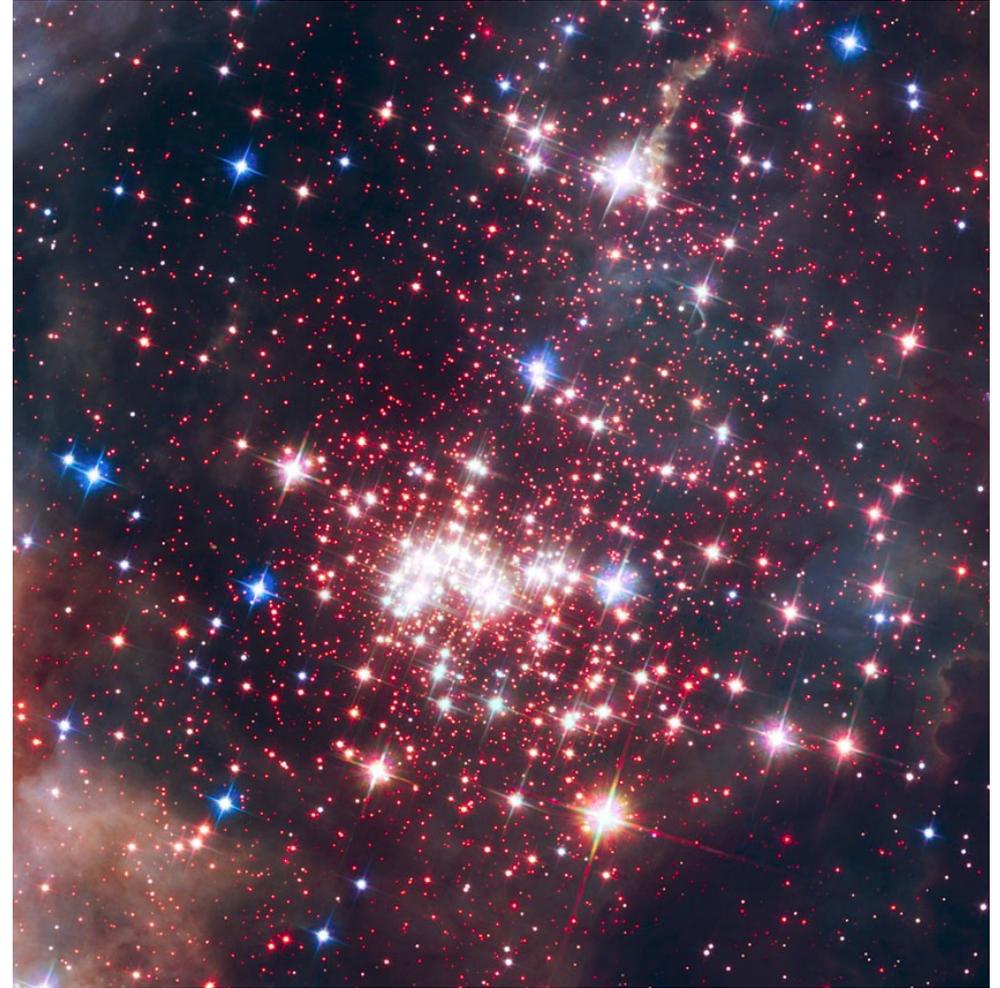
a

Color can be used as a proxy for temperature!

Star Clusters



Globular clusters
(old: 10-12 Gyrs)



Open clusters (young:
few Myrs - few Gyrs)

Star Clusters

Stars in a cluster are all at the same distance

We can use the apparent magnitude to construct the H-R diagram

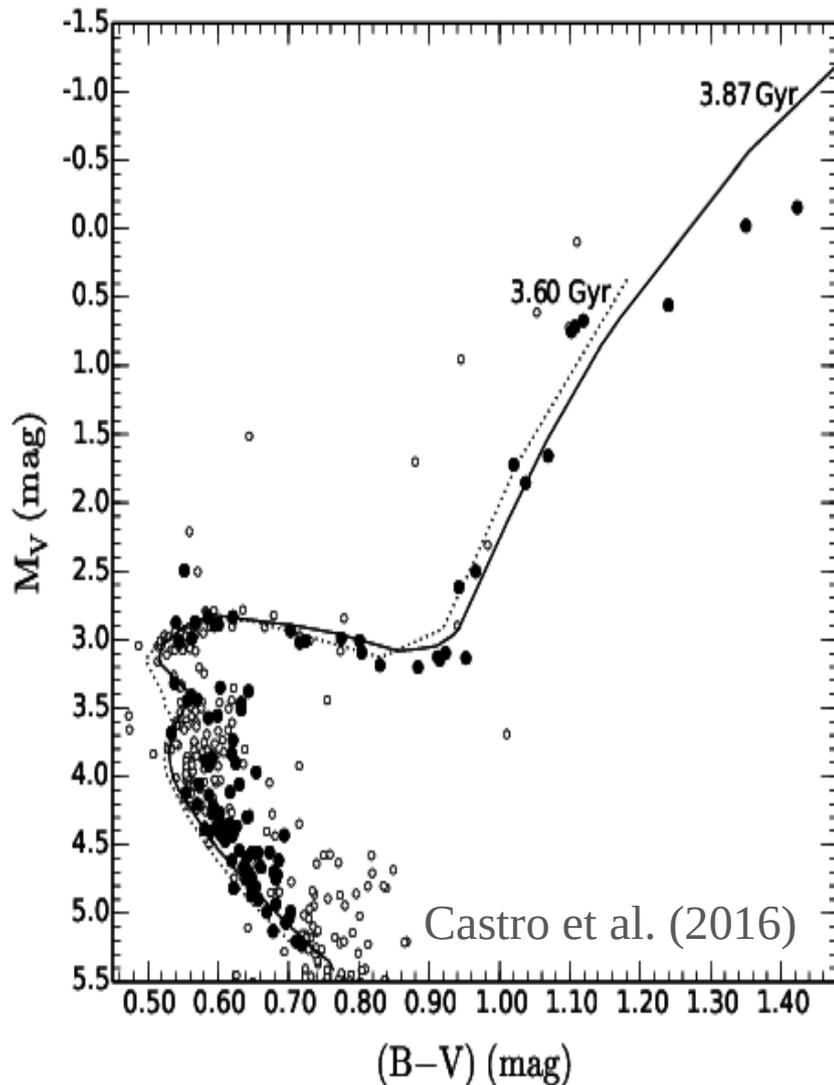
Stars in a cluster have roughly the same age

H-R diagram can be used to trace stellar evolution with time

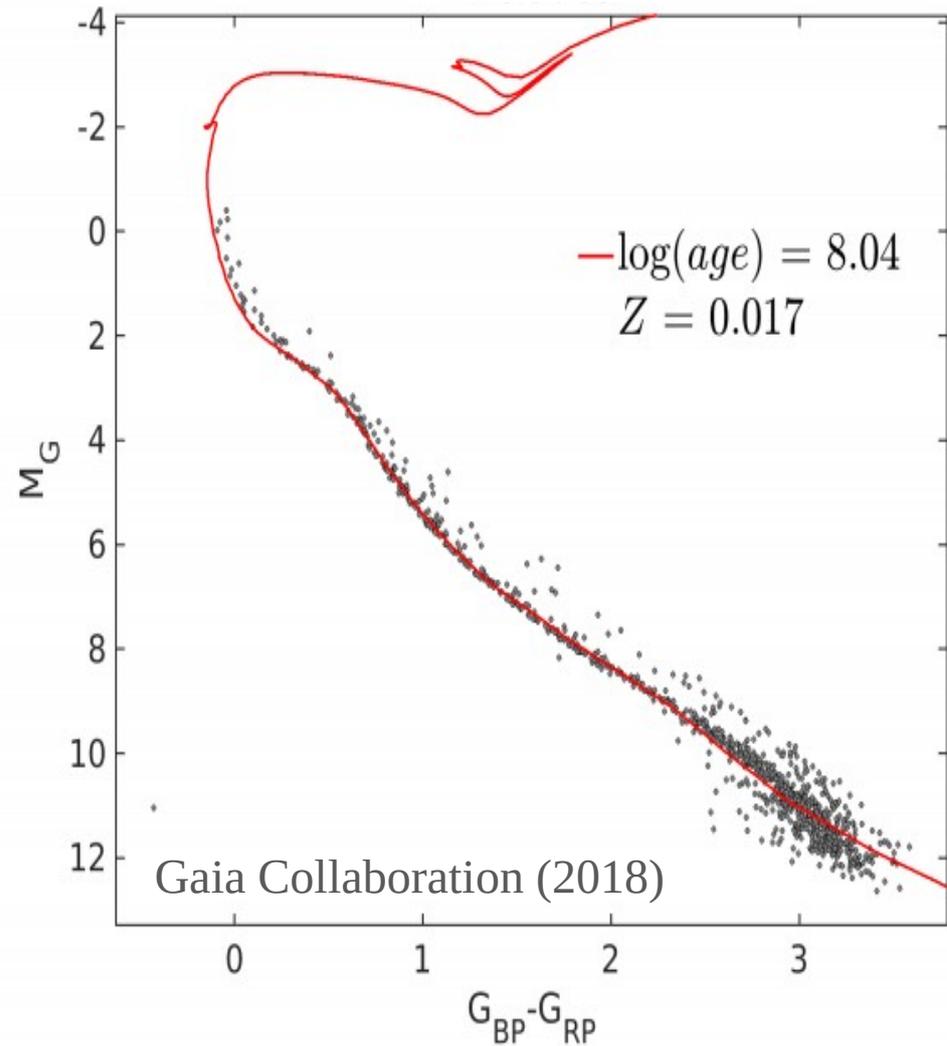


Color-magnitude diagrams of open clusters

M67 (very old open cluster)

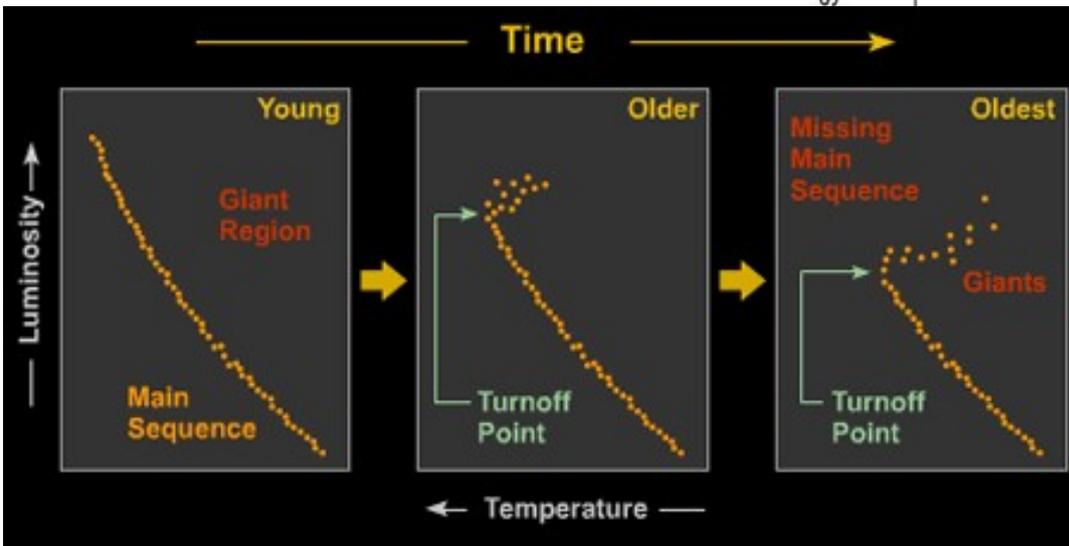
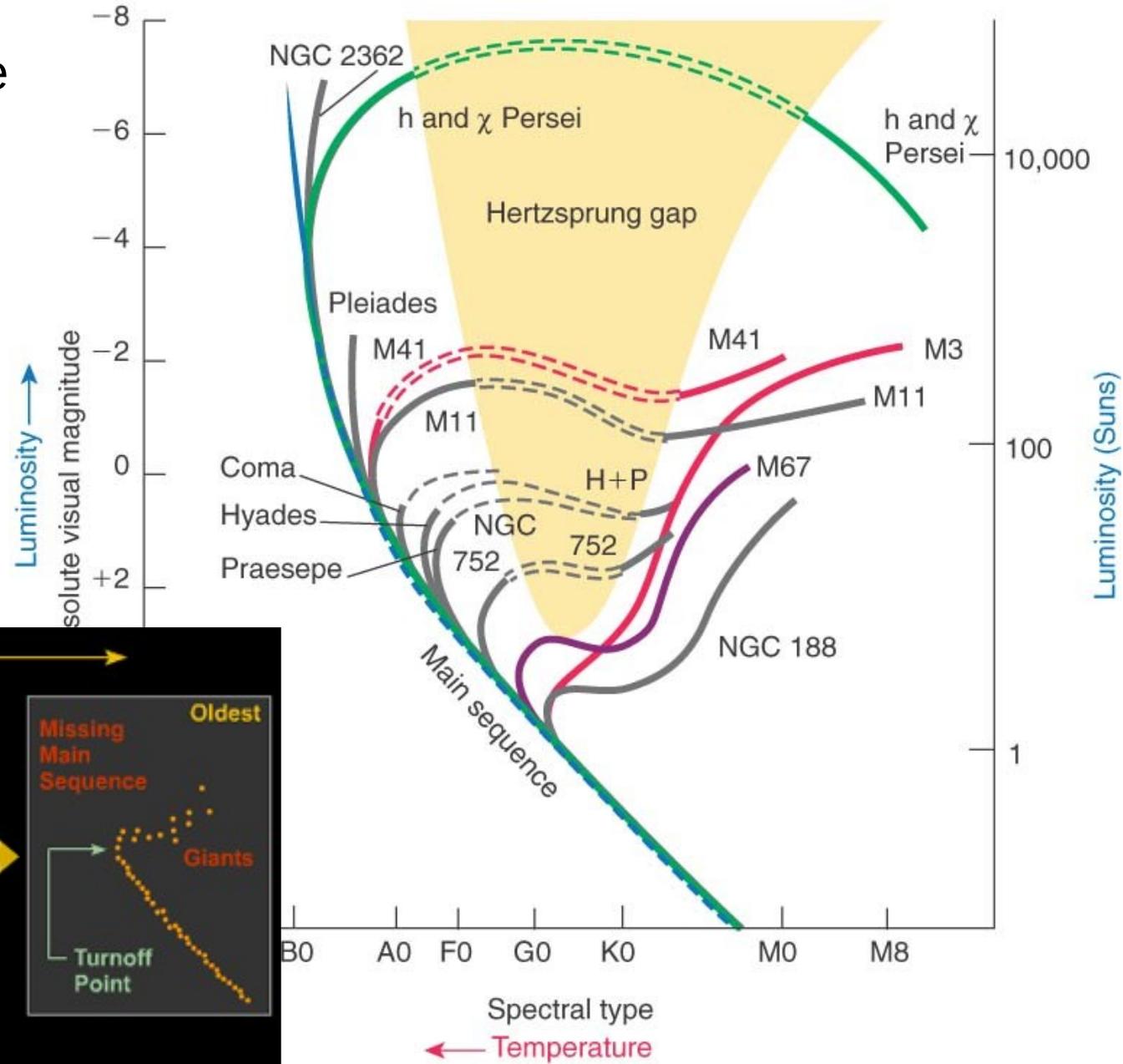


