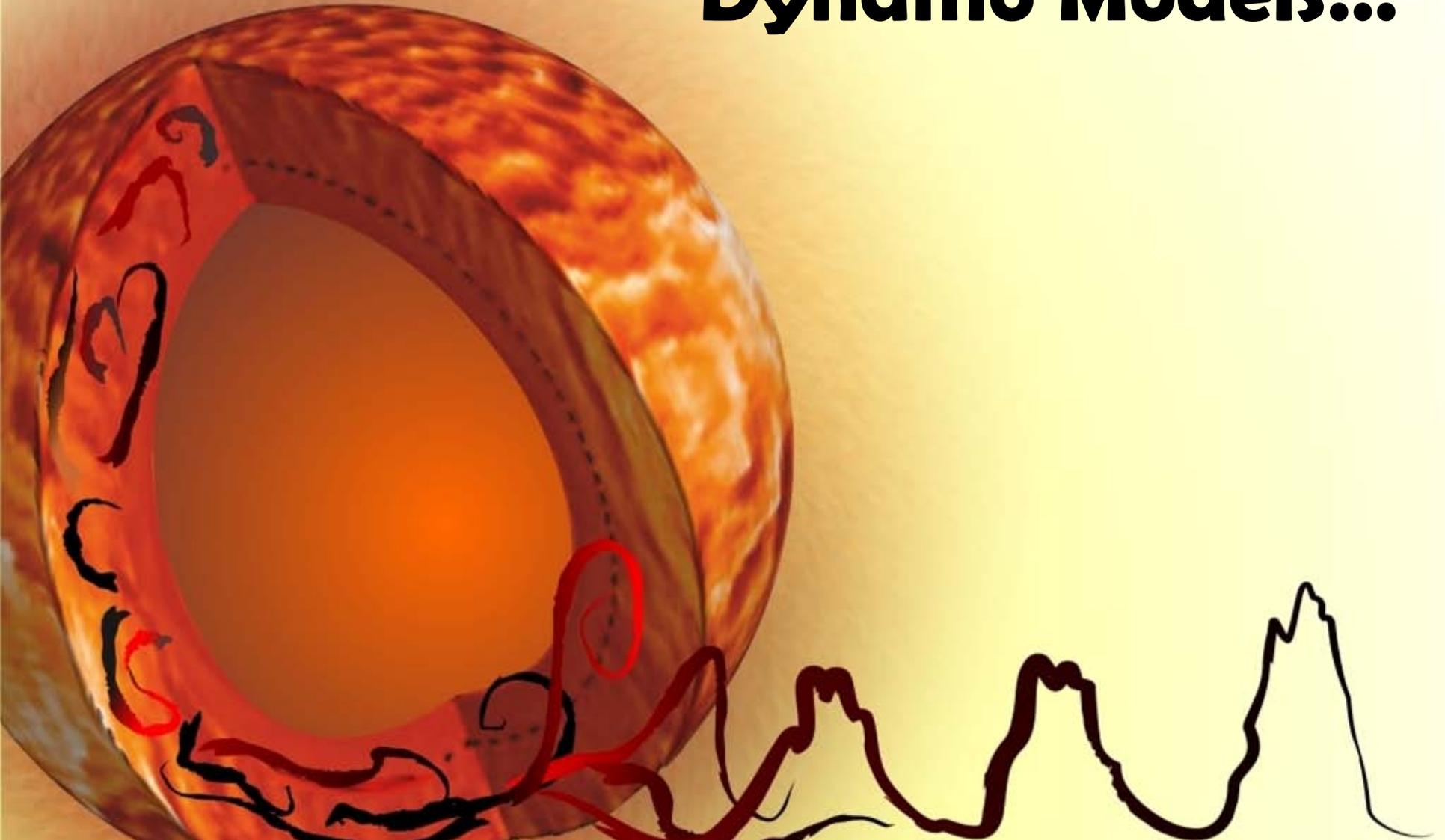


The next step for Solar Dynamo Models...

