A brief introduction to the AdS/CFT conjecture

Jorge Rocha (CENTRA–IST)
AdS/CFT: what is it about?

AdS = anti-de Sitter spacetime
(maximally symmetric solution of the Einstein equations with a negative cosmological constant)

CFT = conformal field theory
(relativistic quantum field theory invariant under scaling transformations)
AdS/CFT: what is it about?

- It is a conjectured mapping between two (apparently) completely different theories:

Some gravitational theories in $d$-dimensional Anti-de Sitter ($\text{AdS}_d$) spacetime

Some conformal field theories (CFT) in $d-1$ dimensions

- Holographic in nature (‘CFT lives at the boundary of AdS’)

- It is a powerful correspondence (strong-weak duality).

- Can extend to situations ≠ CFTs (‘gauge/gravity correspondence’).
Vast literature

- Polchinski’s books “String Theory” (1998)
- Aharony, Gubser, Maldacena, Ooguri & Oz (AGMOO’s) review “Large N field theories, string theory and gravity”, hep-th/9905111
- D’Hoker & Freedman’s TASI lectures “Supersymmetric gauge theories and the AdS/CFT correspondence”, hep-th/0201253
Impact of AdS/CFT on High Energy Physics

Welcome to INSPIRE! INSPIRE is now in full operation and supersedes SPIRES. Please direct questions, comments or concerns to feedback@inspirehep.net.

HEP :: HEPNames :: Institutions :: Conferences :: Jobs :: Experiments :: Help

cited:50-->30000 year:1900-->2012
find /"Phys.Rev.Lett.105/" :: more

Sort by:
latest first desc. times cited 25 results single list

Display results:

HEP 55,162 records found 1 - 25 jump to record: 1

1. The Large N limit of superconformal field theories and supergravity.
   HUTP-98-A097
   e-Print: hep-th/9711200 [hep-th] PDF
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   ATMP Server: Mathematical Reviews
   Detailed record - Cited by 8482 records

2. A Model of Leptons.
   Published in Phys.Rev.Lett. 19 (1967) 1264-1266
   DOI: 10.1103/PhysRevLett.19.1264
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Detailed record - Cited by 7845 records

3. CP Violation in the Renormalizable Theory of Weak Interaction.
   Published in Prog.Theor.Phys. 49 (1973) 652-657
   KUNS-242
   DOI: 10.1143/PTP.49.652
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Detailed record - Cited by 6796 records

4. First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Determination of cosmological parameters.
   Published in Astrophys.J.Suppl. 148 (2003) 175-194
   DOI: 10.1086/377226
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Journal Server: ADS Abstract Service; pdgLive (measurements quoted by PDG)
   Detailed record - Cited by 6557 records
5. Measurements of Omega and Lambda from 42 high redshift supernovae.
   LBNL-41801, LBL-41801
   DOI: 10.1086/307221
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Journal Server; ADS Abstract Service; CERN Library Record; OSTI Information Bridge Server

   Detailed record - Cited by 6436 records

6. Observational evidence from supernovae for an accelerating universe and a cosmological constant.
   Published in Astron.J. 116 (1998) 1009-1038
   DOI: 10.1086/300499
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Journal Server; ADS Abstract Service; Astron.J. Server; CERN Library Record

   Detailed record - Cited by 6183 records

   Published in Astrophys.J. 500 (1998) 525
   DOI: 10.1086/305772
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Journal Server; ADS Abstract Service

   Detailed record - Cited by 6166 records

8. Anti-de Sitter space and holography.
   IASSNS-HEP-98-15
   IASSNS-HEP-98-15
   e-Print: hep-th/9802150 [hep-th] PDF
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   ATMP Server; CERN Library Record; Mathematical Reviews

   Detailed record - Cited by 5695 records

9. A Large mass hierarchy from a small extra dimension.
   Published in Phys.Rev.Lett. 83 (1999) 3370-3373
   MIT-CTP-2860, PUPT-1860, BUHEP-99-9
   DOI: 10.1103/PhysRevLett.83.3370
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Journal Server; ADS Abstract Service; CERN Library Record; Phys. Rev. Lett. Server