Pleiades (much younger)

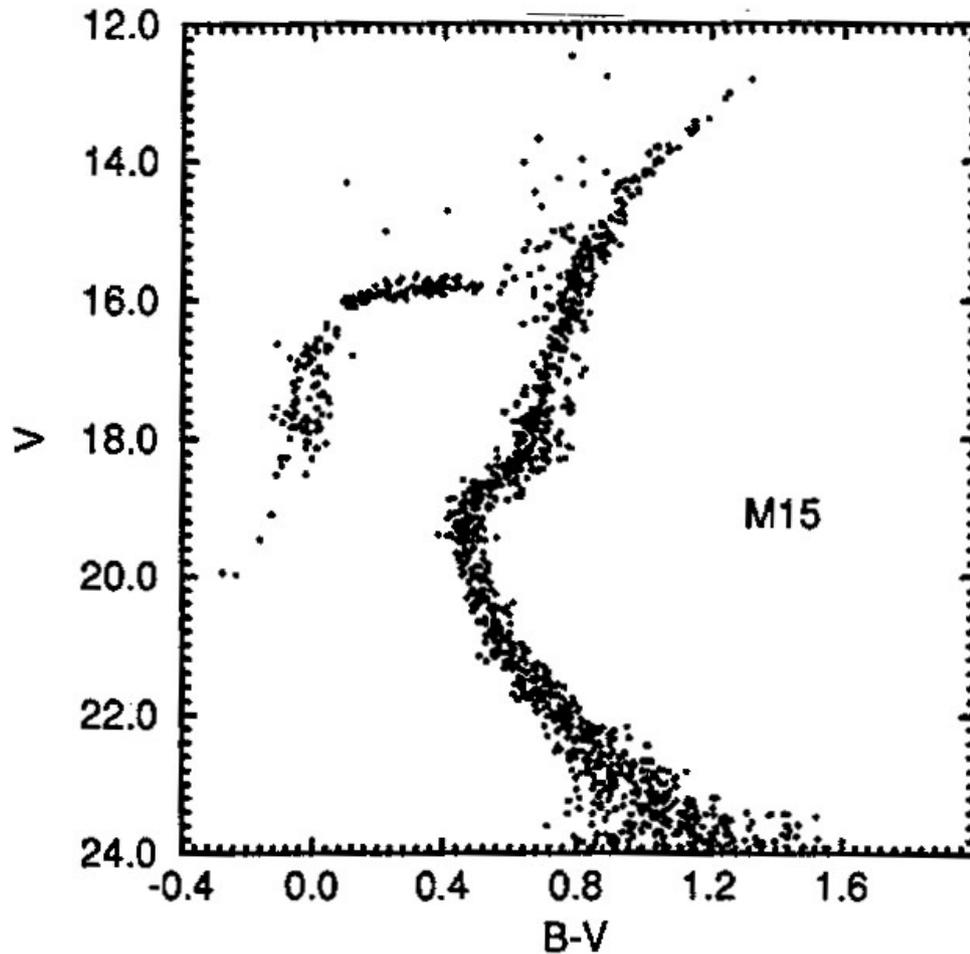


H-R diagrams of star clusters

- Turn-off point indicate **age**



H-R diagrams of star clusters



Globular cluster

Older than open clusters
→ later evolutionary stages visible

HR-diagram of a cluster is like a snapshot in time – all stars have the same age, but are at a different evolutionary stage because of their different masses



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- Basic properties of stars
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- HR-diagram and the stellar main sequence
- **Energy production**
- Stellar evolution



Energy production

We learned from the spectral analysis that stars predominantly contain hydrogen

$$X \equiv \frac{\text{total mass of hydrogen}}{\text{total mass of gas}}$$

$$Y \equiv \frac{\text{total mass of helium}}{\text{total mass of gas}}$$

$$Z \equiv \frac{\text{total mass of metals}}{\text{total mass of gas}}.$$

Solar surface
composition (present):

$$X = 0.7381$$

$$Y = 0.2485$$

$$Z = 0.0134$$

Energy production

- Stars' luminosity is sustained by the **nuclear fusion** process in its center
- Hydrogen is converted to Helium via two main processes:
 - Proton-proton chain (pp-chain)
 - CNO-cycle
- Both processes take **4 H atoms**, and return **1 He atom + energy**

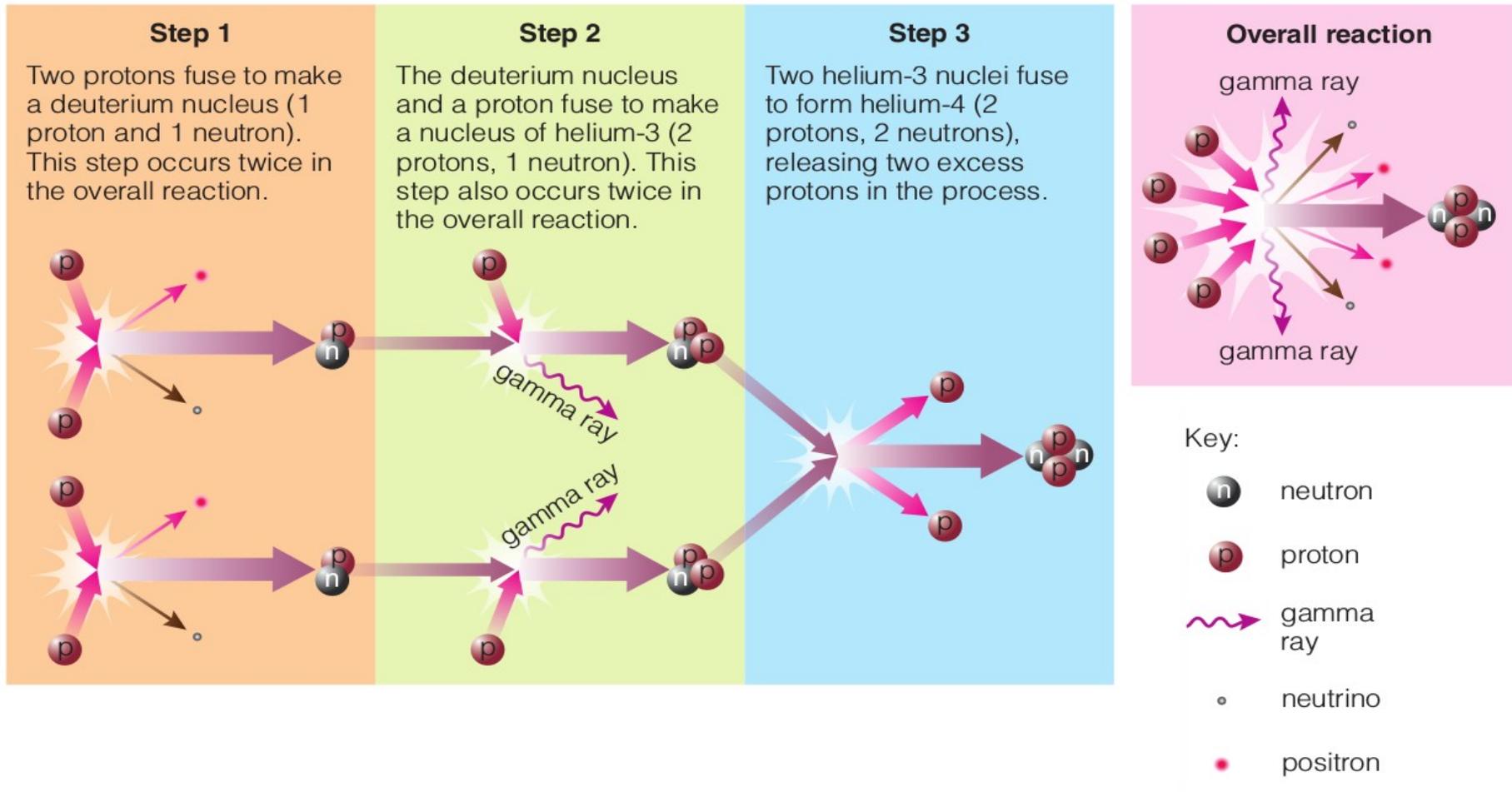
$$\begin{array}{l} m(4H) = 4.03130013u \\ m(He) = 4.002603u \end{array} \left. \vphantom{\begin{array}{l} m(4H) \\ m(He) \end{array}} \right\} \begin{array}{l} \Delta m = 0.028697 \text{ (0.7\%)} \\ E_b = \Delta mc^2 = 26.731 \text{ MeV} \end{array}$$

binding energy of the He nucleus

$$u = 1.66053873 \times 10^{-27} \text{ kg}$$

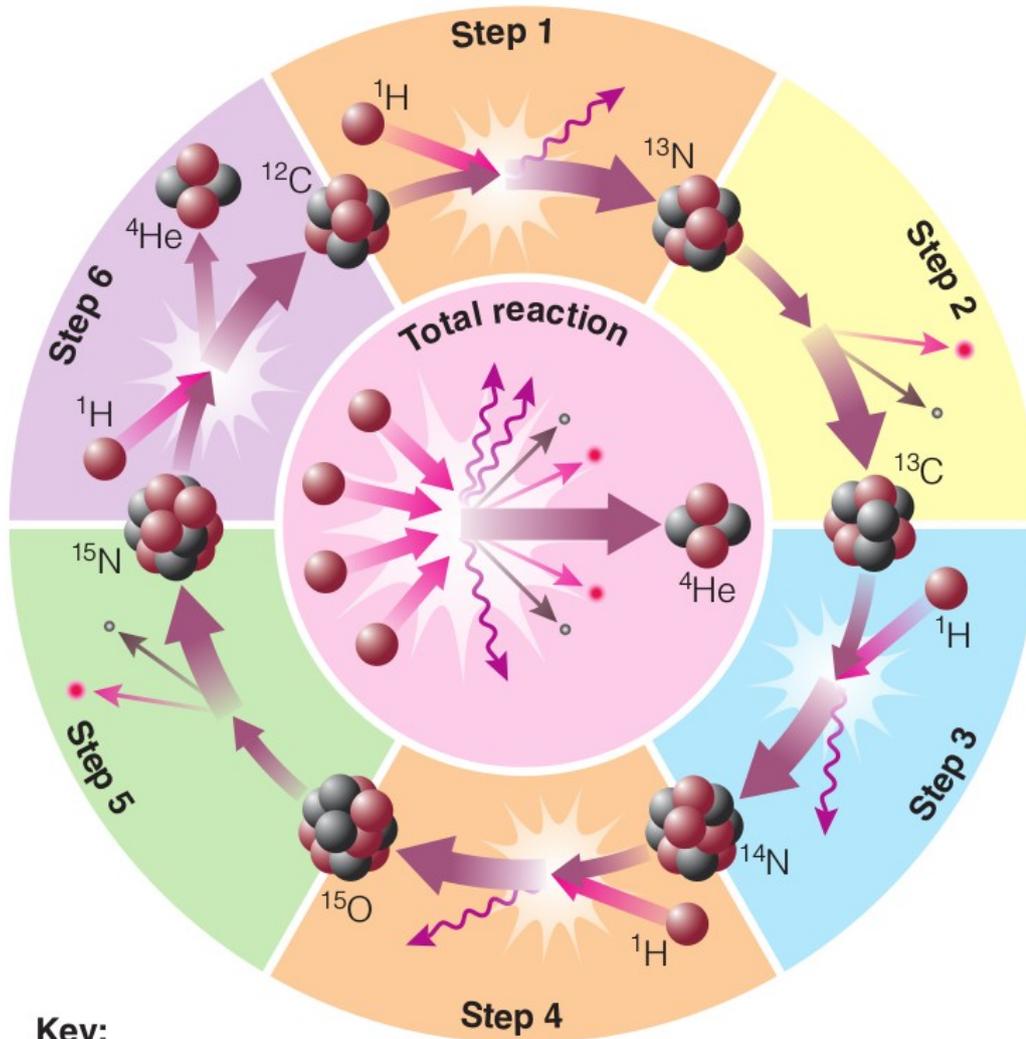
atomic mass unit

Proton-Proton Chain (PP1)



- Requires $T_{\text{core}} > 10^7 \text{ K}$
- Dominant process in Sun and lower mass stars
- Energy generation rate $\epsilon_{\text{PP}} \propto T^4$

