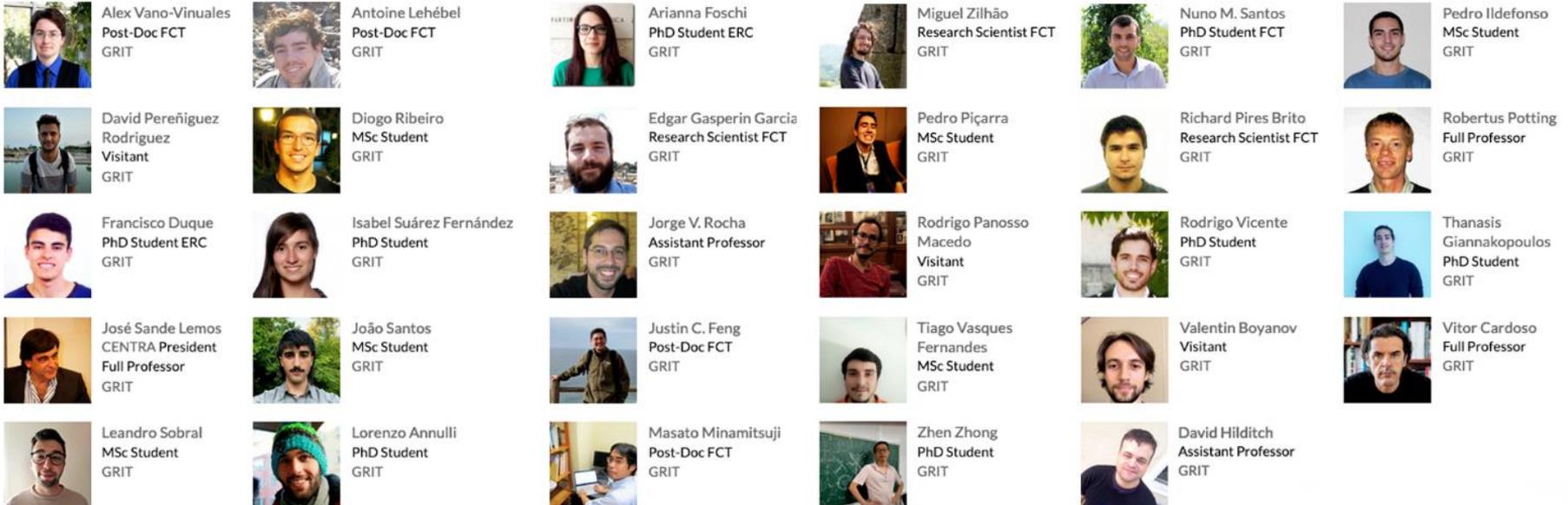


# Black holes: an overview



∞ Vítor Cardoso ∞  
(CENTRA/IST)

# GRIT/CENTRA



The largest (& coolest) gravity group in country

# GRIT/CENTRA

Black holes, neutron stars, General Relativity:

Cosmic Censorship

Dark matter

Gravitational collapse

Gravitational waves

...beyond Einstein

We are leading the Working Groups on Science Interpretation and Fundamental Physics for LISA and The Event Horizon Telescope

# Gravity & BHs hold everything together!

Astrophysics (GW-emission, GRB engine, recoil, structure formation)

Fundamental issues (Censorship, information loss, etc)

Particle physics (DM, engines for HEP, constraints on light states, etc)

High-energy physics (AdS/CFT & TeV gravity, trans-planckian problem)

Condensed matter (AdS/CMT)

Fluids (holography, turbulence, acoustic analogues)

Gravity itself...learn content of field equations and be prepared...

$$\frac{\Phi_{\text{Newt}}}{c^2} = \frac{GM_{\odot}}{c^2 \text{1AU}} \sim 10^{-8} \qquad = \frac{GM_{\odot}}{c^2 r_{\text{periastron}}} \sim 10^{-6} \qquad = \frac{GM_{\odot}}{c^2 r_H} \sim 1$$

Solar system tests

Milisecond binary pulsar

Binary BHs

**Particle physics:**

Subnuclear physics

Atomic physics

**Length**



Extrapolating GR to strong-field regime could be premature...

Similar to describing QCD with quantum mechanics...

# References

S. Chandrasekhar,  
*The Mathematical theory of black holes*  
(Oxford University Press)

M. Maggiore,  
*Gravitational Waves*  
(Cambridge University Press)

*E. Poisson & C. M. Will,*  
*Gravity (Newtonian, Post-Newtonian, Relativistic)*  
(Cambridge University Press)

S. Shapiro and S. Teukolsky,  
Black holes, white dwarfs and neutron stars  
(Wiley)

E. Berti, V. Cardoso & A. Starinets,  
*Quasinormal modes of black holes and black branes*  
Classical and Quantum Gravity 26: 163001 (2009)

R. Brito, V. Cardoso & P. Pani,  
*Superradiance*  
Lecture Notes 971: 1-293 (Springer-Verlag 2020)

V. Cardoso and P. Pani, Living Reviews 22: 1 (2019)

L. Barack et al, arXiv: 1806.05195 [gr-qc]

[www.blackholes.ist.utl.pt](http://www.blackholes.ist.utl.pt)

# Origins of the Classical Definition

42 *Mr. MICHELL on the Means of discovering the*

16. Hence, according to article 10, if the semi-diameter of a sphaere of the same density with the sun were to exceed that of the sun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its surface a greater velocity than that of light, and consequently, supposing light to be attracted by the same force in proportion to its vis inertiaë, with other bodies, all light emitted from such a body would be made to return towards it, by its own proper gravity.

Escape velocity from a mass  $M$  and radius  $R$

$$v_{esc} = \left( \frac{2GM}{R} \right)^{1/2}$$

hence for an object so compact that

$$R \leq R_s = \frac{2GM}{c^2}$$

(Schwarzschild radius)

$$v_{esc} \geq c \Rightarrow \text{black!}$$

(full description requires general relativity)

Object	Mass	$R_s$
Atom	$10^{-26}$ Kg	$10^{-51}$ cm
Human being	70 Kg	$10^{-23}$ cm
Earth	$6 \times 10^{24}$ Kg	0.89 cm
Sun	$2.0 \times 10^{30}$ Kg	3.0 Km
Galaxy	$10^{11} M_\odot$	$10^{-2}$ ly
Universe (closed)	$10^{23} M_\odot$	$10^{10}$ ly



# Some History

- 1784** John Michell and Pierre-Simon Laplace (1796) propose the existence of “dark stars”...that lock in light (in Newtonian mechanics)
- 1915** Albert Einstein publishes his first paper on General Relativity
- 1916** Karl Schwarzschild finds the first nontrivial solution
- 1930** Subrahmanyan Chandrasekhar suggests that a massive star can collapse into something denser (1930)
- 1939** Robert Oppenheimer and Hartland Snyder predict that massive stars can collapse into “black holes”
- 1963** Roy Kerr finds solution for rotating BHs, Schmidt identifies quasar
- 1964** UHURU orbiting X-ray obs. identifies 300 x-ray stars, Cygnus X-1

# Some History

**1967** John Wheeler coins the term “black hole”

**1970s** Golden years I: Hawking evaporation, uniqueness, etc

**1990s** LIGO begins construction

**2002** VLBI missions track region near horizon

**2005s** Golden years II: BHs on computers, GW templates

**2016** LIGO measures gravitational waves from BHs

**2019** Event Horizon Telescope and GRAVITY instrument “see” BHs

**2030s** LISA, Einstein Telescope, Cosmic Explorer, etc

# Uniqueness: the Kerr solution

Theorem (Carter 1971; Robinson 1975):

A stationary, asymptotically flat, vacuum solution must be Kerr

$$ds^2 = \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} dt^2 + \frac{2a(r^2 + a^2 - \Delta) \sin^2 \theta}{\Sigma} dt d\phi - \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \sin^2 \theta d\phi^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2$$

$$\Sigma = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 + a^2 - 2Mr$$

Describes a rotating BH with mass  $M$  and angular momentum  $J=aM$ ,  $a < M$

“In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein’s equations of general relativity provides the *absolutely exact representation* of untold numbers of black holes that populate the universe.”

*S. Chandrasekhar*, The Nora and Edward Ryerson lecture, Chicago April 22 1975

In the limit  $r \rightarrow \infty$  reduces to Minkowski in polar coordinates

In the limit  $a \rightarrow 0$  reduces to Schwarzschild

In the limit  $M \rightarrow 0$  reduces to Minkowski in spheroidal coords\*

It is *stationary*: it does not depend on time explicitly

It is *axisymmetric*: it does not depend on  $\phi$  explicitly

It is not static: not invariant under time reversals  $t \rightarrow -t$

\* *locally, see Gibbons & Volkov arXiv:1705.07787*

It is invariant under the simultaneous inversions  
as expected from rotating object

$$\begin{aligned} t &\rightarrow -t \\ \phi &\rightarrow -\phi \end{aligned}$$

Curvature invariant:

$$K = \frac{48M^2(r^6 - 15a^2r^4 \cos^2 \theta + 15a^4r^2 \cos^4 \theta - a^6 \cos^6 \theta)}{(r^2 + a^2 \cos^2 \theta)^6}$$

No curvature singularity at  $\Delta = 0$  , it is removable

Horizon at  $r_{\pm} = M \pm \sqrt{M^2 - a^2}$

Upper limit on rotation:  $j \equiv a/M \leq 1$

Note that in SI units, this limit is  $j = \frac{Jc}{GM^2} \leq 1$

The Sun has  $j=1.12$  and a spinning top has  $j=10^{19}$

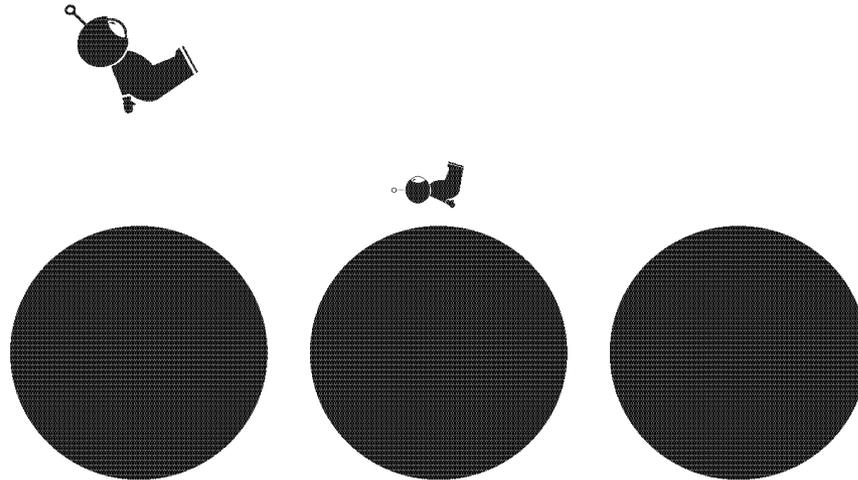
# Black holes have no hair

*J.A. Wheeler, 1971*

One star of matter and another of anti-matter produce the same BH.