   Detailed record - Cited by 5456 records

10. Wilkinson Microwave Anisotropy Probe (WMAP) three year results: implications for cosmology.
    Published in Astrophys.J.Suppl. 170 (2007) 377
    DOI: 10.1086/513700
    References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
    Journal Server; ADS Abstract Service; pdgLive (measurements quoted by PDG)
Impact of AdS/CFT on High Energy Physics

11. **Gauge theory correlators from noncritical string theory.**
   DOI: 10.1016/S0370-2693(98)00377-3
   e-Print: hep-th/9802109 [hep-th]
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Journal Server: ADS Abstract Service; CERN Library Record; Mathematical Reviews
   Detailed record - Cited by 5009 records

12. **Review of Particle Physics.**
   DOI: 10.1016/j.physletb.2008.07.018
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Journal Server: Particle Data Group (PDG) Server; pdgLive (measurements quoted by PDG)
   Detailed record - Cited by 4888 records

13. **Review of Particle Physics.**
   Particle Data Group Collaboration (W.M. Yao (LBL, Berkeley) et al.). 2006.
   DOI: 10.1088/0954-3899/33/1/001
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Detailed record - Cited by 4677 records

   DOI: 10.1016/j.physletb.2004.06.001
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Journal Server: pdgLive (measurements quoted by PDG)
   Detailed record - Cited by 4644 records

15. **An Alternative to compactification.**
   Published in Phys.Rev.Lett. 83 (1999) 4690-4693
   MIT-CTP-2874, PUPT-1867, BUHEP-99-13
   DOI: 10.1103/PhysRevLett.83.4690
   e-Print: hep-th/9906064 [hep-th]
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Detailed record - Cited by 4594 records

16. **The Hierarchy problem and new dimensions at a millimeter.**
   SLAC-PUB-7769, SU-ITP-98-13
   DOI: 10.1016/S0370-2693(98)00466-3
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Journal Server: ADS Abstract Service; CERN Library Record; SLAC Document Server
   Detailed record - Cited by 4573 records
Impact of AdS/CFT on High Energy Physics

17. Partial Symmetries of Weak Interactions.
   Published in Nucl.Phys. 22 (1961) 579-588
   DOI: 10.1016/0029-5582(61)90469-2

   Published in Nucl.Phys. B126 (1977) 298
   LPTENS-77-6
   DOI: 10.1016/0550-3213(77)90384-4

19. Particle Creation by Black Holes.
   DOI: 10.1007/BF02345020

20. Weak Interactions with Lepton-Hadron Symmetry.
   Published in Phys.Rev. D2 (1970) 1285-1292
   DOI: 10.1103/PhysRevD.2.1285

21. The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems.
   SLAC-PUB-2576
   DOI: 10.1103/PhysRevD.23.347

   (4028) Particle Data Group Collaboration (Kaoru Hagiwara et al.). 2002.
   DOI: 10.1103/PhysRevD.66.010001
AdS/CFT: not so crazy...

- A quantum theory of gravity should contain a spin-2 massless particle (‘graviton’). The Weinberg-Witten theorem forbids such particles in QFTs with usual stress tensors.
  - The graviton and the CFT should live on different spaces.

- The Holographic Principle implies that the quantum theory of gravity must have a number of degrees of freedom which scales like the area, not the volume.
  - The quantum gravity theory should live in one more dimension than the CFT.

- It is convenient to slice up QFTs according to energy scale $u$. The Renormalization Group Equations tell us how coupling constants depend on $u$.
  - The extra dimension should be identified with energy scale.
**AdS = anti-de Sitter spacetime**

- In vacuum: \( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} = 0 \)

- Maximally symmetric constant curvature: \( R = \frac{2d}{d-2} \Lambda \)

- Global parametrization: \( ds^2 = -R^2 \cosh^2 \left( \frac{\rho}{R} \right) dt^2 + d\rho^2 + R^2 \sinh^2 \left( \frac{\rho}{R} \right) d\Omega_{d-2}^2 \)

- Boundary at \( \rho = \infty \) with topology \( \mathbb{R} \times S^{d-2} \).