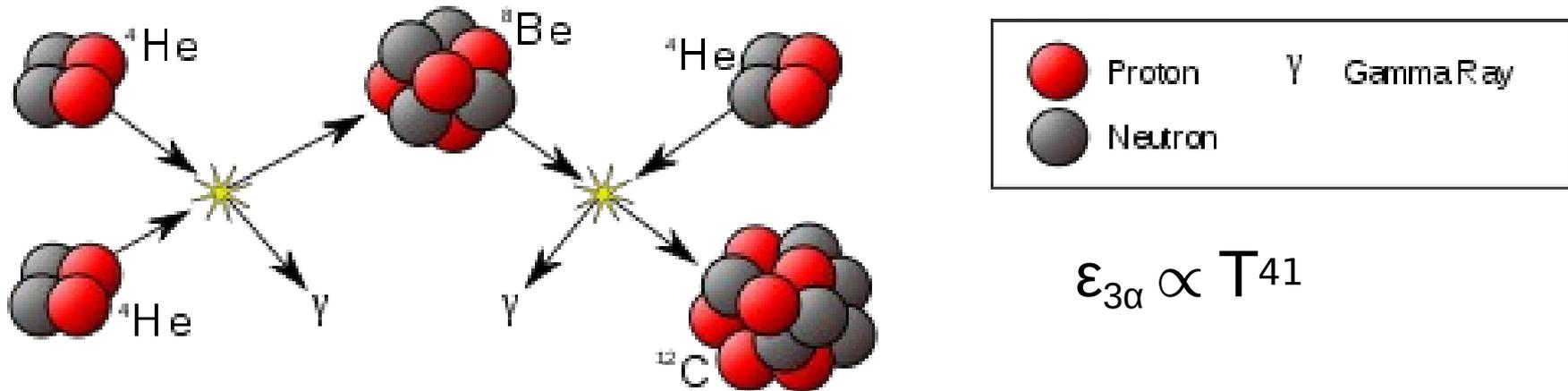
CNO Cycle



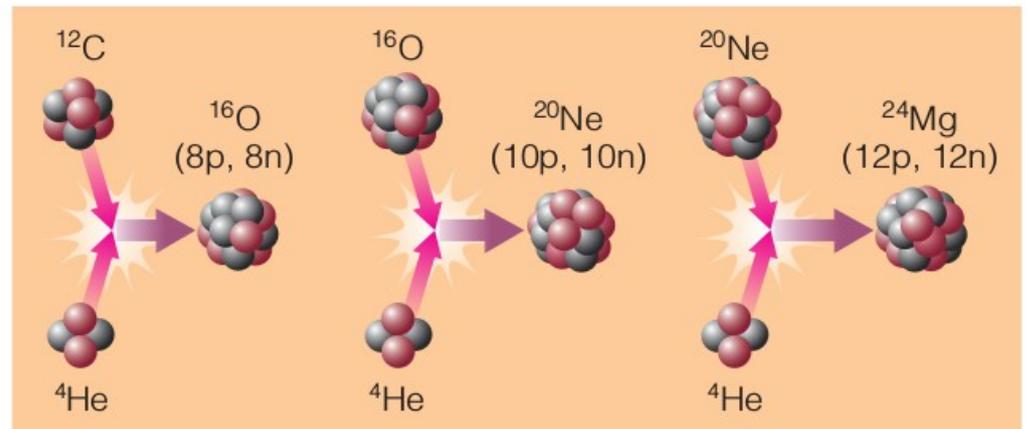
- C, N, O serve as catalysts in the reaction, globally they're not destroyed or produced
- Net energy output same as in PP chain, but much faster
- Require higher core temperature to initiate ($\sim 15\text{MK}$)
- Dominant in more massive stars (transition at $1.5 M_{\odot}$)
- CNO cycle is much faster than the pp-chain: massive stars live shorter
- $\epsilon_{\text{CNO}} \propto T^{20}$

Fusion of heavier nuclei

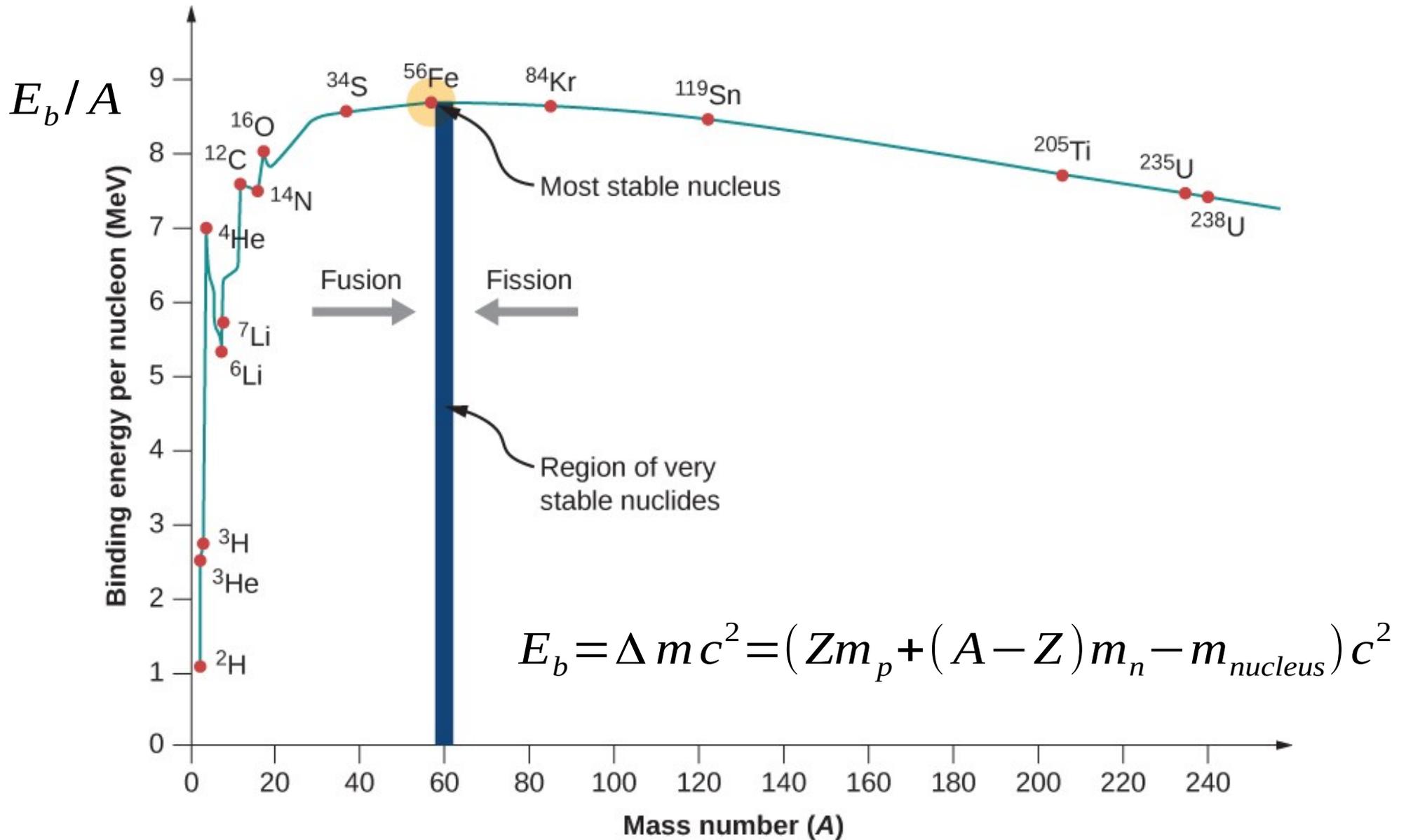
- **Triple alpha process** (fusion of He into C)



- **Capture of He**
(production of heavier nuclei)



Binding energy

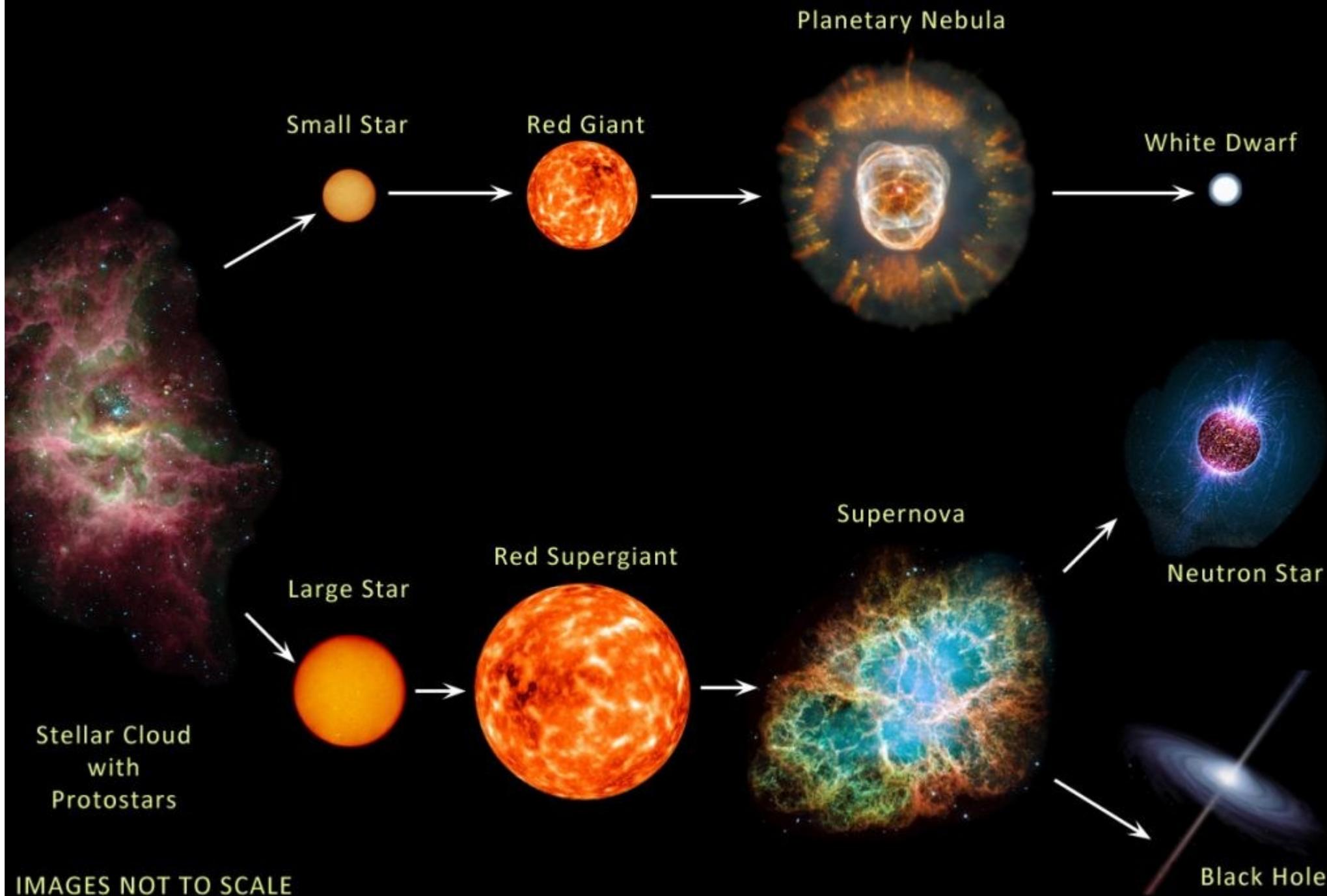


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EVOLUTION OF STARS



IMAGES NOT TO SCALE

Stellar evolution

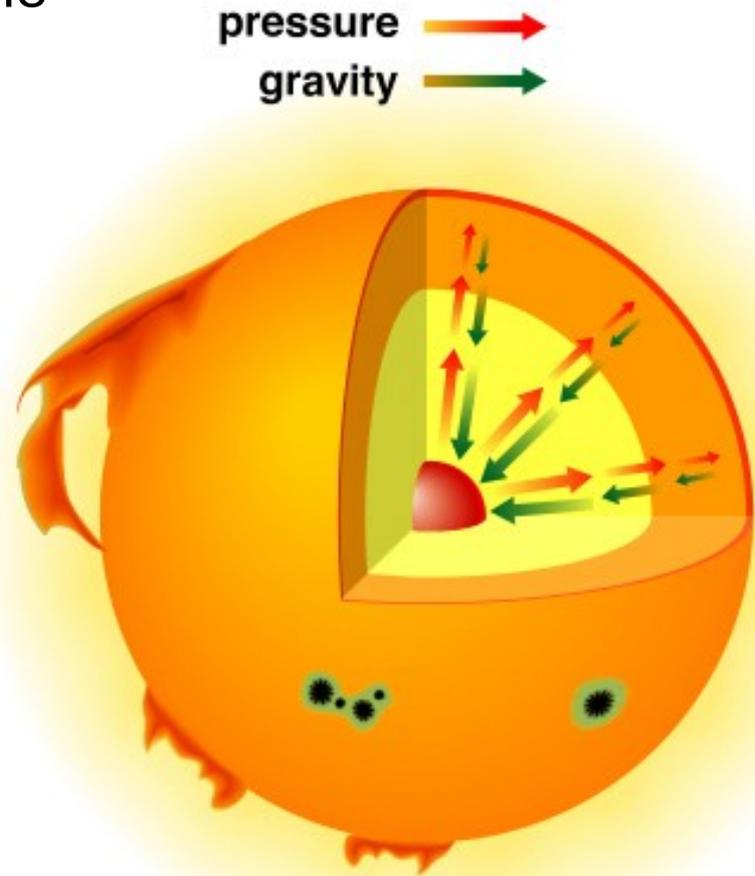
- Important to remember: The **mass** and the **composition** structure throughout a star uniquely determine its radius, luminosity, and internal structure, as well as its subsequent evolution
 - (Known as the Vogt-Russell theorem)
- Why evolution? Because it is a consequence of the change in composition due to nuclear burning

Hydrostatic Equilibrium

- **Gravity** works from outside towards the center of the star
- Gravity must be compensated by an opposing force to prevent collapse → **pressure**