BH shares 3 common quantities with progenitor (if no radiative processes):

**mass, rotation (and electric charge)**



# The anatomy of black holes

Once they form they are left with **mass, spin, charge**

# The anatomy of black holes

Once they form they are left with ~~mass, spin, charge~~

- If  $Q > 10^{-18} Q_{\max}$ , EM overwhelms gravity, causing charge separation & neutralizing the BH. Need only

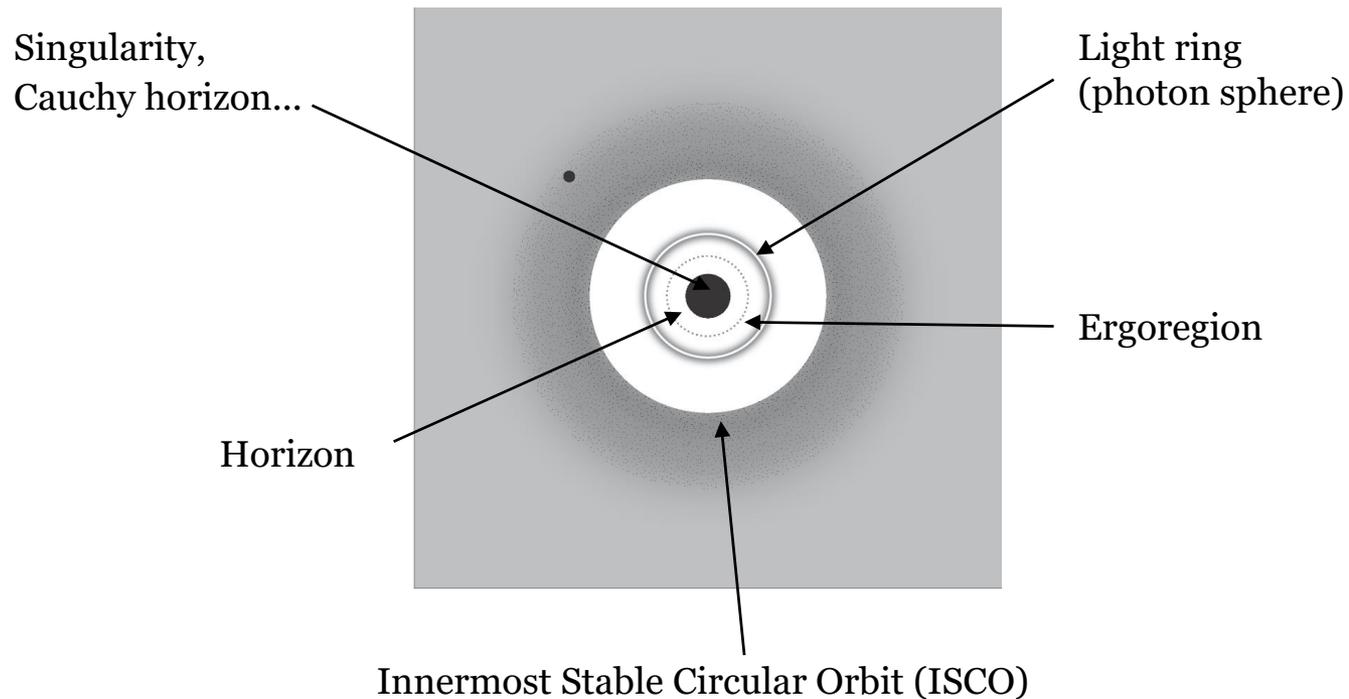
$$M_{\text{plasma}} = 10^{15} \frac{M_{BH}}{M_{\odot}} \text{ g}$$

- Even in vacuum, Schwinger emission limits the maximum Q

$$Q \leq 10^{-5} \frac{M_{BH}}{M_{\odot}} Q_{\max}$$

*(Eardley and Press, 1975; Cardoso+ JCAP05:054 (2016))*

# The anatomy of black holes

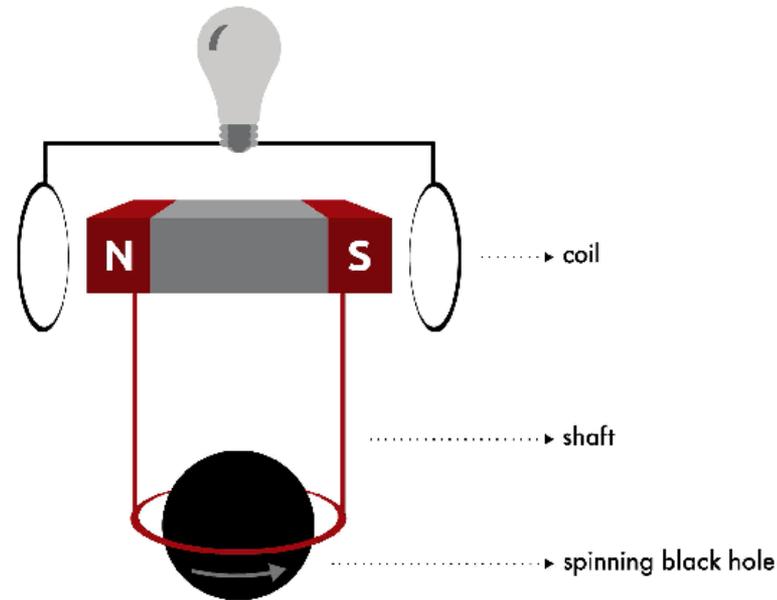


Any evidence for existence of these features is welcome

Cardoso & Pani, Living Reviews in Relativity 22: 1 (2019)

*Image: Ana Carvalho*

# Energy source?



*Image: Ana Carvalho*

Brito, Cardoso & Pani, *Superradiance* (Springer-Verlag, 2020)

*I only wish to make a plea for “black holes” to be taken seriously and their consequences to be explored in full detail. For who is to say, without careful study, that they cannot play some important part in the shaping of observed phenomena?*

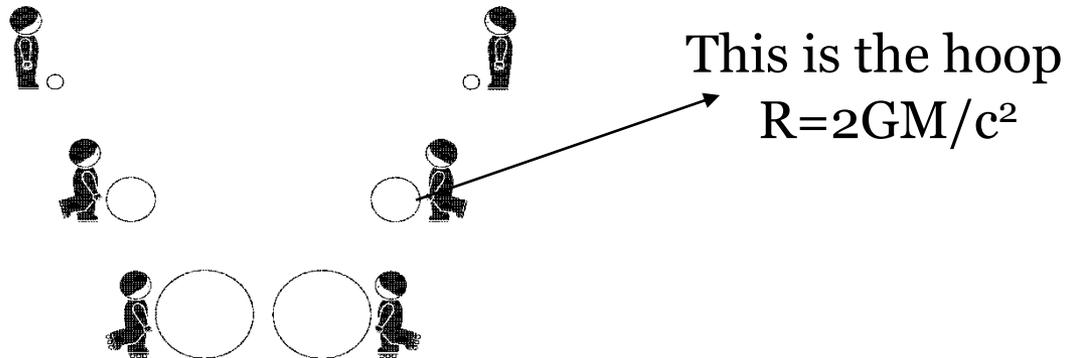
Penrose, *Gravitational Collapse: the role of General Relativity* (1969)

# Hoop Conjecture

(Thorne 1972)

“An imploding object forms a BH when, and only when, a circular hoop with circumference  $2\pi$  the Schwarzschild radius of the object can be made that encloses the object in all directions.”

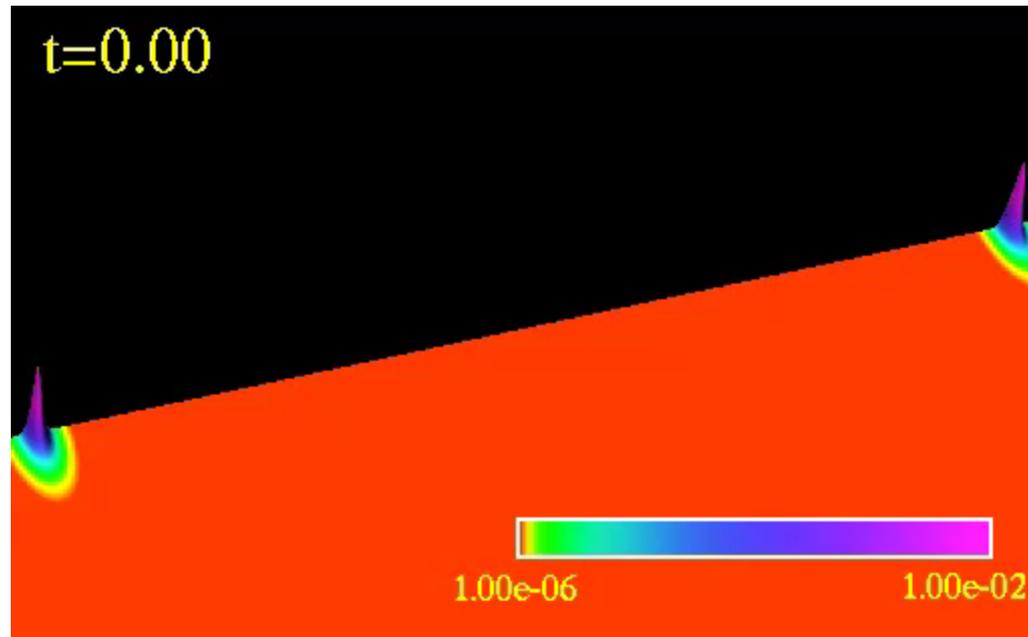
## Large amount of energy in small region