- Another parametrization (not global): \( ds^2 = \frac{R^2}{r^2} dr^2 + \frac{r^2}{R^2} (-dt^2 + d\vec{x}^2) \)
Minkowski is the maximally symmetric solution of Einstein eqs with $\Lambda = 0$:

$$ds^2 = -dt^2 + d\vec{x}^2 = -dt^2 + dr^2 + r^2 d\Omega^2_{d-2}$$

- Minkowski is globally hyperbolic but AdS is not.
- Data must be specified on the boundary of AdS.
Quantum field theory (QFT) is the framework for constructing quantum mechanical models of systems classically represented by an infinite number of degrees of freedom (fields).

Example: the standard model of particle physics.

Unitary scale invariant QFTs are also conformally invariant.

- **CFT** = conformal field theory

**Conformal group**

- $M_{\mu \nu}$ Lorentz rotations
- $P_\mu$ Translations
- $D$ Dilatations
- $K_\mu$ Special conformal transformations
CFT local operators and correlators

\[
\text{conformal group} \quad \begin{cases} 
M_{\mu \nu} & \text{Lorentz rotations} \\
P_{\mu} & \text{Translations} \\
D & \text{Dilatations} \\
K_{\mu} & \text{Special conformal transformations}
\end{cases}
\]

Generators satisfy an algebra which is \( \text{SO}(d,2) \supset \text{SO}(d) \times \text{SO}(2) \).

Classify local operators by their transformation properties under \( \text{SO}(d) \times \text{SO}(2) \).

\[
O_{\Delta}(\lambda x^\mu) = \lambda^{-\Delta} O_{\Delta}(x^\mu)
\]

Correlation functions are highly constrained by conformal symmetry:

\[
\left\langle O_{\Delta_1}(x_1) O_{\Delta_2}(x_2) \right\rangle = \frac{\delta_{\Delta_1,\Delta_2}}{|x_1 - x_2|^{2\Delta_1}}
\]
Recap: string theory basics

- ST is a theory of strings, but not only!
  
  It is inhabited by other creatures and among the most important are D-branes.

- What parameters do we have in ST?

  - $\alpha'$ controls stringy effects. String tension is $T = \frac{1}{2\pi \alpha'}$
  
  - $g_s$ controls quantum effects. (‘string coupling constant’)
  
  - Splitting/joining of closed (open) introduces a factor of $g_c$ ($g_o$)

  $$g_o^2 \sim g_c = g_s \alpha'^2$$
Recap: D-branes

- D-branes are very special objects. They preserve half of the SUSYs' BPS states → no force between parallel branes
- Dp-branes couple to (p+1)-form potentials.
- Tension of Dp-brane: \( \tau_p \propto g_s^{-2} \alpha'^{-1-p} \)
- World-volume effective action of a single Dp-brane is (p+1)-dimensional Super Yang-Mills with gauge group U(1).

For N coincident branes there is symmetry enhancement:

\[ U(1)^N \rightarrow U(N) \]
‘Deriving’ the correspondence

○ Consider type IIB superstring theory with a stack of $N$ D3-branes.

○ Two descriptions of the same system:
Taking limits (cleverly)

- Can use $N$ as a parameter, in addition to $g_s$ and $\alpha$.

- Weak coupling limit: $g_s \to 0$, $N$ fixed. Back-reaction can be ignored and we just get 4d SYM with gauge group SU(N) ‘living on the branes’

- ‘t Hooft limit: $g_s \to 0$, $N \to \infty$, $\lambda = g_s N$ fixed. Back-reaction ($\propto g_s N$) cannot be neglected. D-branes source a classical supergravity solution.
Throat geometry

\[ ds^2 = f^{-1/2}(-dt^2 + d\tilde{x}^2) + f^{1/2}(dr^2 + r^2d\Omega_5^2), \quad f = 1 + \frac{R^4}{r^4}, \quad R^4 = 4\pi g_s N\alpha'^2 \]

For \( r >> R \) we get flat spacetime.