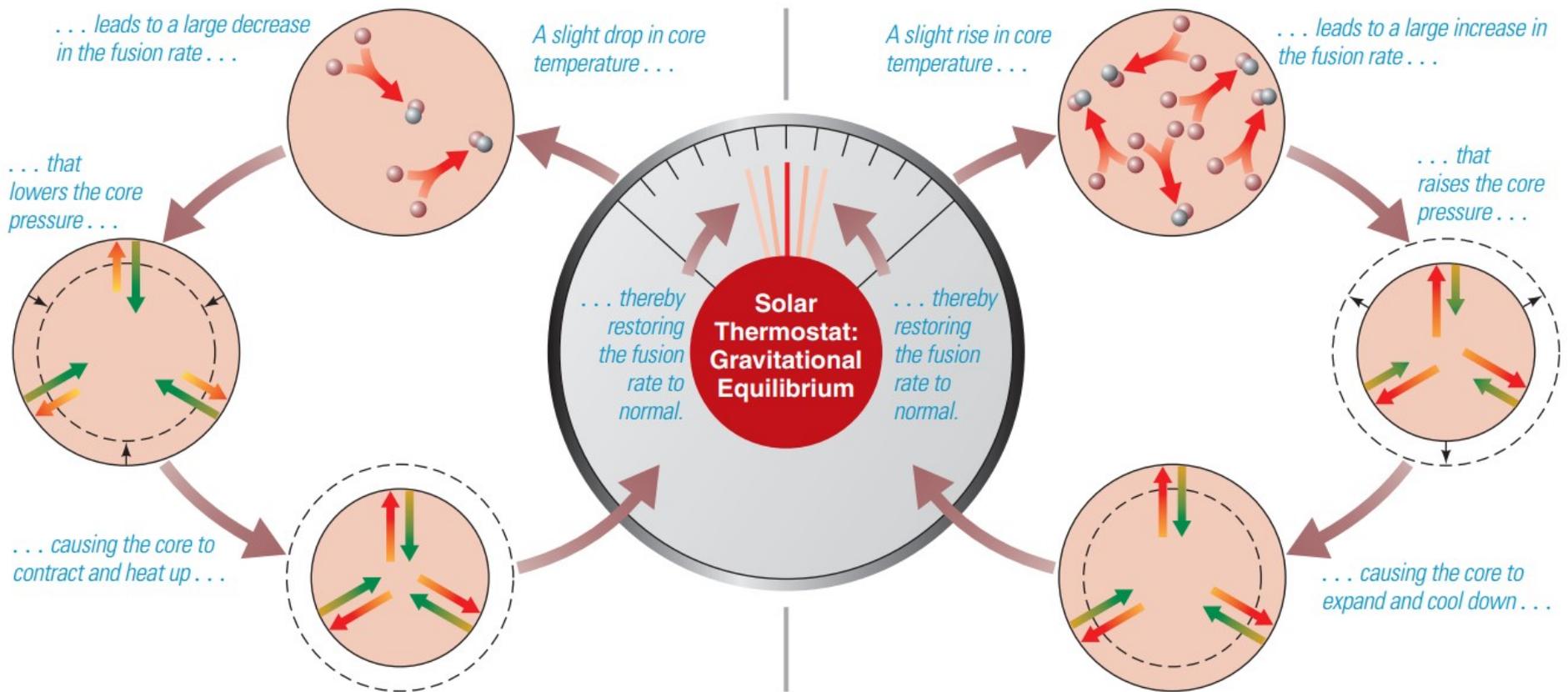
$$\frac{dP}{dr} = -G \frac{M_r \rho}{r^2} = -\rho g$$

$$g \equiv \frac{GM_r}{r^2} \quad (\text{local acceleration of gravity at radius } r)$$



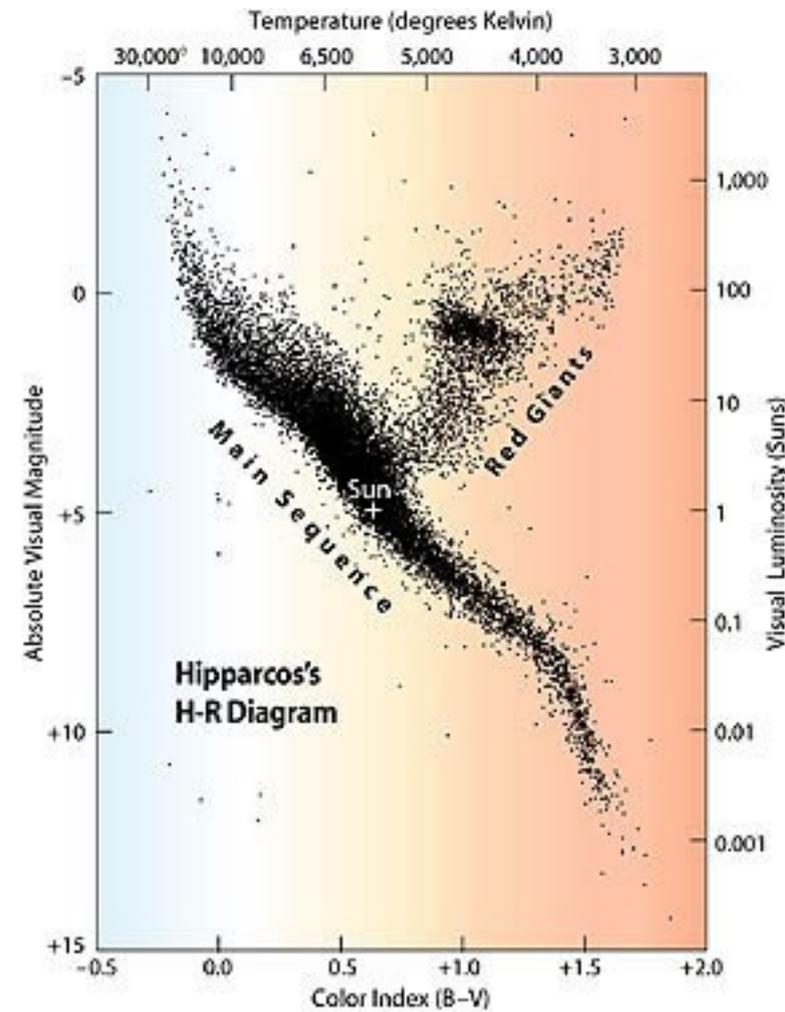
Stellar thermostat

- How does a star respond to small changes in the core temperature?



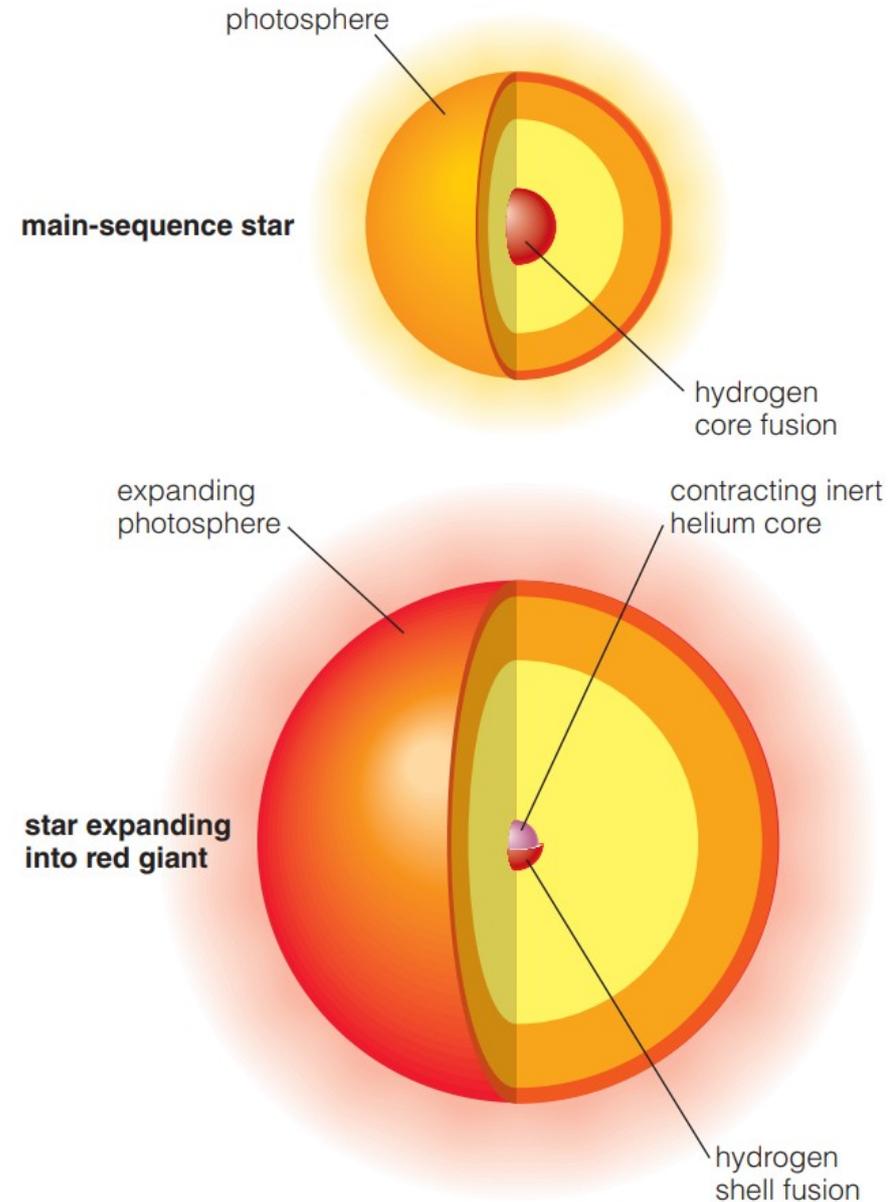
Main sequence evolution

- **Increase of luminosity on the MS**
 - Fusion: 4 H atoms \rightarrow 1 He atom
 - Total number of particles drops down with time
 - Causes the core to shrink
 - Shrinking increases core temperature and fusion rate
- Sun increased its luminosity by 30%
- MS is a band, not line



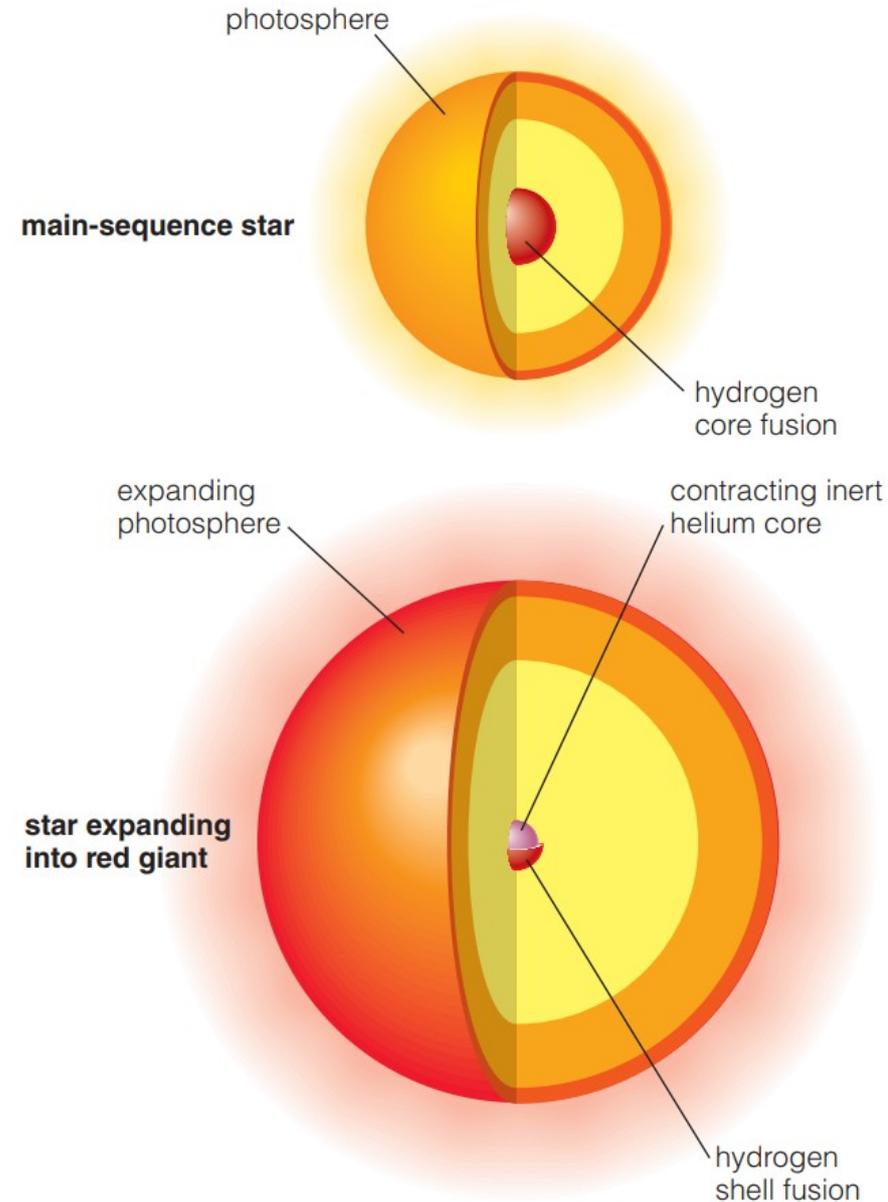
Post-MS evolution

- All H in the core converted into He
→ fusion stops
- Core cannot stand the gravity pull,
and starts to shrink
- Inert He core and hydrogen shell
fusion
- **Broken thermostat** – H-burning
shell can't do anything to inflate the
He core, only keeps dumping more
He onto it