Size of electron:  $10^{-17}$  cm  
Schwarzschild radius:  $10^{-55}$  cm

# The end of short-distance physics

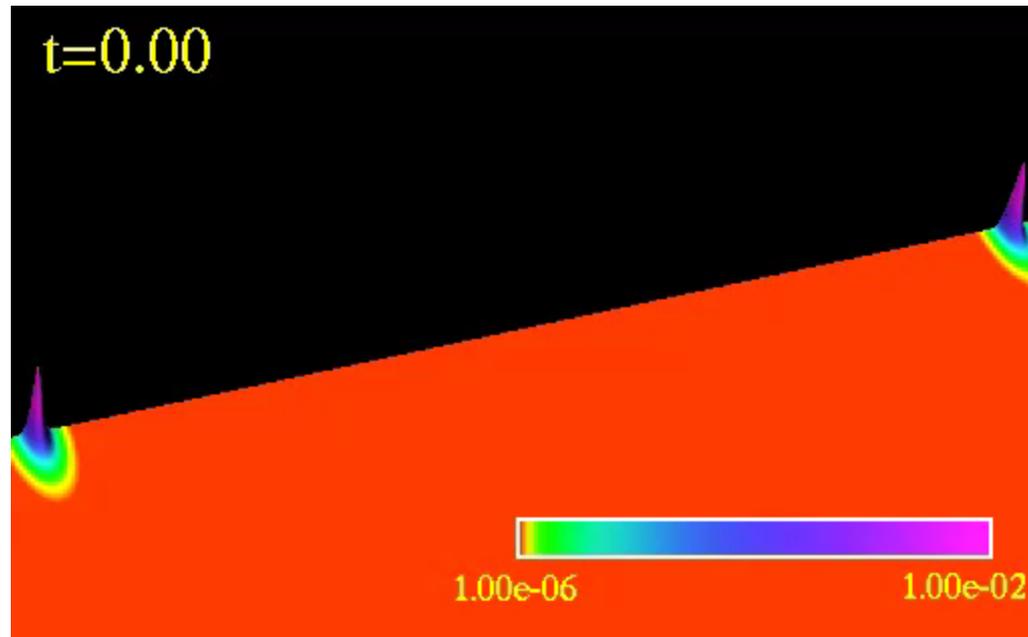
Lorentz boost = 2



Choptuik & Pretorius, Phys.Rev.Lett. 104:111101 (2010)

# The end of short-distance physics

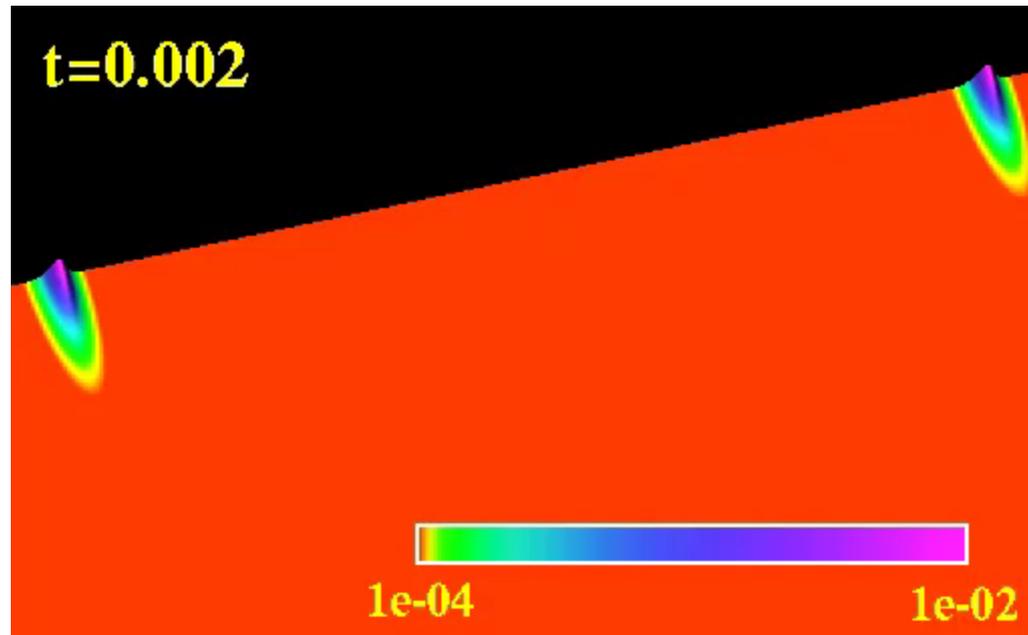
Lorentz boost = 2



Choptuik & Pretorius, Phys.Rev.Lett. 104:111101 (2010)

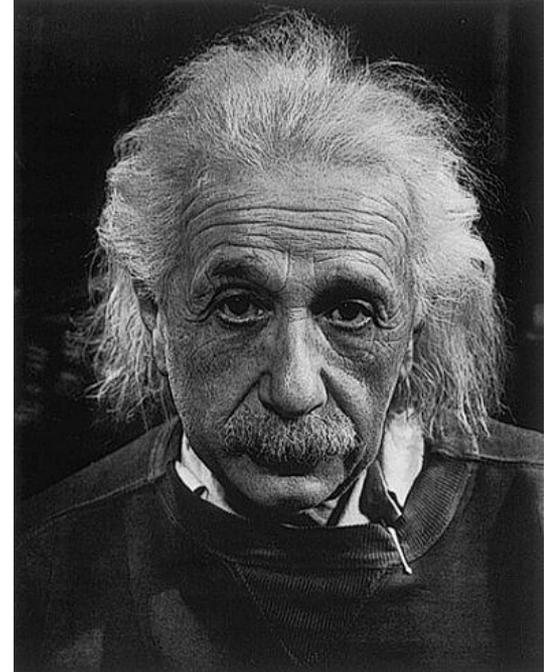
# The end of short-distance physics

Lorentz boost = 4

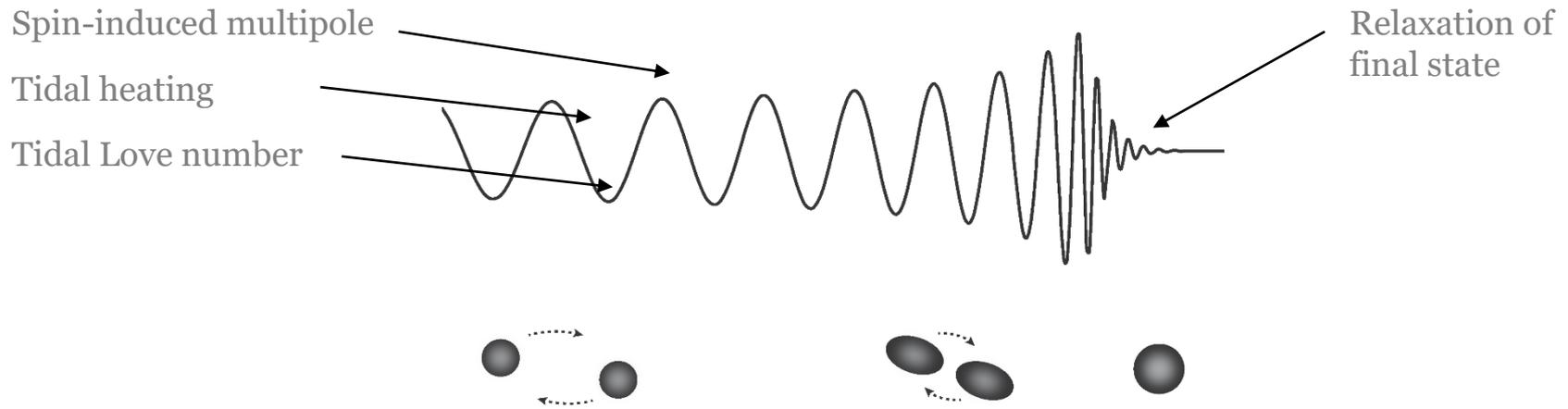


Choptuik & Pretorius, Phys.Rev.Lett. 104:111101 (2010)

In 1916, Einstein shows that GWs are a consequence of the linear theory.



# Gravitational waves & two-body systems



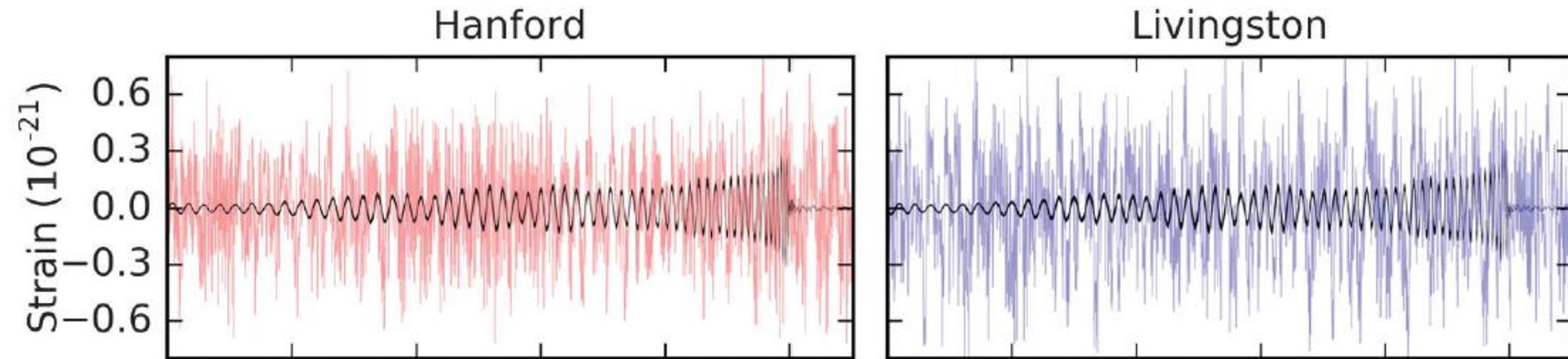
But BHs in GR are simple objects:

Multipolar structure entirely dependent on mass and spin

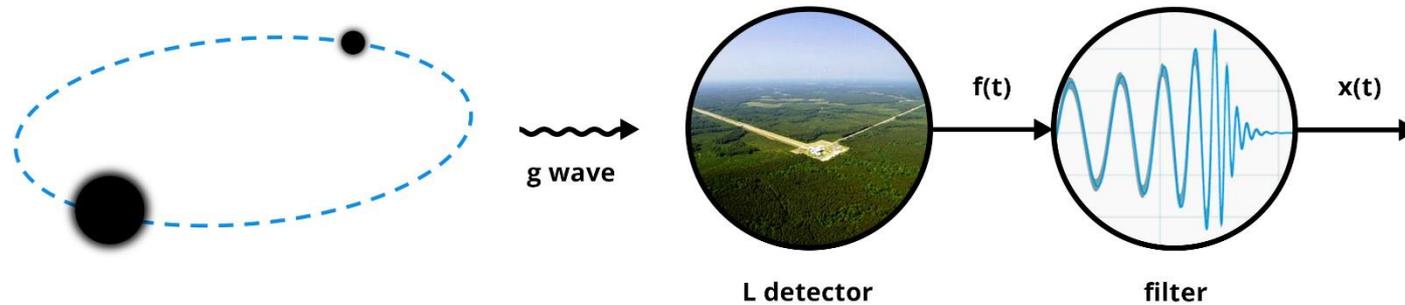
Tidal Love numbers vanish (black holes don't "polarize")

Relaxation depends only on mass and spin...

# The needle in the haystack problem



# Matched-Filtering

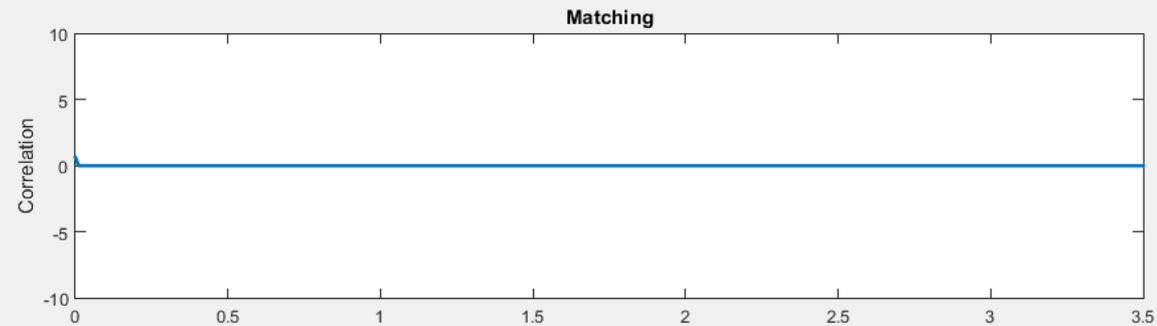
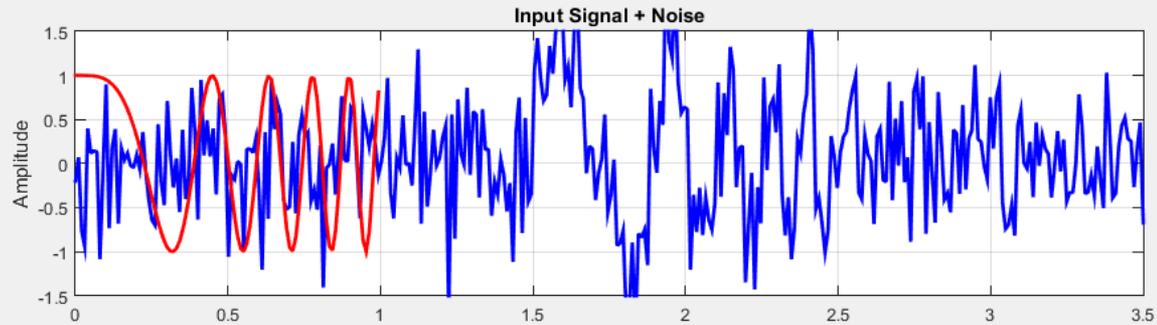
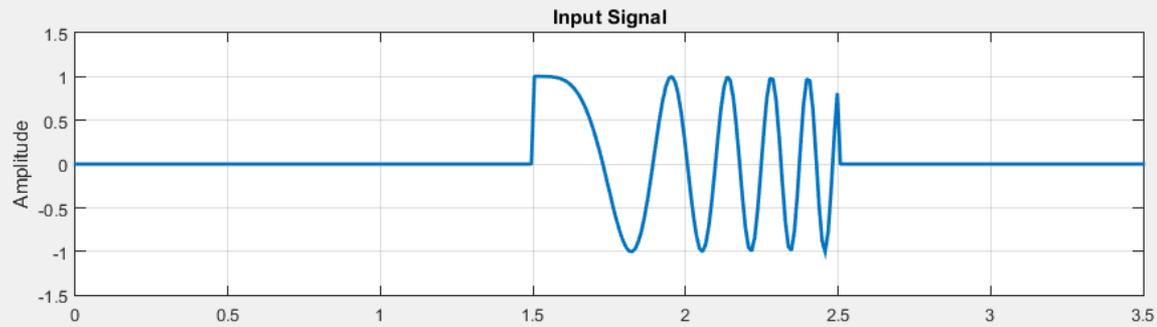


The detector output  $f(t) = h(t) + n(t)$

where  $n(t)$  is the noise. For stationary Gaussian noise, process signal with filter  $K(t)$  against data stream producing number

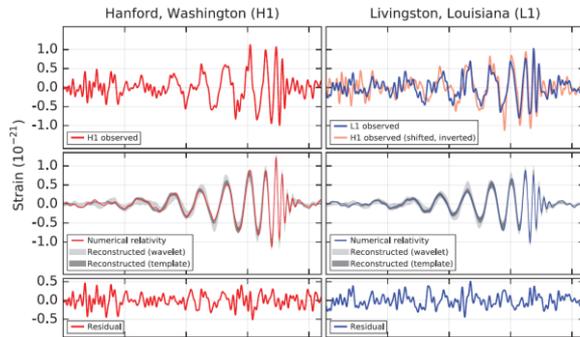
$$\frac{S}{N} = \frac{\text{expected value of } X \text{ with signal}}{\text{rms value of } X \text{ with no signal}} = \frac{\langle X \rangle}{\sqrt{\langle X^2 \rangle_{h=0}}}$$

Optimum filter  $K$  (Wiener, or matched) maximizes SNR and is the signal  $h$  itself!

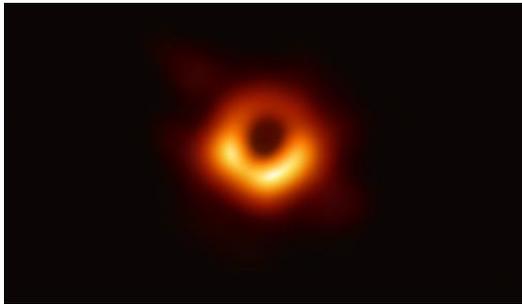


3% Mismatch: 10% lost events!....LIGO uses ~250000 templates for CBC searches

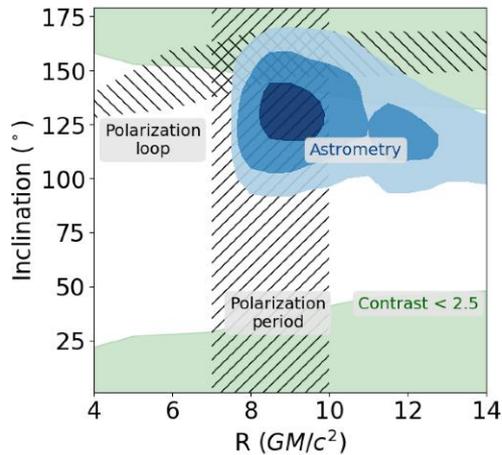
# Black holes exist



LIGO/Virgo Collaboration PRL116:061102 (2016)



EHT Collaboration ApJL 875: 1 (2019)



GRAVITY Collaboration AA 635: A143 (2020)

# Fundamental questions

a. BH seeds, BH demography, galaxy co-evolution (how many, where, how?)

*See review Barack+ arXiv:1806.05195*

b. What is the graviton mass or speed?

*Baker+ PRL119: 251301 (2017); See review Barack+ arXiv:1806.05195*

c. Is cosmic censorship preserved?

*Sperhake+ PRL103:131102 (2009); Cardoso+ PRL120:031103 (2018)*

d. Is it a Kerr black hole? Can we constrain alternatives?

*Berti+ 2016; Cardoso & Gualtieri 2016*

e. Can GWs from BHs inform us on fundamental fields/DM?

*Arvanitaki+ PRD95: 043001 (2016); Brito+ PRL119:131101 (2017)*

f. Is the final - or initial - object really a black hole?