For \( r << R \) we get \( \text{AdS}_5 \times \text{S}^5 \) geometry:

\[ ds^2 \approx \frac{R^2}{r^2} dr^2 + \frac{r^2}{R^2}(-dt^2 + d\tilde{x}^2) + R^2 d\Omega_5^2 \]
The decoupling limit

- For low energies, $E \ll \alpha'$, only massless modes can be excited.
- 10d supergravity describing the massless modes in the bulk and in the asymptotically flat region both decouple.

4d SU(N) SYM
with $g_{YM}^2 = 2\pi g_s$

IIB string theory on $AdS_5 \times S^5$
with $R^4 = 4\pi \lambda \alpha'^2$
Strong/weak duality

\[
\frac{R^4}{\alpha'^2} = 4\pi \lambda \quad \text{and} \quad g_s = \frac{\lambda}{2\pi N}
\]

- \(O(\alpha')\) corrections \(\leftrightarrow\) \(O(\lambda^{-1/2})\) corrections
- \(O(g_s)\) corrections \(\leftrightarrow\) \(O(N^{-1})\) corrections (at fixed \(\lambda\))

This is both a blessing and a curse!
Matching of symmetries

- The $\text{AdS}_d$ metric can be obtained from the embedding of a hyperboloid in $(d+1)$-dimensional flat spacetime (‘with two time coordinates’).
  - The isometry group of $\text{AdS}_d$ is $\text{SO}(2,d-1)$

- The isometry group of $\text{S}^5$ is $\text{SO}(6)$, the rotation group.

- The isometry group of $\text{AdS}_5 \times \text{S}^5$ is $\text{SO}(2,4) \times \text{SO}(6)$.

- This precisely matches the conformal symmetry and R-symmetry of SYM
  - (rotations of the six scalar fields)
The precise statement of the correspondence is the equality of the partition functions of the two theories:

\[ \int D\phi(z, \bar{x}) e^{-S[\phi(z, \bar{x})]} \equiv Z_{AdS}[\bar{\phi}(\bar{x})] = Z_{CFT}[\bar{\phi}(\bar{x})] \equiv \left\langle e^{\int d^d x \bar{\phi}(\bar{x}) O(\bar{x})} \right\rangle \]

Boundary conditions in AdS are associated with sources for the field operators.

Computing correlators is straightforward:

\[ \langle O_{\Delta_1} (x_1) O_{\Delta_2} (x_2) \ldots \rangle = \frac{\partial^n Z[\phi_{\Delta_i}]}{\partial \phi_{\Delta_1} (x_1) \partial \phi_{\Delta_2} (x_2) \ldots} \bigg|_{\phi_{\Delta_i} = 0} \]
Applications: modeling QCD

- 4d SYM is conformal but QCD is not. Actually they are very different theories: QCD is asymptotically free, exhibits confinement and is not SUSY.

- It is known how to construct gravitational duals for confining theories.

Confinement/deconfinement phase transition

Hawking-Page phase transition
Applications: Heavy ion collisions

- Transport coefficients (e.g. viscosity) can be computed from correlation functions.

AdS/CFT is a great tool to study quark-gluon plasmas (strongly coupled systems) that are observed at particle accelerators.
Applications: Holographic superconductors

We have a theory (BCS, 1957) that correctly describes low $T_c$ superconductors. Weakly coupled Cooper pairs of electrons form and condense below temperature $T_c$.

But in 1986 a new class of high $T_c$ superconductors was discovered! These are not well understood: the electron pairing mechanism is strong coupling.

AdS/CFT is appropriate to address this problem and holographic superconductors have been theoretically constructed with very similar qualitative properties as real world high $T_c$ superconductors.
Applications: Hydrodynamics and turbulence

The fluid-gravity correspondence shows that the Navier-Stokes equations can be obtained from the Einstein equations (in the regime of long wavelength perturbations of AdS black holes).

Can we understand turbulence?
Conclusions

- Is String Theory the correct theory of quantum gravity? It is certainly an excellent candidate (arguably the best).

- We are perhaps still far from confirming the ‘reality’ of String Theory but it has already produced an extremely important legacy:

  the AdS/CFT correspondence.