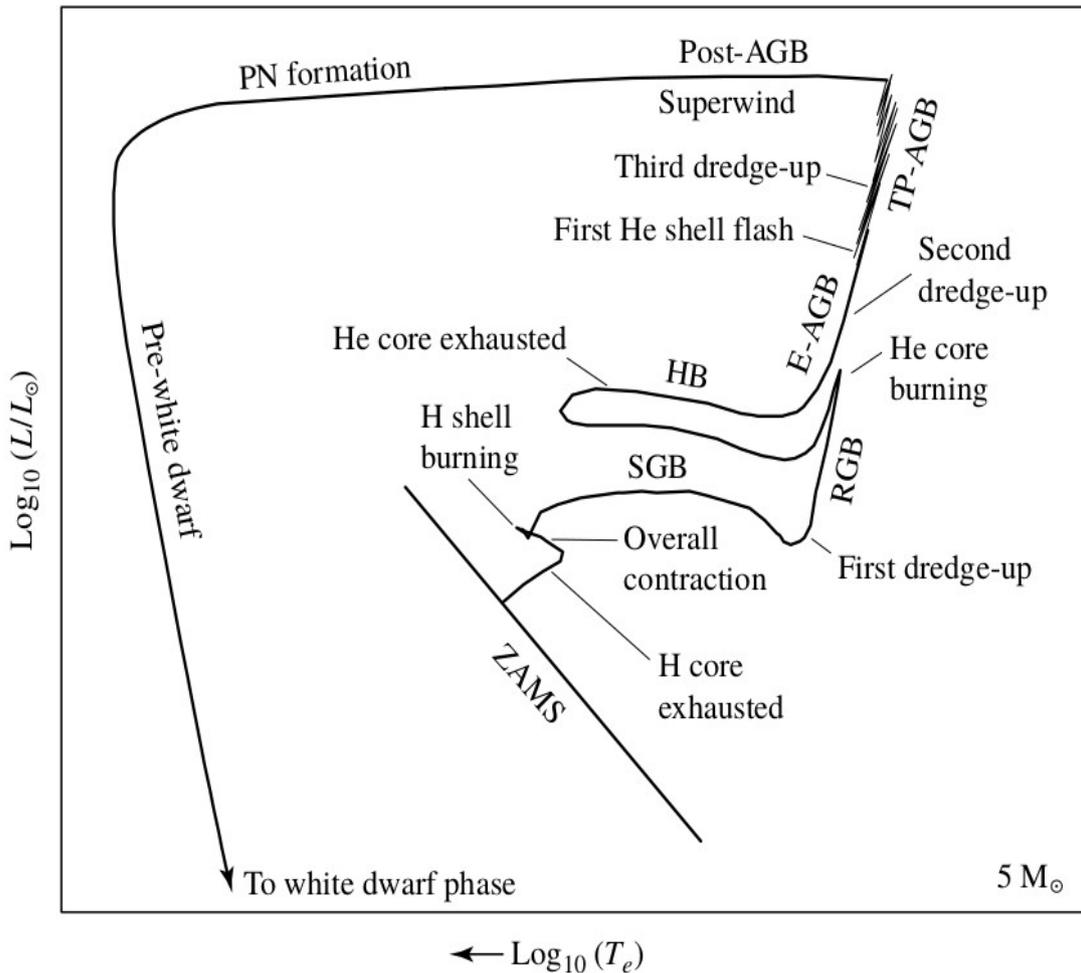
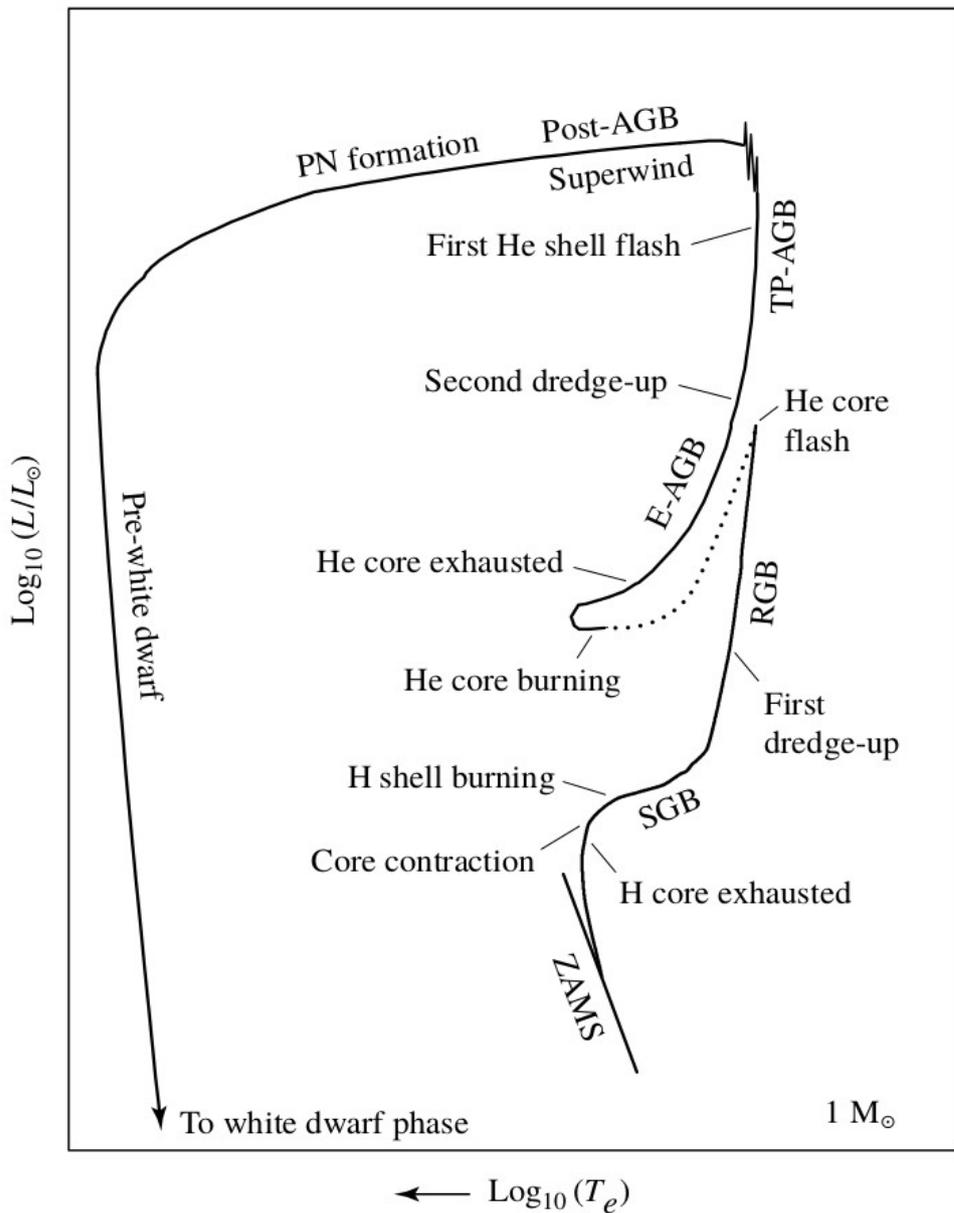


Post-MS evolution

- The H-shell heats up and the fusion rate goes up → the outer layers expand so that the luminosity rises to match this
- At some point the He-core collapses as it cannot support the mass sitting on top of it → release of gravitational energy and envelope expansion
- Star is now a **RED GIANT**

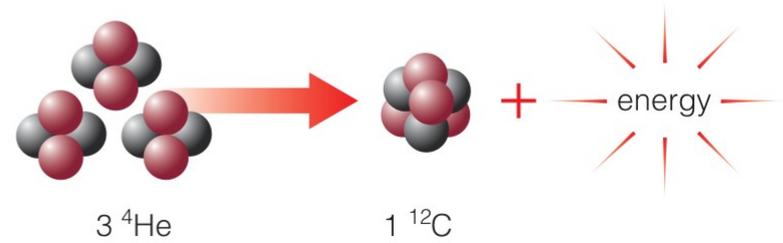


Evolution of stars of different masses on the HR-diagram



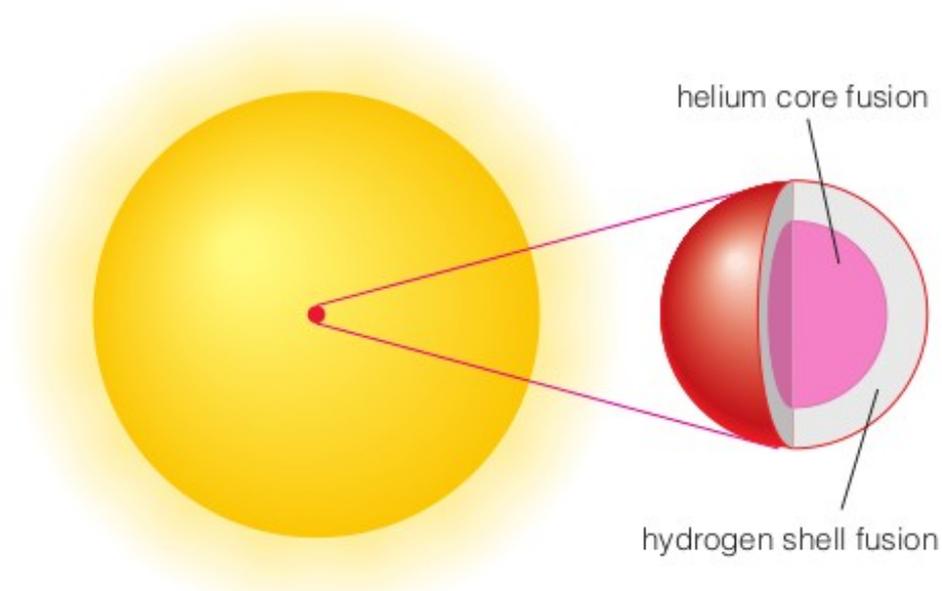
Post-MS evolution

- At ~ 100 MK \rightarrow He-fusion



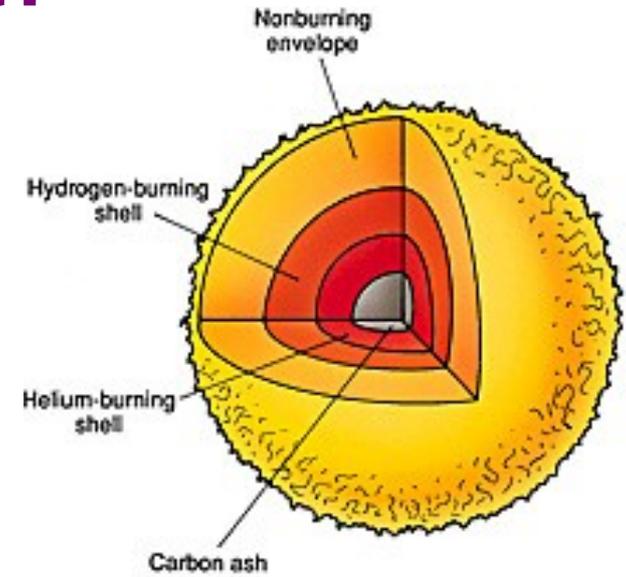
- Enormous production of energy

$$\epsilon_{3\alpha} \propto T^{41}$$



What happens next?

- Finish off He → core is made of inert **carbon**
- Helium and hydrogen fuse in shells (double-shell burning giant)
- The two shells heat up → outer layers further expand



What happens next?

mass $< 4M_{\odot}$

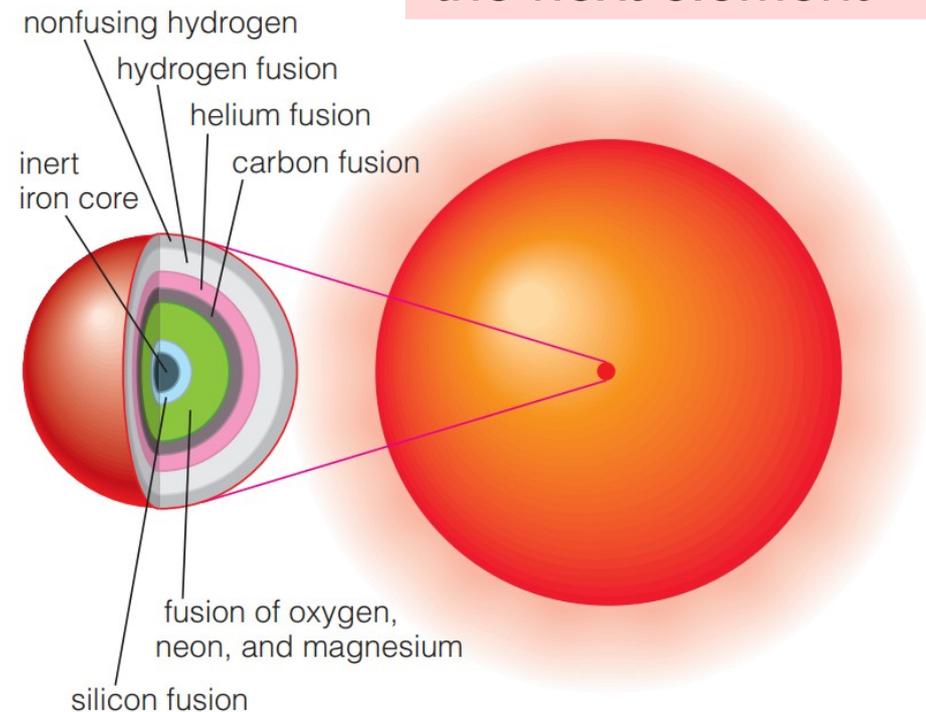
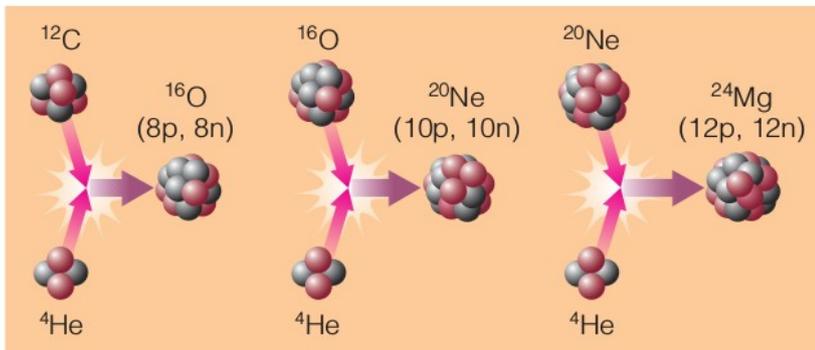
Core never gets hot enough to start fusing carbon/oxygen

$4M_{\odot} < \text{mass} < 8M_{\odot}$

Core never gets hot enough to start fusing carbon/oxygen, but can produce Ne, Mg

mass $> 8M_{\odot}$

ignite C at ~ 600 MK
Each time an element is depleted, the core collapses and heats up, until it becomes hot enough to start fusing the next element