*Cardoso+ PRL116: 171101 (2016); Nature Astronomy 1: 586 (2017)*

Answer requires understanding of theoretical framework, PDEs, precise modelling,  
challenging simulations & challenging data analysis techniques

# Cosmic censorship

I. Strong: future is always predictable

(maximal Cauchy development ends at singularities)

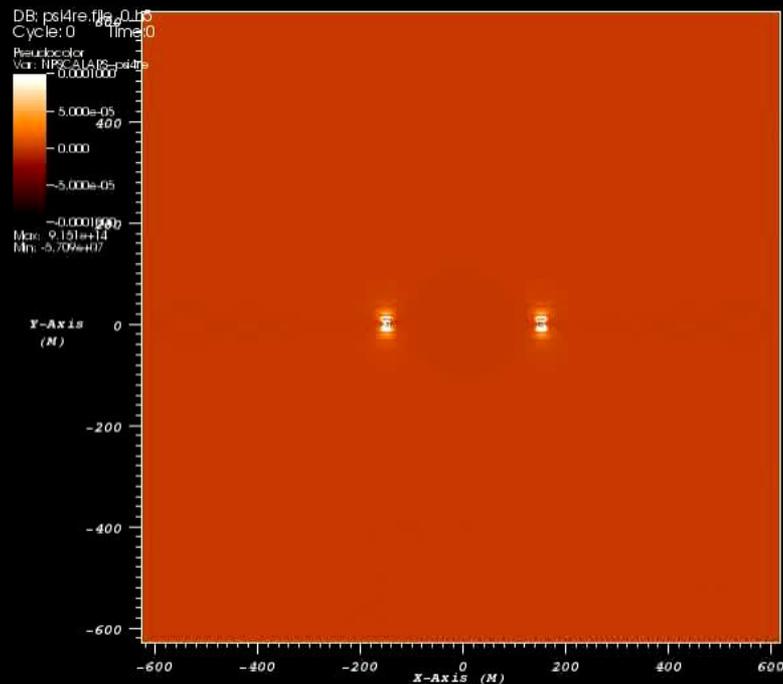
II. Weak: singularities are always hidden behind horizons

Penrose 1978; see 2018 Physics Viewpoint by H. Reall

$$\frac{cJ}{GM^2} \leq 1 \quad \text{or} \quad a \leq M$$

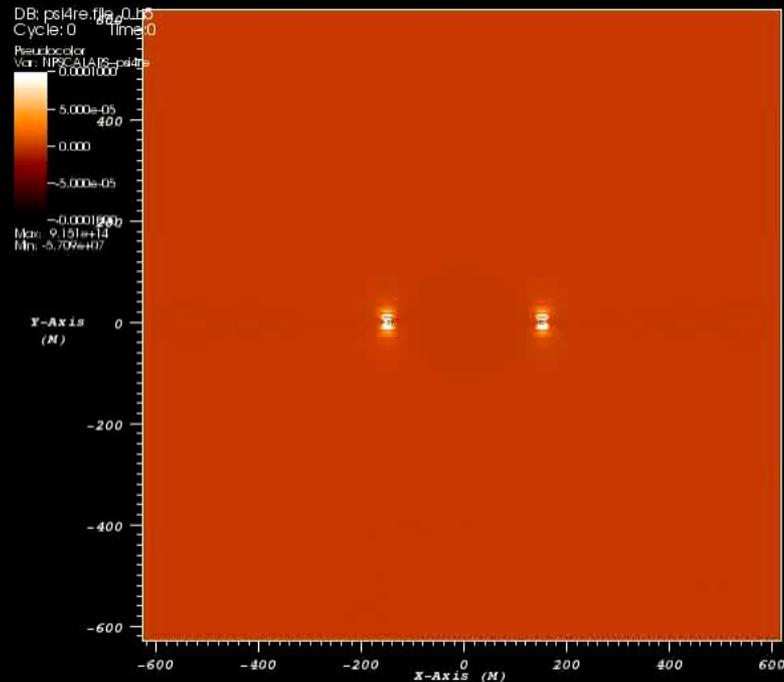
*Black holes have small angular momentum (very compact objects)*

# Extreme CC violation attempts



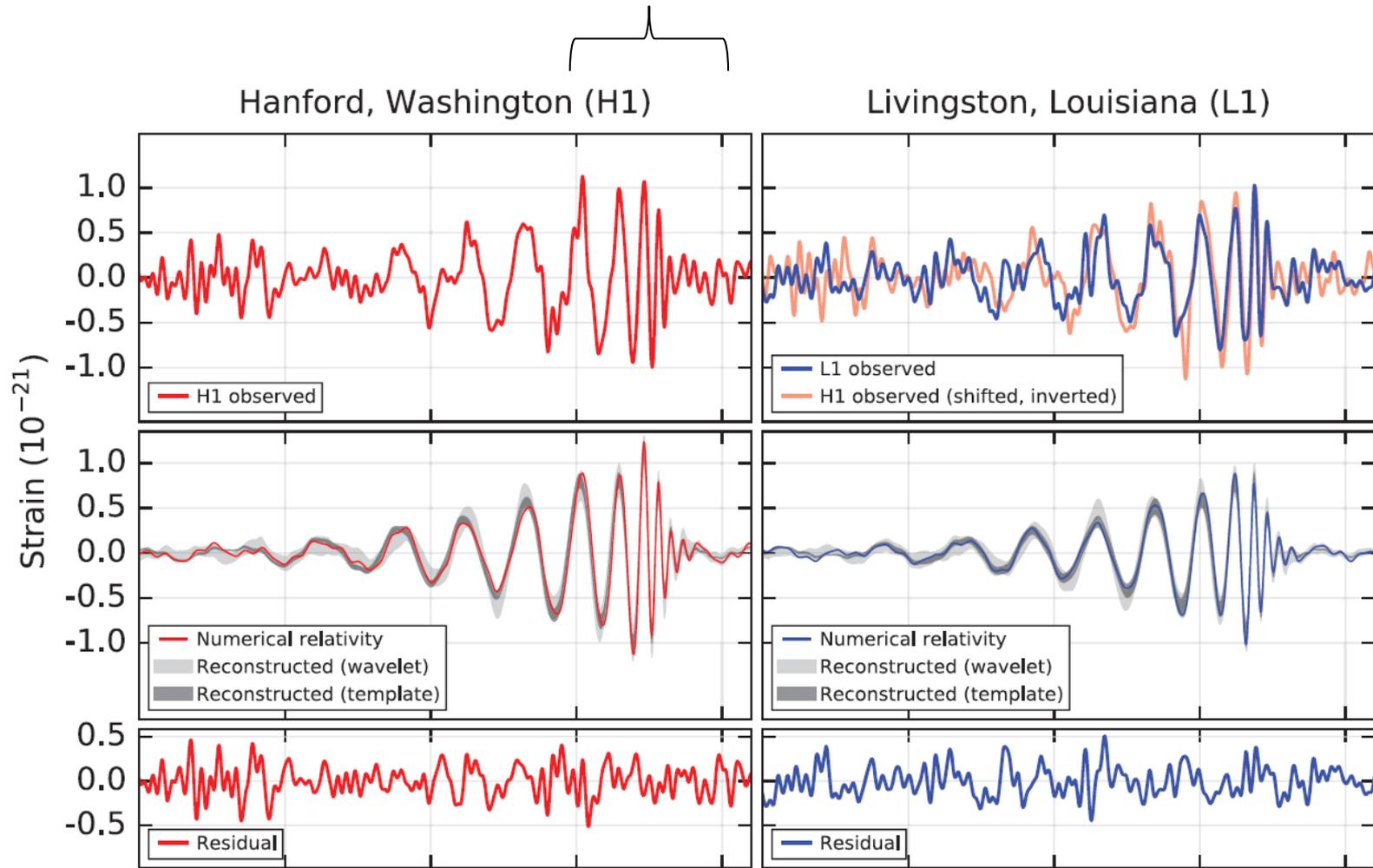
User: gperhake  
Mon Feb 23 13:57:53 2009

# Extreme CC violation attempts



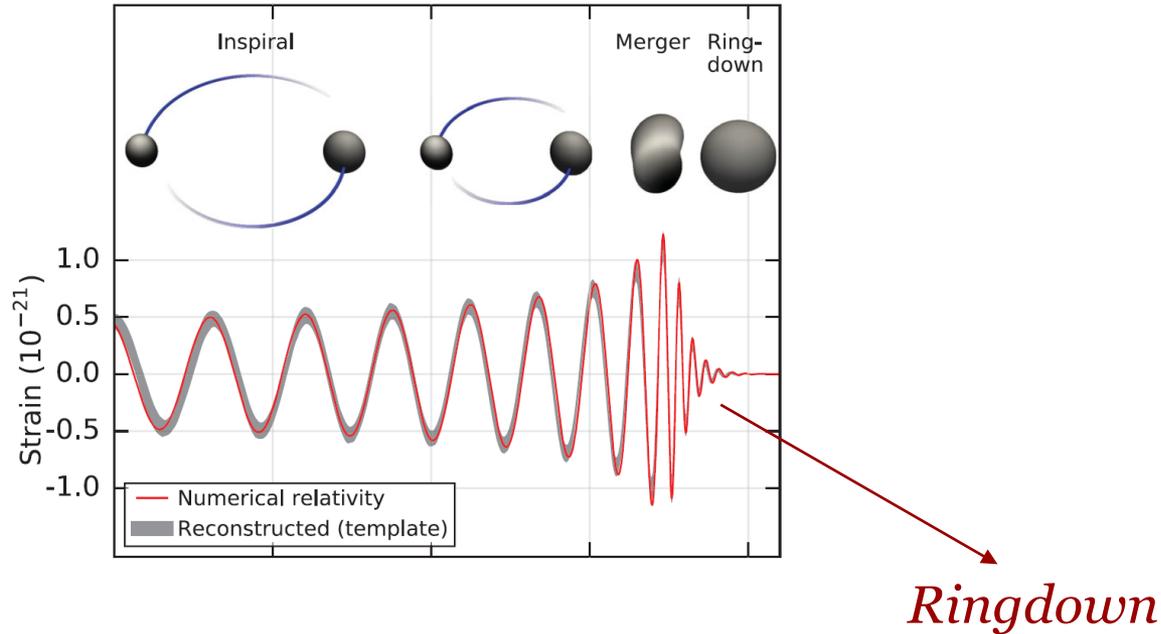
User: gperhake  
Mon Feb 23 13:57:53 2009

0.05 secs (~80 Schwarzschild radius)