Asymptotic Giant Branch star and its dust shells

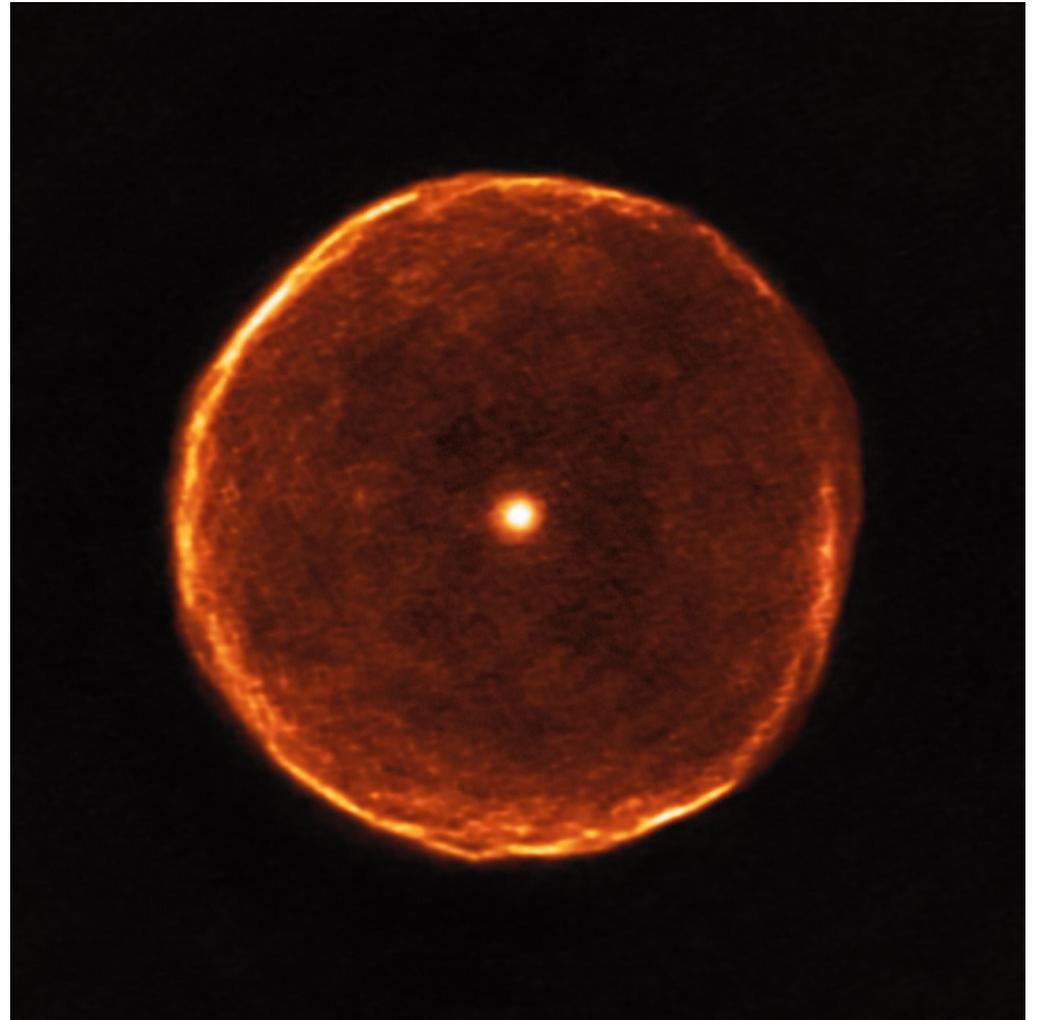
Stars $< 8M_{\odot}$ become huge and lose lots of material through stellar winds

U Antliae observed by ALMA (mm-wavelengths, trace cool dust)

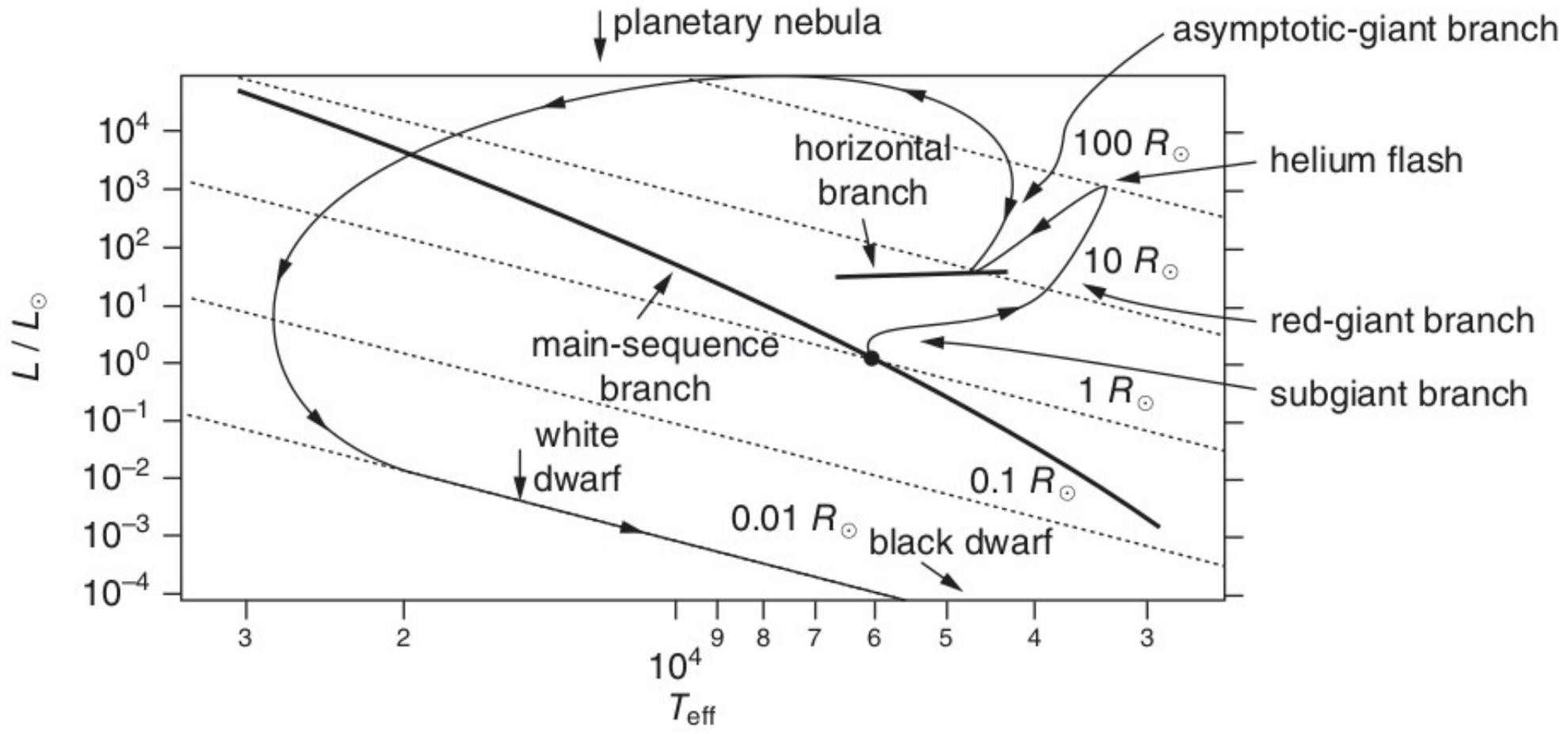
Image size: 2.18×2.18 arcmin²

- Material ejection some 2700 yr ago
- Shell radius $0.05 \text{ pc} \approx 10300 \text{ AU}$

<http://www.sci-news.com/astronomy/alma-bubble-u-antliae-05246.html>

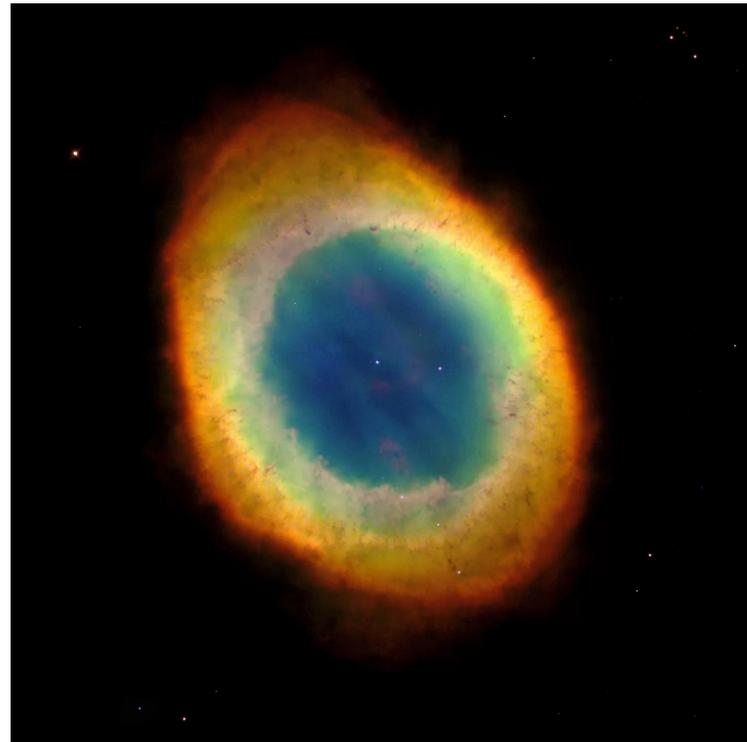


Solar type star evolution (low-mass stars)



Formation of planetary nebula

- Once most material is expelled, the hot core is exposed → **white dwarf**
- Hot core makes the expanding shell glow



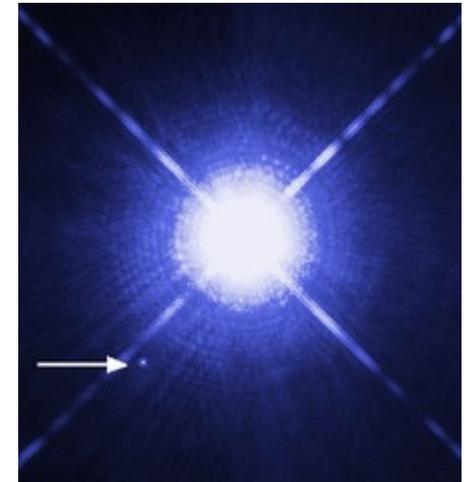
Final stage – White dwarf

- Initially hot, but cools down (no fusion)
- Gravitational stability through degenerate electron gas
- **Mass** $< 1.4 M_{\odot}$ (Chandrasekar limit)

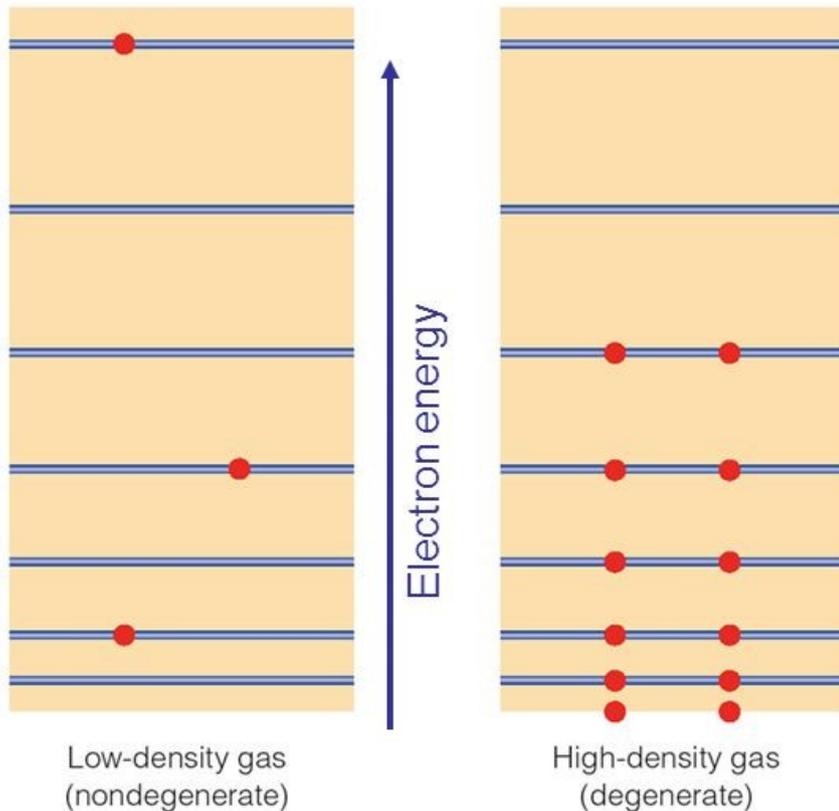
degenerate gas: add mass \rightarrow compression \rightarrow electrons start to move faster (speed of light is the limit)

- **Radius** ~ 1 Earth radius
- **Density** 10^9 kg/m^3
 - a teaspoon of WD material would weight several tons on Earth!

Sirius B



Degenerate matter

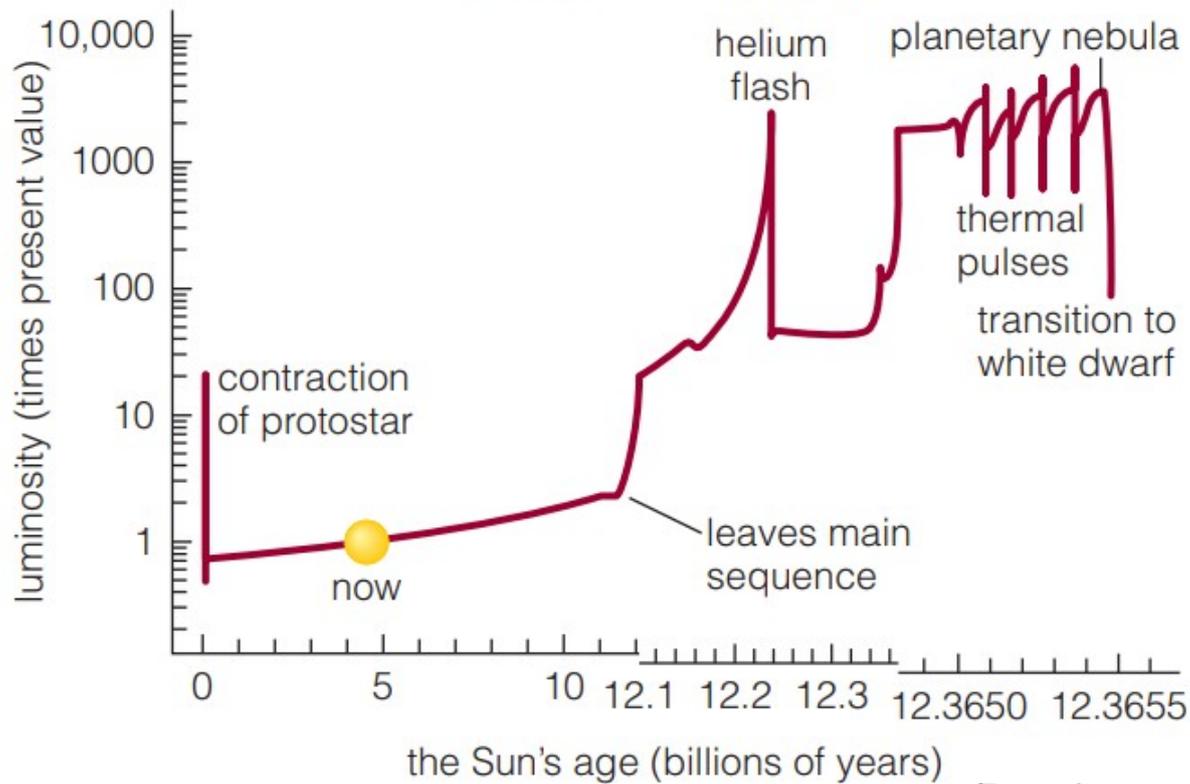


- **Pauli exclusion principle:** only one fermion per each quantum state
- **Condition for degeneracy**