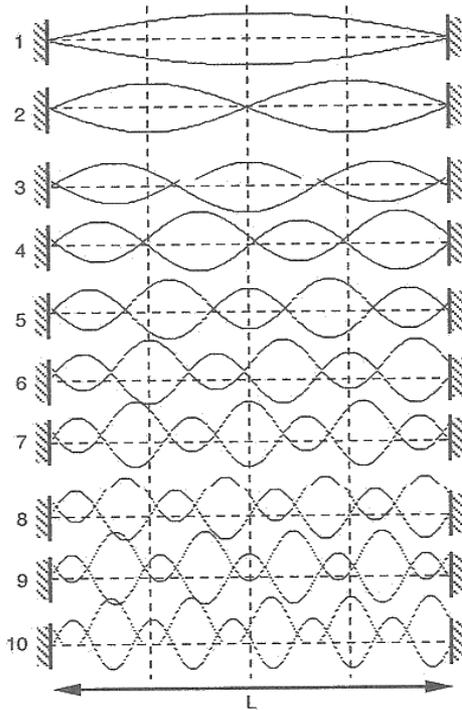
Abbott et al, Phys.Rev.Lett.116:061102 (2016)

# BH spectroscopy: testing the Kerr nature



$$h = \sum_k A_k e^{-\omega_I^{(k)} t} \sin \left( \omega_R^{(k)} t \right)$$

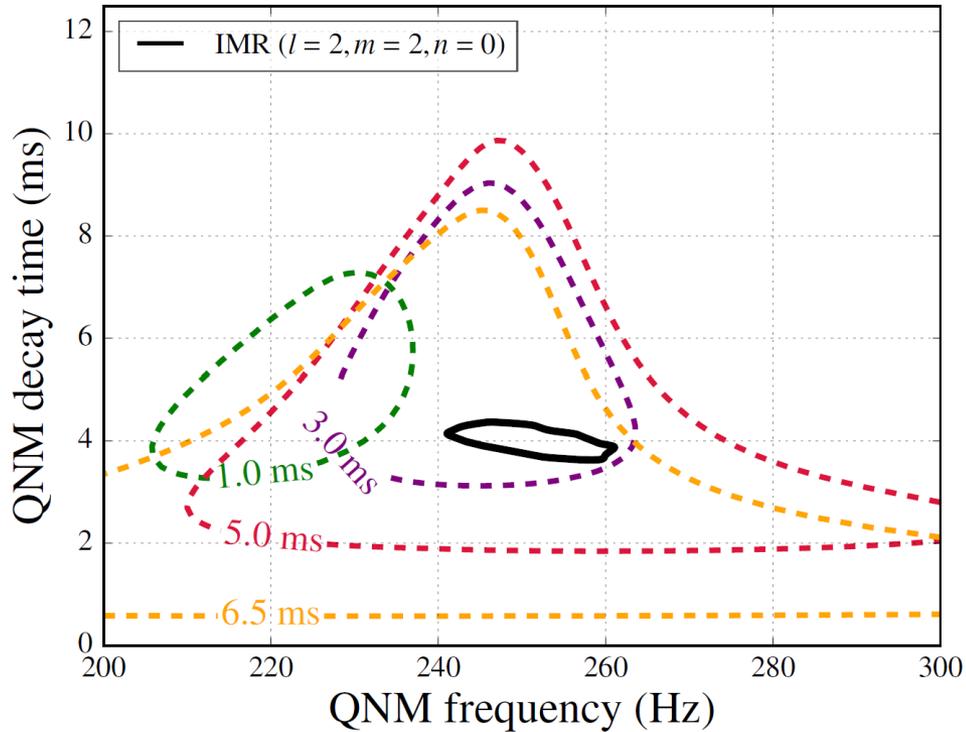
# Tests of the no-hair hypothesis



$$f = \frac{nv}{2L}, \quad n = 1, 2, 3\dots$$

Measure fundamental mode, determine length  $L$ .  
Measure first overtone, test if it's a string...

# Black hole spectroscopy



90% posterior distributions.

Black solid is 90% posterior of QNM as derived from the posterior mass and spin of remnant

# GWs and dark matter I

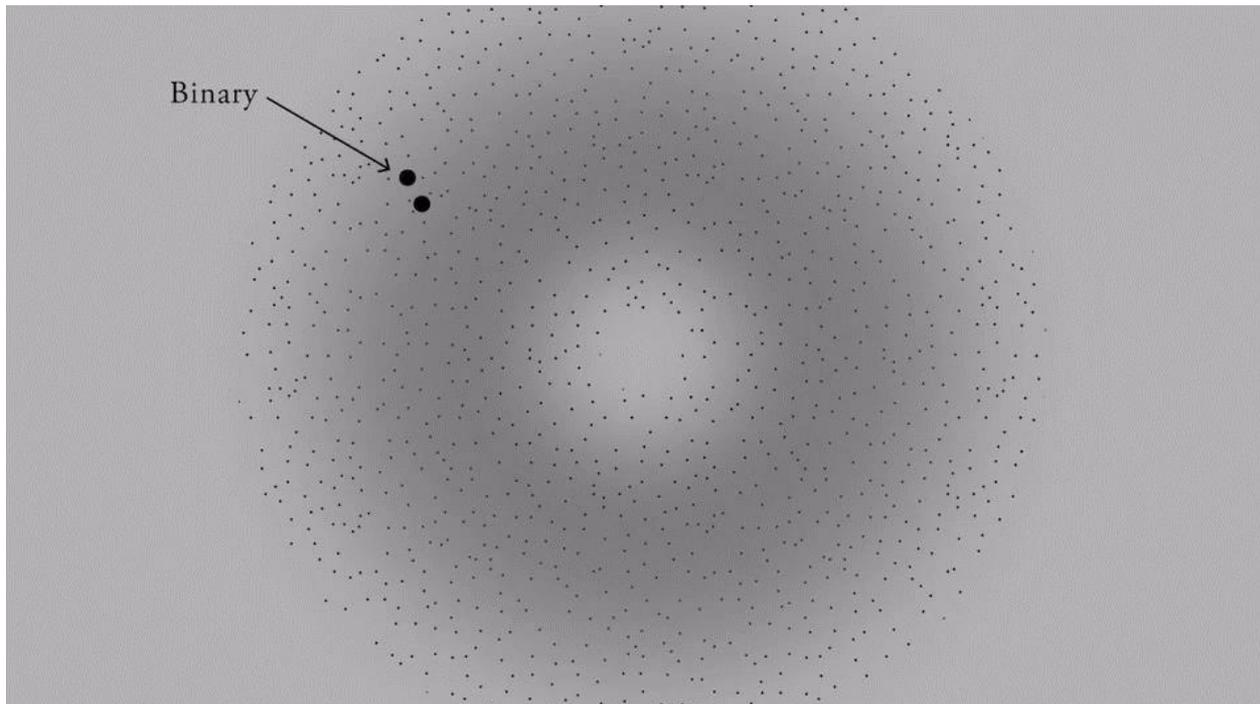
DM not strong-field phenomenon, but GW observations may reveal a “mundane” explanation in terms of heavy BHs.

*Bird + PRL116:201301 (2016)*

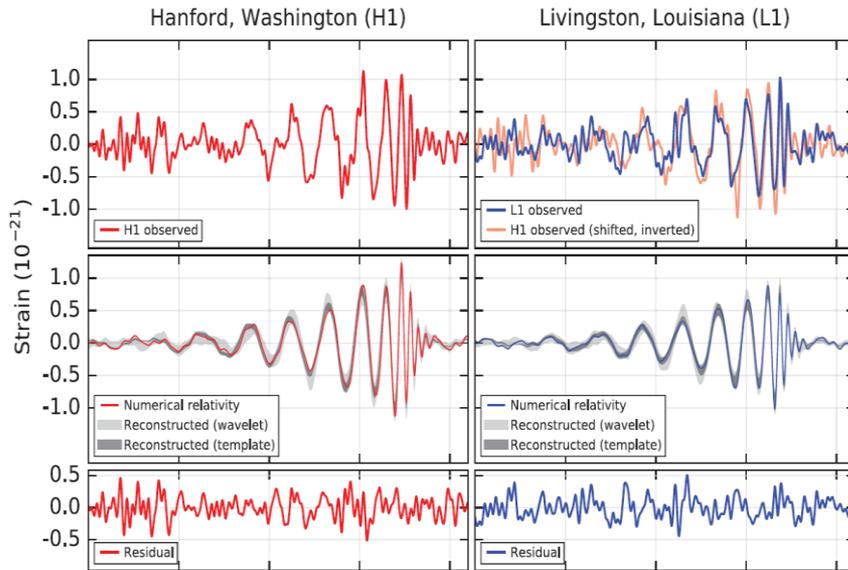
Inspiral occurs in DM-rich environment and may modify the way inspiral proceeds, given dense-enough media: accretion and gravitational drag play important role.

*Eda + PRL110:221101 (2013); Macedo + ApJ774:48 (2013); Cardoso+ AA644: A147 (2020);*

*Kavanagh + PRD102:083006 (2020); Annulli + PRD102: 063022 (2020)*



# The nature of dark compact objects



$$f_{GW}^{-8/3}(t) = \frac{(8\pi)^{8/3}}{5} \left( \frac{G\mathcal{M}}{c^3} \right)^{5/3} (t_0 - t)$$

$$\mathcal{M} = (\mu^3 M^2)^{1/5}$$

Two unknowns, need frequency at two instants. Result:  $M \sim 65$  suns

Use Kepler's law, separation at collision is  $\sim 500$  Km... same using ringdown...

Massive, compact object indeed!

Why is this enough?

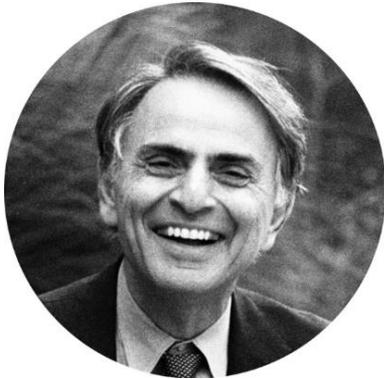
BHs are end-point of gravitational collapse, using EoS thought to prevail.

No other massive, dark object has been seen to arise from collapse of known matter.

# Quantifying the evidence for black holes

*Cardoso & Pani, Living Reviews in Relativity (2019)*

1. BH exterior is pathology-free, interior is not.
2. Quantum effects not fully understood. Non-locality to solve information paradox? Is BH just a fuzzball? BH area quantization? (Mathur 2005; Bekenstein & Mukhanov 1995; Giddings 2017)
3. Tacitly assumed quantum effects at Planck scales. Planck scale could be significantly lower (*Arkani-Hamed+ 1998; Giddings & Thomas 2002*). Even if not, many orders of magnitude standing, surprises can hide.



*“Extraordinary claims require extraordinary evidence.”*  
Carl Sagan

4. *Dark matter exists, and interacts gravitationally. Are there compact DM clumps?*
5. Physics is experimental science. We can test exterior. Aim to quantify evidence for horizons. Similar to quantifying equivalence principle.

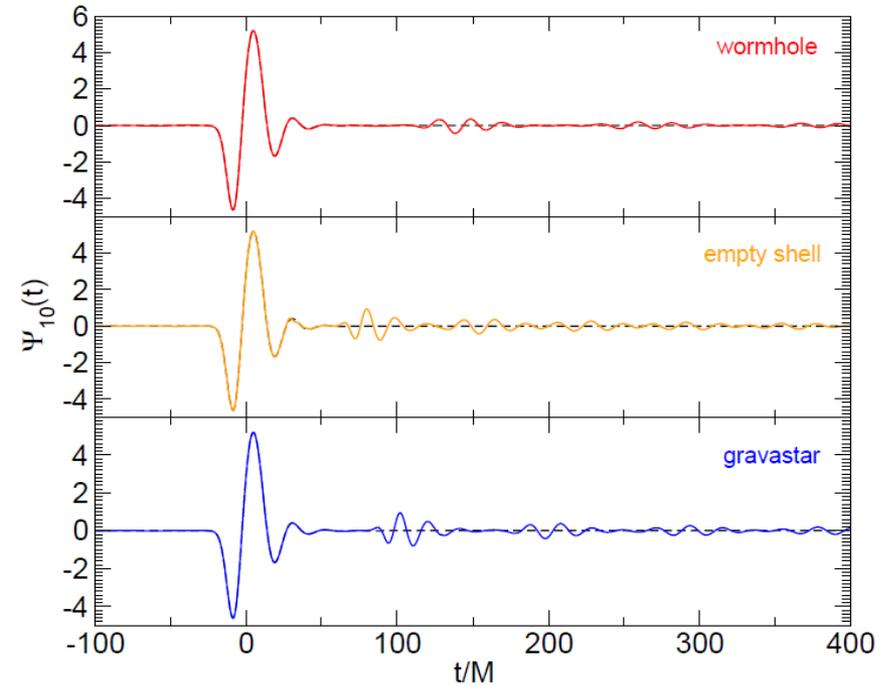
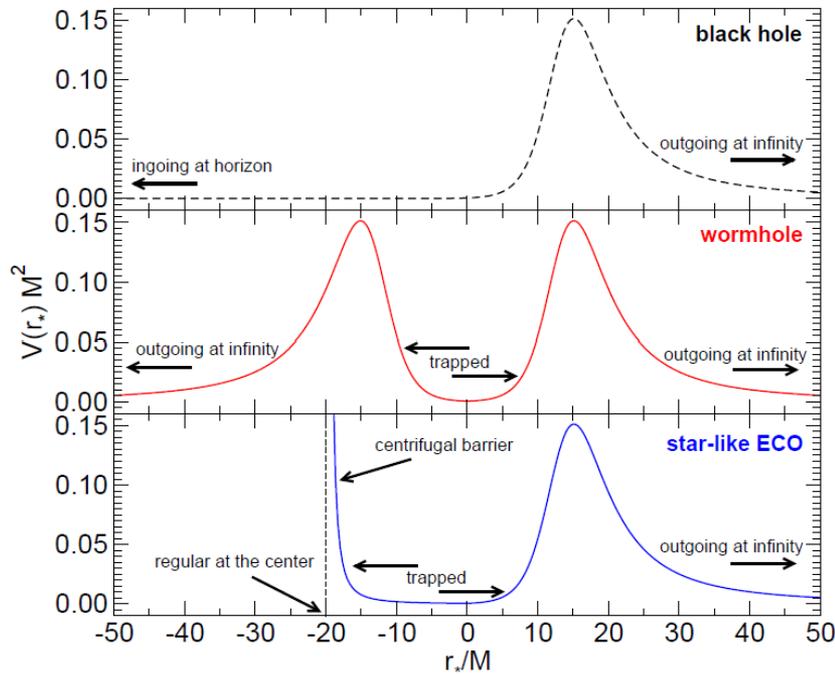
# Some challenges

*Cardoso & Pani, Living Reviews in Relativity 22: 1 (2019)*

- i. Well-posed alternatives yielding ultracompact solutions?
- ii. Formation mechanism for alternatives?
- iii. Are these BH mimickers dynamically stable? Timescales?
- iv. How do they look like? Is GW or EM signal similar to BHs?
- v. Observationally, how close do we get to horizons?

# Post-merger: echoes

more than just w-modes



Cardoso + PRL116:171101 (2016); Cardoso and Pani, Nature Astronomy 1: 2017  
Cardoso and Pani, Living Reviews in Relativity 22:1 (2019)

Searches for echoes were conducted by the LIGO/Virgo Collaboration arXiv:2010:14529

# The evidence for black holes

Cardoso and Pani, Living Reviews in Relativity 22:1 (2019)

	Constraints		Source
	$\epsilon(\lesssim)$	$\frac{\nu}{\nu_\infty}(\gtrsim)$	
1.	$\mathcal{O}(1)$	1.4	Sgr A* & M87
2.	$\mathcal{O}(0.01)$	10	GW140915
3.	$10^{-4.4}$	158	All with $M > 10^{7.5} M_\odot$
4.	$10^{-14}$	$10^7$	Sgr A*
5.	$10^{-40}$	$10^{20}$	All with $M < 100 M_\odot$
	Effect and caveats		
1.	Uses detected structure in “shadow” of SgrA and M87. Spin effects are poorly understood; systematic uncertainties not quantified.		
2.	Uses same ringdown as BH and lack of echoes. ?		
3.	Lack of optical/UV transients from tidal disruption events. Assumes: all objects are horizonless, have a hard surface, spherical symmetry, and isotropy.		
4.	Uses absence of relative low luminosity from Sgr A*, compared to disk. Spin effects and interaction of radiation with matter poorly understood; assumes spherical symmetry.		
5.	Uses absence of GW stochastic background (from ergoregion instability). Assumes: hard surface (perfect reflection); exterior Kerr; all objects are horizonless.		

# Conclusions: exciting times!

Gravitational wave astronomy *will* become a precision discipline, mapping compact objects throughout the entire visible universe.

Black holes remain the most outstanding object in the universe. BH spectroscopy will allow to test GR and provide strong evidence for the presence of horizons... improved sensitivity pushes putative surface closer to horizon, like probing short-distance structure with accelerators. BHs can play the role of perfect laboratories for particle physics, or high energy physics.



“But a confirmation of the metric of the Kerr spacetime (or some aspect of it) cannot even be contemplated in the foreseeable future.”

*S. Chandrasekhar, The Karl Schwarzschild Lecture, Astronomischen Gesellschaft, Hamburg, 18 Sept. 1986*

**Thank you**



# Questions

Realistic inspiral, with galactic potential, DM halo, etc?

Kicks due to DM, accretion disks, etc?

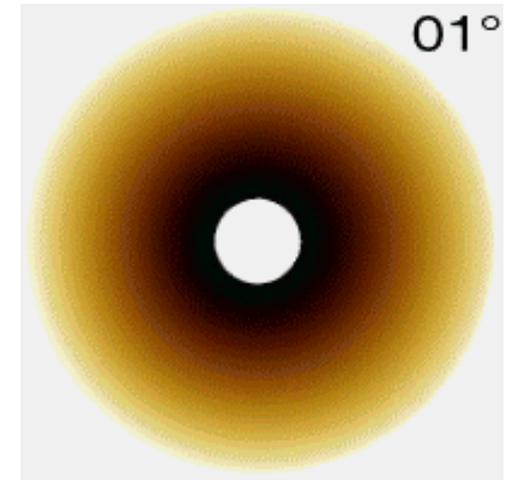
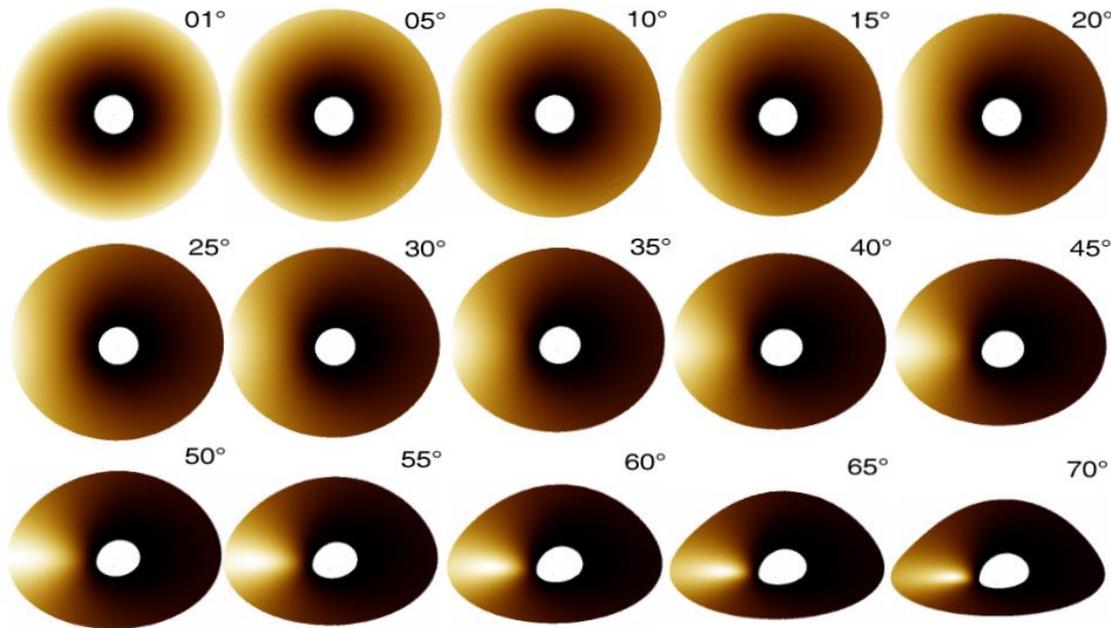
Extremely relativistic systems:  $c^5/G$ ...Dyson bound?