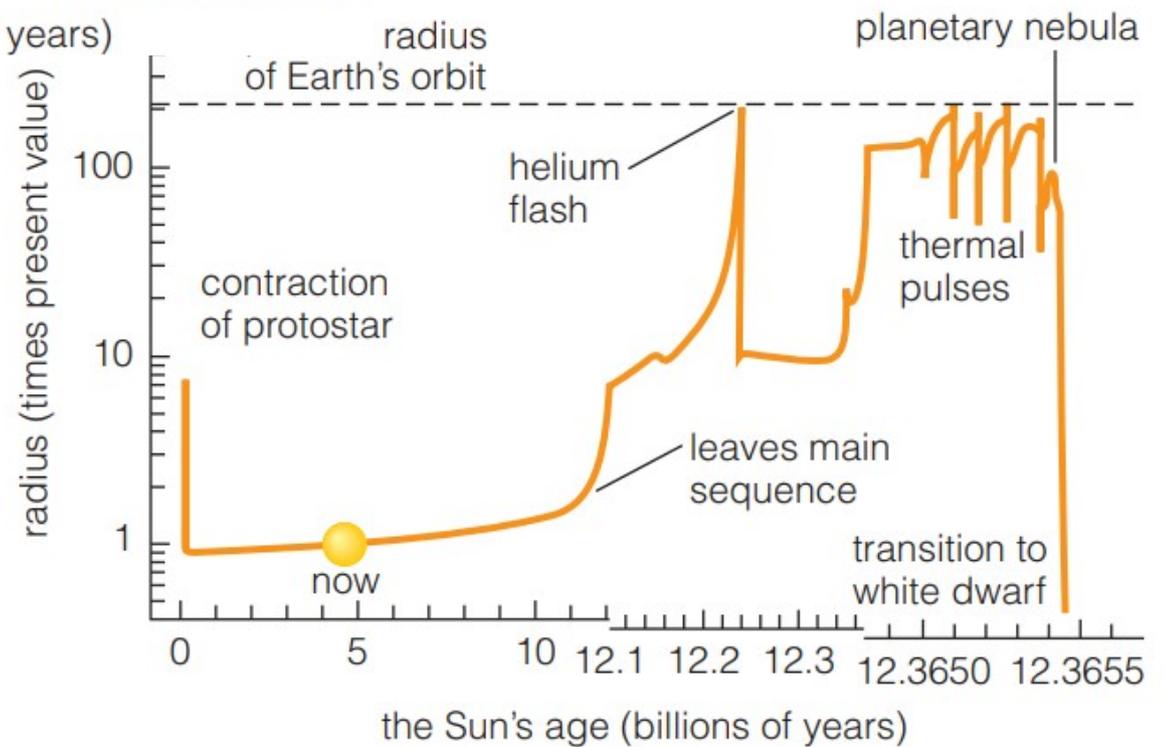
$$\frac{T}{\rho^{2/3}} < D$$

- Very dense gas can become degenerate
- Resists compression (pressure)
- Pressure does NOT depend on temperature

The Sun's Luminosity



The Sun's Radius



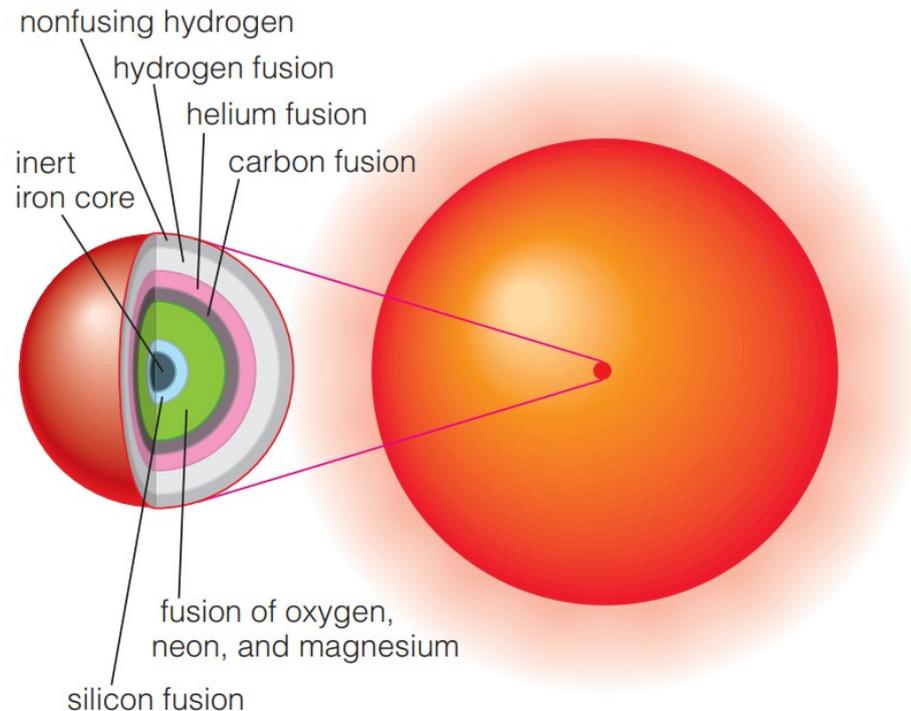
Massive star evolution

Mass > 8 M_☉

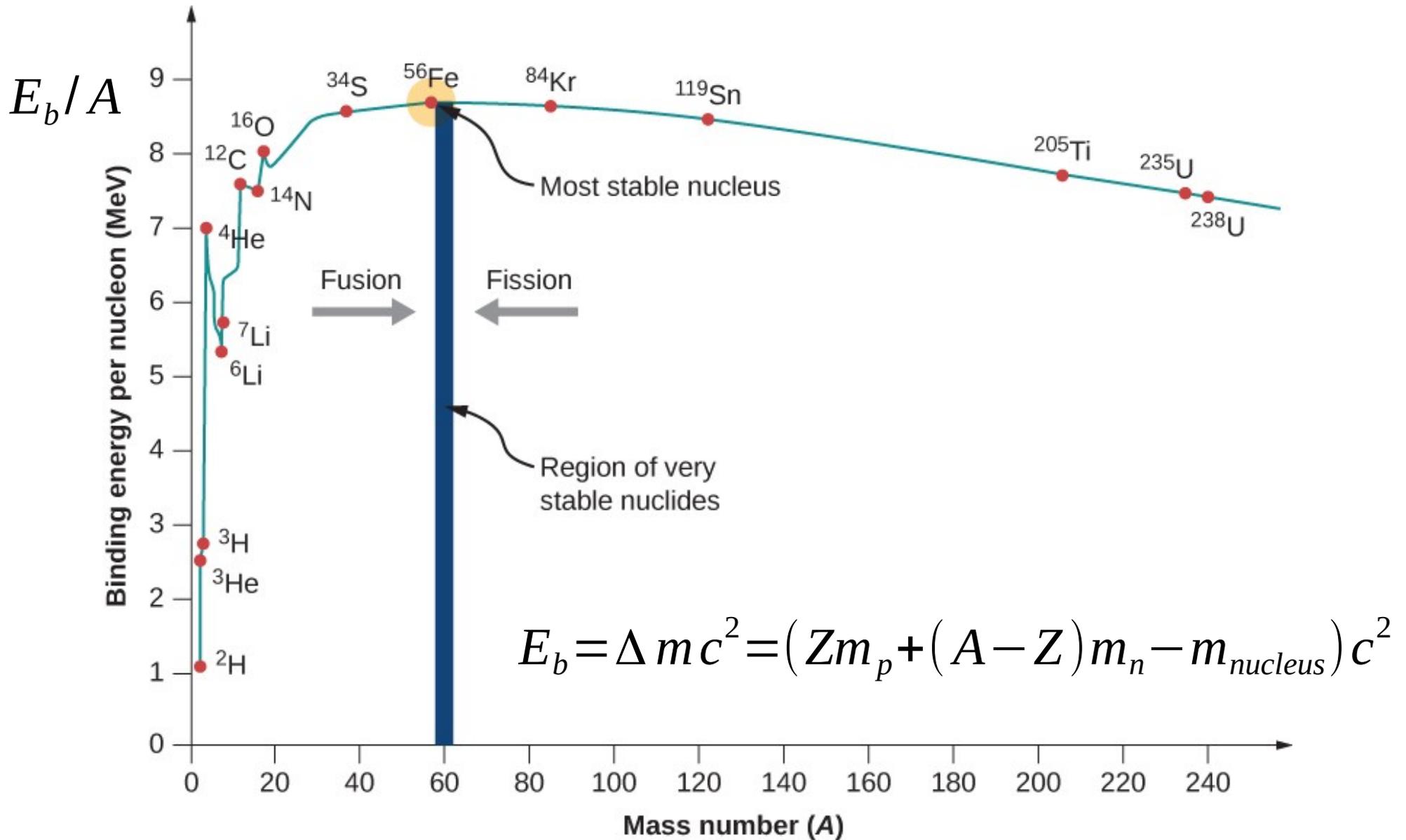
“Onion-like” structure

Final stage:

- Fe piles up in the core
- At this stage the star is a supergiant ($L \sim 10^6 L_{\odot}$, $R > 1000 R_{\odot}$)



Binding energy



Massive star evolution

- Nuclear reaction times for massive stars ($\sim 25 M_{\odot}$)

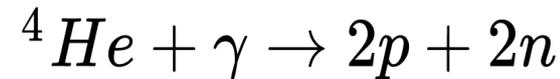
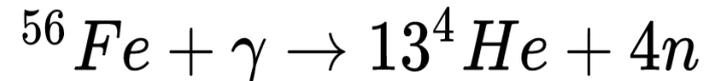
Table 6.5 Major phases of nuclear burning[#].

Burning phase	Elements produced	Central temperature	Timescale
H	He	6.0×10^7 K	7×10^6 yr
He	C, O	2.0×10^8 K	5×10^5 yr
C	O, Ne, Mg	9.0×10^8 K	600 yr
Ne	O, Mg, Si	1.7×10^9 K	0.5 yr
O	Si, S	2.3×10^9 K	6 d
Si	Fe-peak	4.0×10^9 K	1 d

[#]Theoretical results for a $25-M_{\odot}$ star from Arnould, M. and Samyn, M., *La physique nucléaire en astrophysique*, EDP Sciences, Les Ulis (2002).

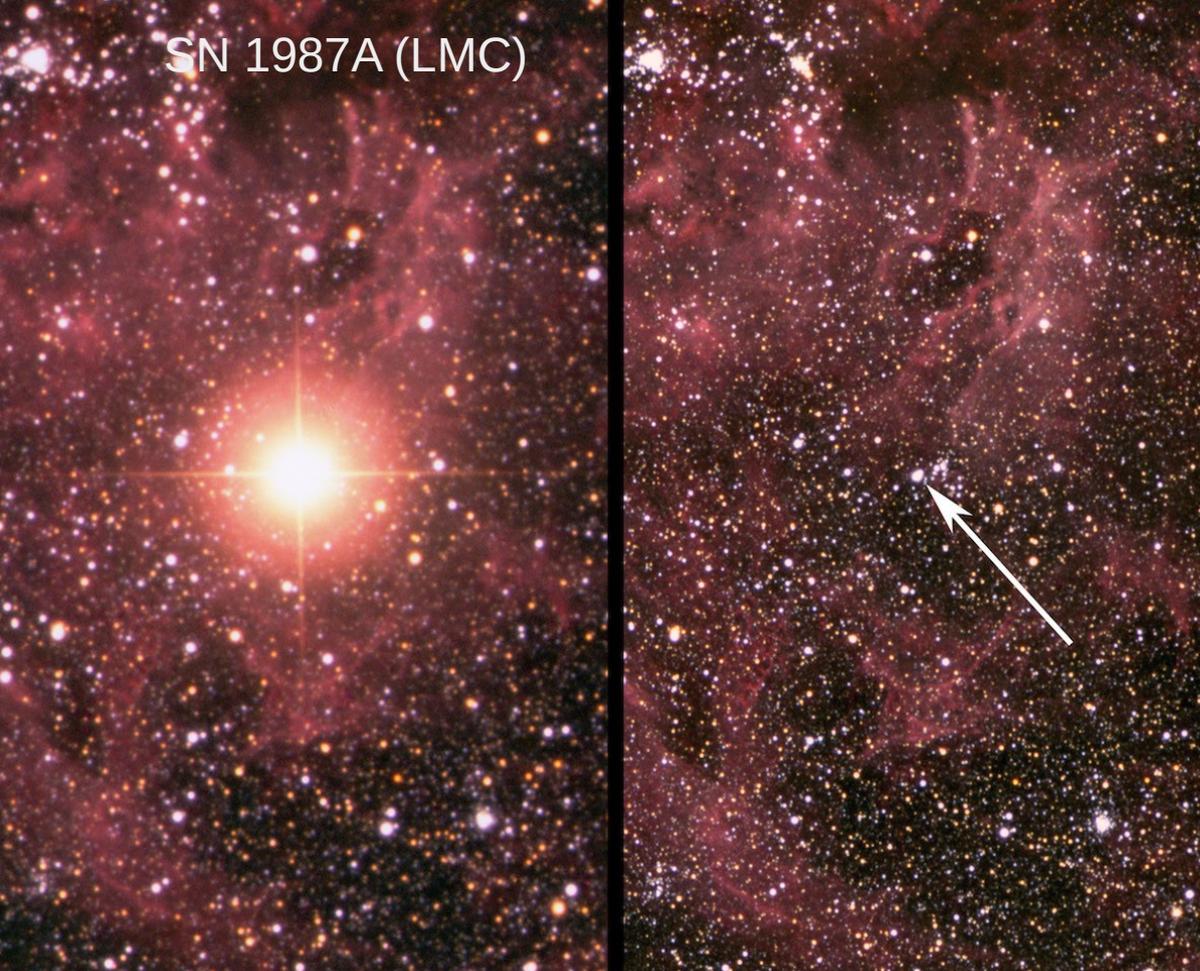
Supernova explosion

- Fe core contracts and heats up → photons have sufficient energies to start desintegrating the Fe atoms