Collisions at exactly the speed of light?

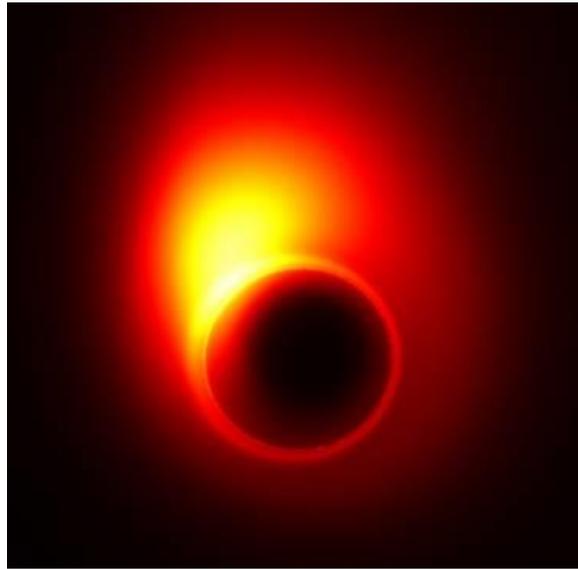
Are these calculations generalizable to other theories?

What is the final characteristic ringing? Imprint of horizons?

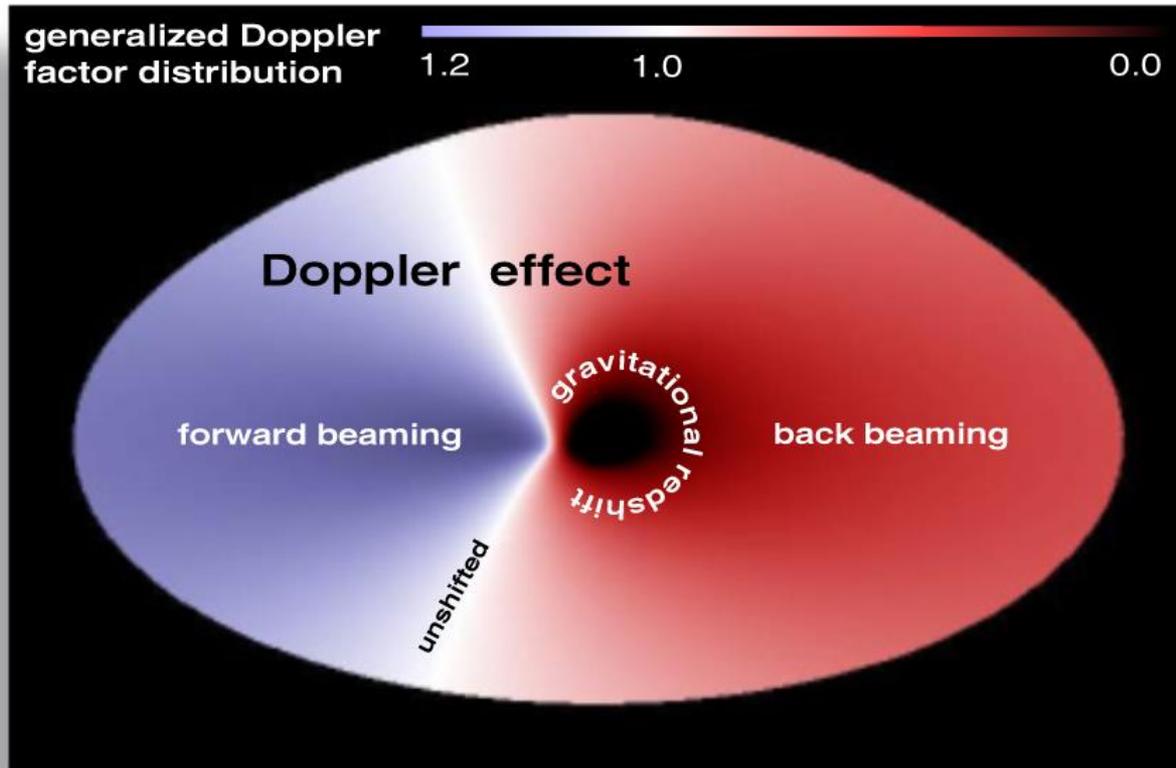
# black hole shadow



müller 2002, [http://www.lsw.uni-heidelberg.de/users/amueller/astro\\_sl.html#kbhrt](http://www.lsw.uni-heidelberg.de/users/amueller/astro_sl.html#kbhrt)



# black hole - render disk images



$$i = 60^\circ$$

$$a = 0.99$$

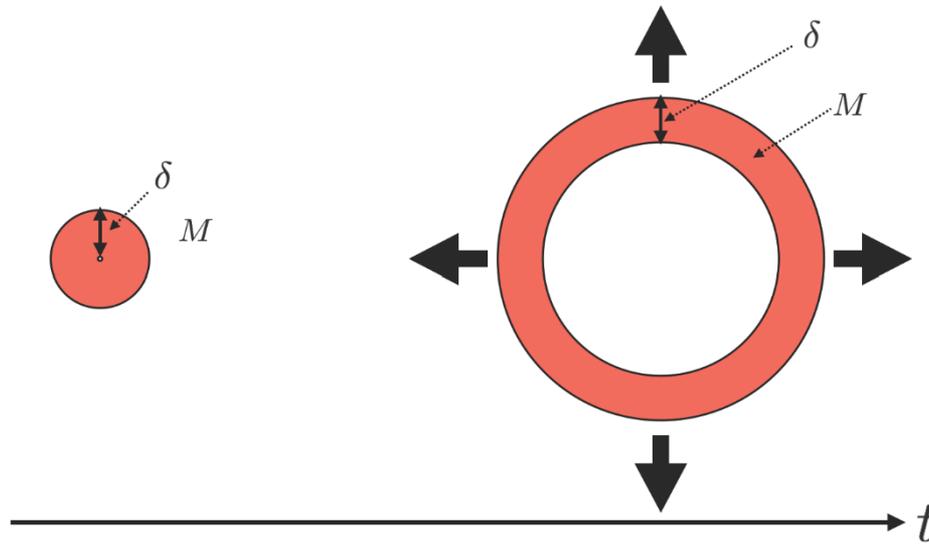
$$r_{\text{in}} = r_{\text{H}}$$

$$r_{\text{out}} = 30.0$$

keplerian  
kinematics

Classical Doppler effect, relativistic beaming and gravitational redshift effects influence any emission in black hole systems!

# On the maximum luminosity



$$\text{Luminosity} = \frac{Mc^2}{\delta/c} < \frac{c^5}{G}$$

K. Thorne, Gravitational Radiation (North-Holland 1983)

F. J. Dyson, Interstellar Communication (Benjamin, NY, 1963)

Gibbons & Barrow MNRAS 446:3874 (2015);

Cardoso+ PRD97:084013 (2018)

# On the maximum luminosity

Event	Peak luminosity
Three Gorges dam	$3 \times 10^{-43} \mathcal{L}_P$
Most powerful laser	$3 \times 10^{-38} \mathcal{L}_P$
Tsar Bomba	$3 \times 10^{-27} \mathcal{L}_P$
Solar luminosity	$1 \times 10^{-30} \mathcal{L}_P$
$\gamma$ -ray bursts	$1 \times 10^{-5} \mathcal{L}_P$
Inspiralling BHs	$2 \times 10^{-3} \mathcal{L}_P$
High-energy BH collision	$2 \times 10^{-2} \mathcal{L}_P$
Critical collapse	$2 \times 10^{-1} \mathcal{L}_P$
End point of BH evaporation	$\mathcal{L}_P$

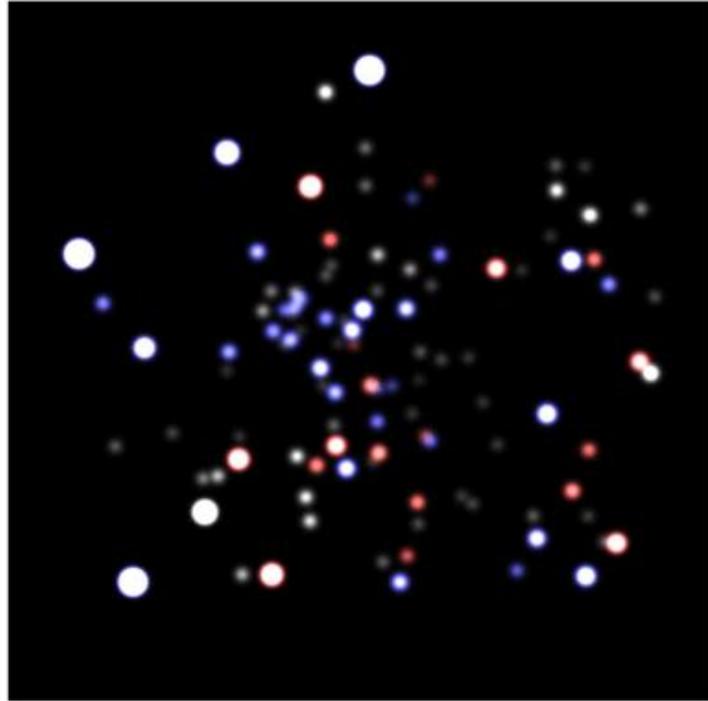
$$\mathcal{L}_P = \frac{c^5}{G} = 3.6 \times 10^{52} \text{ W}$$

*Conjecture:*

Maximum possible luminosity in past-regular spacetime is Planck luminosity  $\mathcal{L}_P$

K. Thorne, Gravitational Radiation (North-Holland 1983); Cardoso+ PRD97:084013 (2018)

# Black holes exist



*Credit: ESO/MPE (2010)*