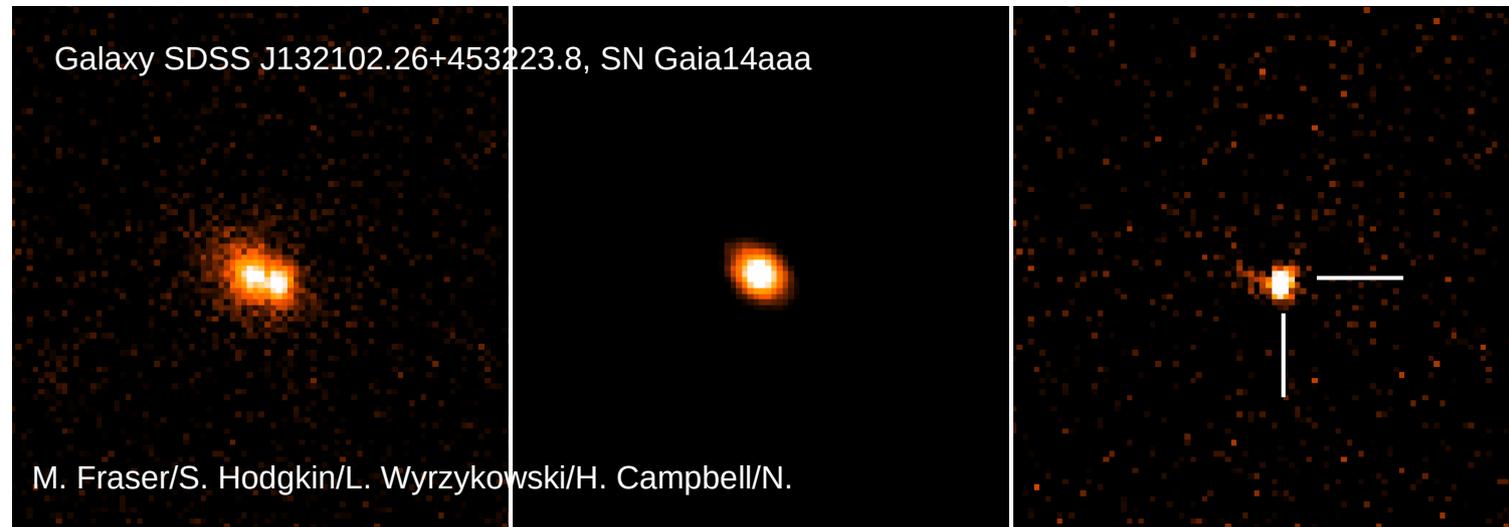
- Collapse continues, densities só large that $p + e^- \rightarrow n + \nu_e$
- Neutron “ball” formed, collapse stops due to **neutron degeneracy pressure**
- Collapse of the core is stopped, and the material falling on top of it rebounds (pressure waves towards outside, build up into a shock wave)
- Release of an enormous amount of energy which drives star’s outer layers out in space → **supernova**

SN 1987A (LMC)



The energy output can be so large to outshine an entire galaxy

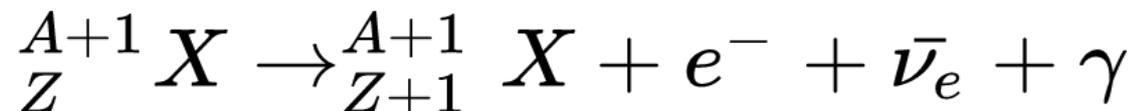
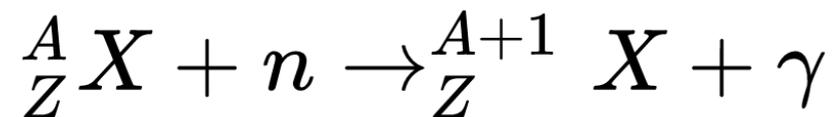
Galaxy SDSS J132102.26+453223.8, SN Gaia14aaa



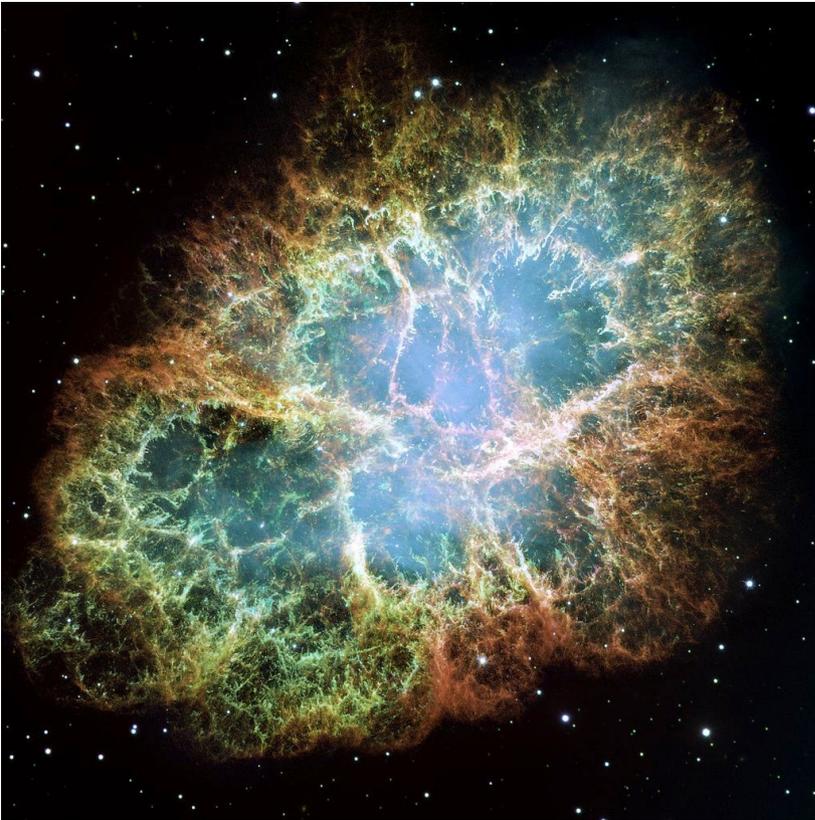
M. Fraser/S. Hodgkin/L. Wyrzykowski/H. Campbell/N.

Supernova explosion

- Final object:
 - $M_* < 25 M_{\odot} \rightarrow$ **neutron star** (supported by the neutron degeneracy pressure)
 - $M_* > 25 M_{\odot} \rightarrow$ **stellar black hole**
 - not even the degeneracy pressure can stop the collapse
 - the object whose mass has collapsed to a singularity of infinite density
- Formation of isotopes and elements heavier than Fe (neutron capture and β^- decay)

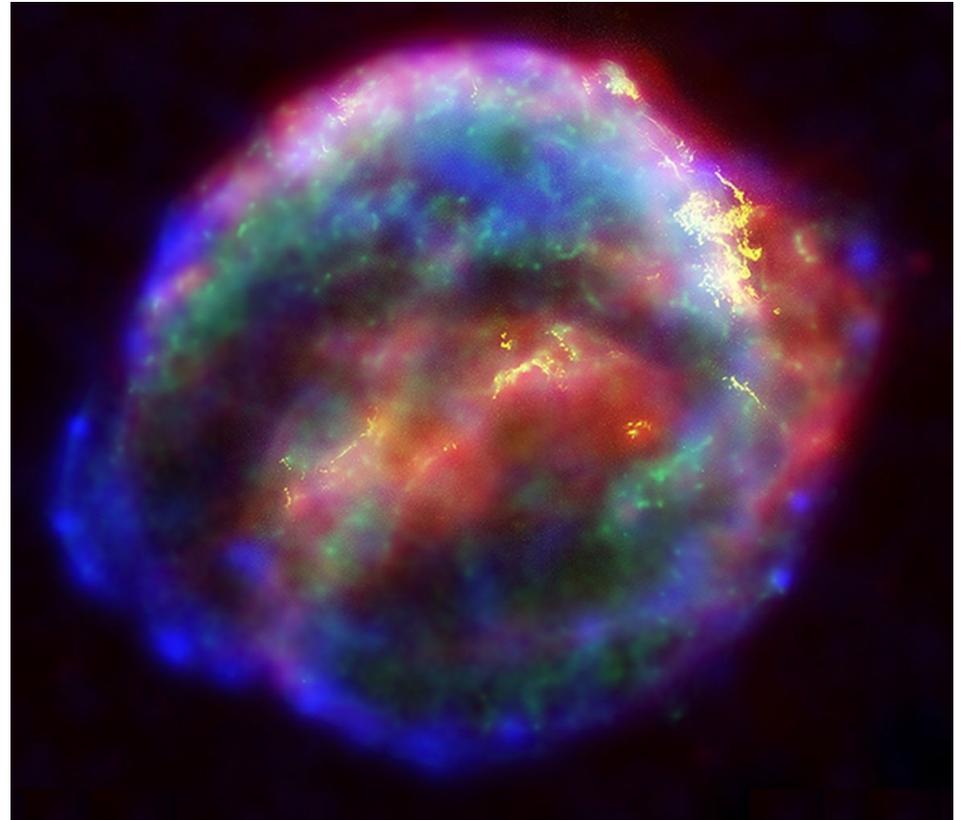


Supernova remnants



Crab nebula

- SN 1054
- Expanding at ~ 1500 km/s



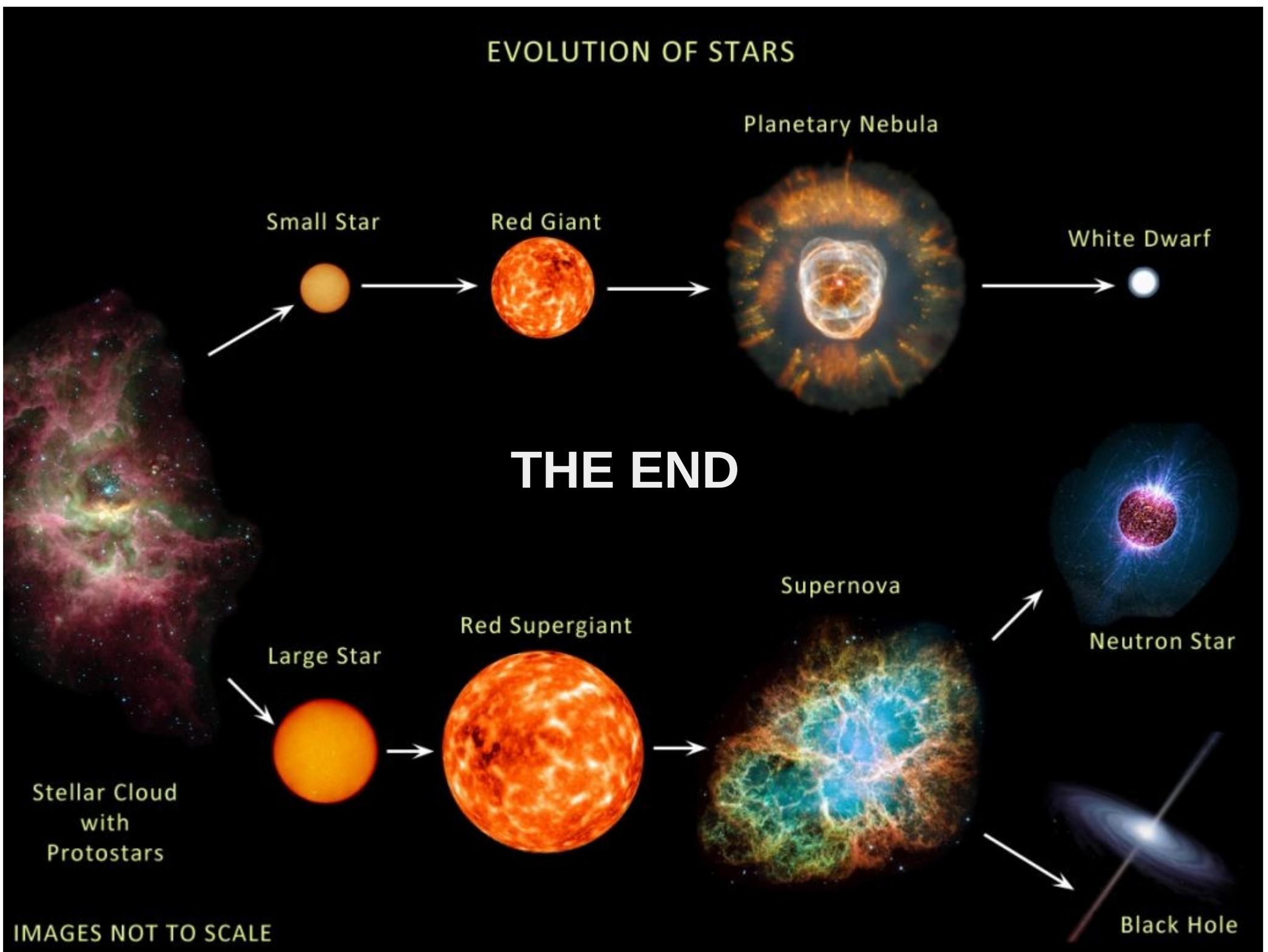
Cassiopeia A

- ~ 300 yrs ago, but no record
- Expanding at ~ 5000 km/s

Neutron stars

- Supported by the neutron degeneracy pressure
- Cools off with time (similar to WDs)
- **Mass** $< 2.2 M_{\odot}$ (Tolman-Oppenheimer-Volkoff limit)
- **Radius** ~ 10 km
- **Density** 10^{18} kg/m³
 - a teaspoon would weight 100 Mt on Earth!

EVOLUTION OF STARS



Small Star

Red Giant

Planetary Nebula

White Dwarf

THE END

Large Star

Red Supergiant

Supernova

Neutron Star

Stellar Cloud
with
Protostars

Black Hole

IMAGES NOT TO SCALE