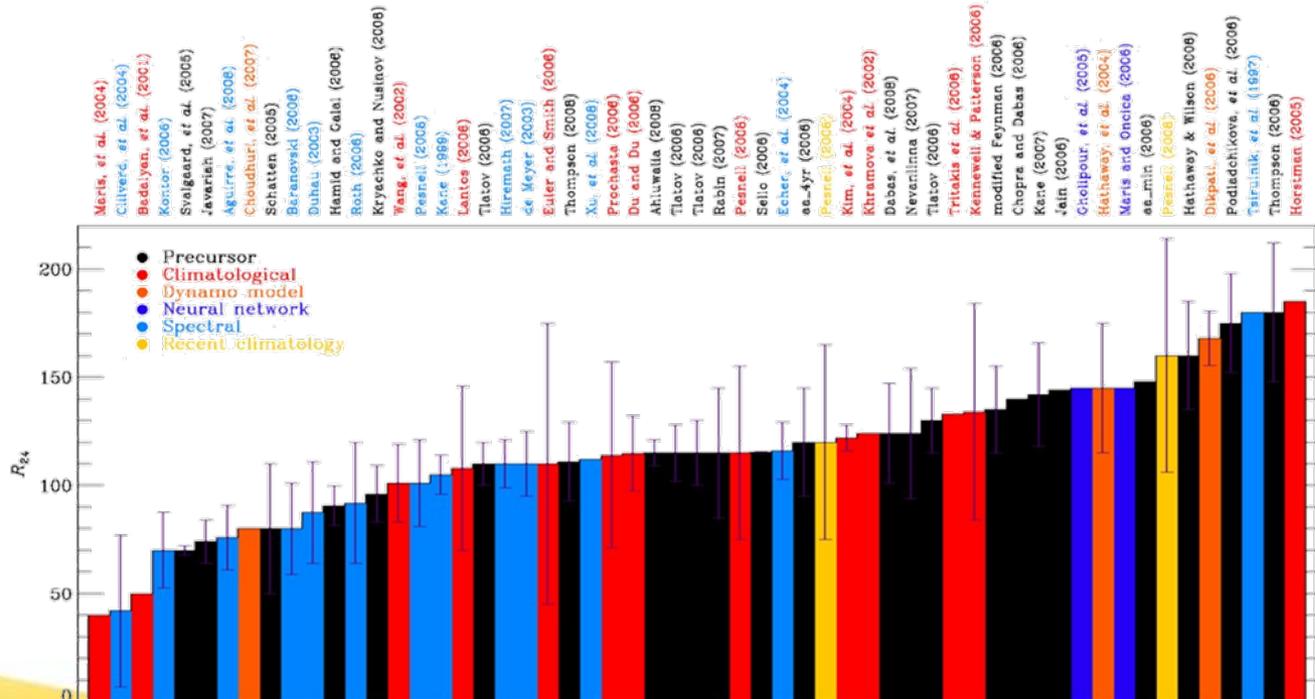


Space Weather, Solar Cycle and Predictability

- ❑ Space weather conditions many activities:
 - satellite's operation, astronaut activities, power and telecommunications networks, high latitude airplanes and space tourism (in the near future).
- ❑ Solar magnetic activity controls space weather
- ❑ Solar cycle variability is one of the most difficult characteristics to understand/predict
- ❑ Prediction techniques are divided into three categories:
 - precursors, extrapolation (statistical) and model based

Most Promising?

**Solar
Dynamo
Models**



Solar Dynamo Models

Common to all models: MHD induction equation, mean field theory (scale separation)

$$\partial \mathbf{B} / \partial t = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Axisymmetry: Large scale fields separated into poloidal and toroidal comp.

Magnetic field

$$\mathbf{B} = B_{\phi} \hat{e}_{\phi} + (\nabla \times A_p \hat{e}_{\phi})$$

Velocity field

$$\mathbf{u} = \Omega \hat{e}_{\phi} + \mathbf{v}_p$$

Small scale properties included in the coefficients of the dynamo equations

Different source terms and locations define the “dynamo zoo”

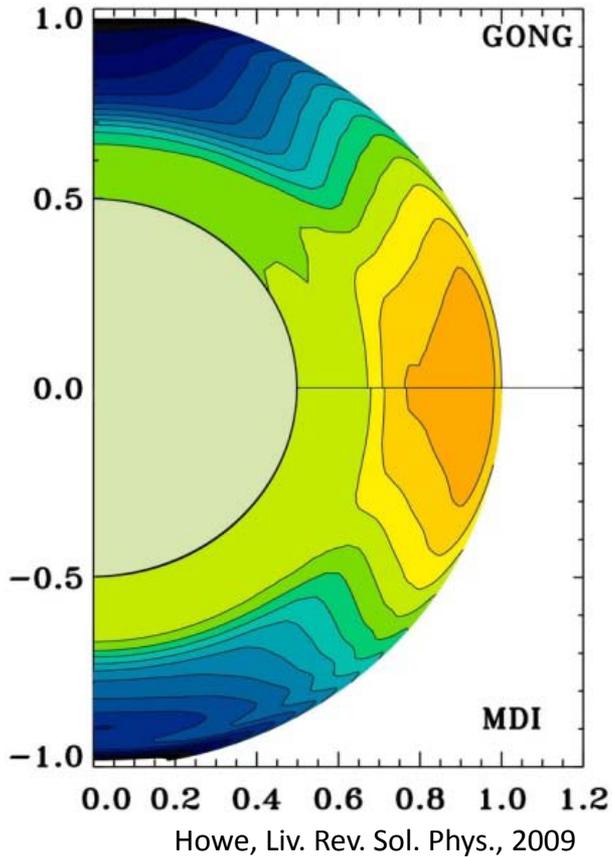
- $\alpha\Omega$ dynamos - Solar dif. rotation and Coriolis effect
- $\alpha^2\Omega$ dynamos - Solar dif. Rotation, Coriolis force and turbulence
- Flux transport dynamos - Solar dif. Rotation, meridional circulation and (usually) active regions decay

99% of these models run in the kinematic regime, i.e., it's assumed that the flows control the magnetic fields. No back reaction from the field into the flow.



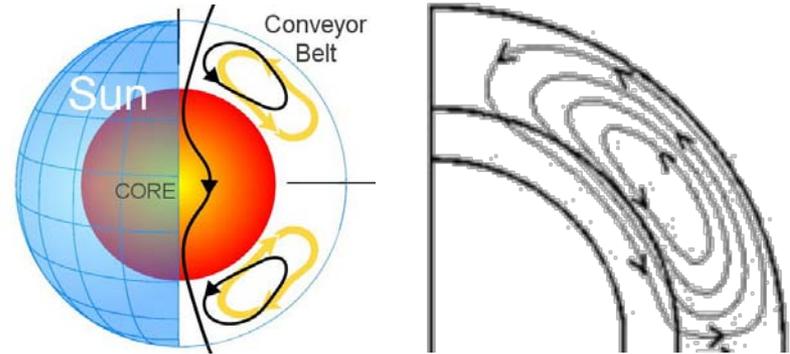
Large scale plasma velocity fields in the Sun

Differential rotation (strong flow)

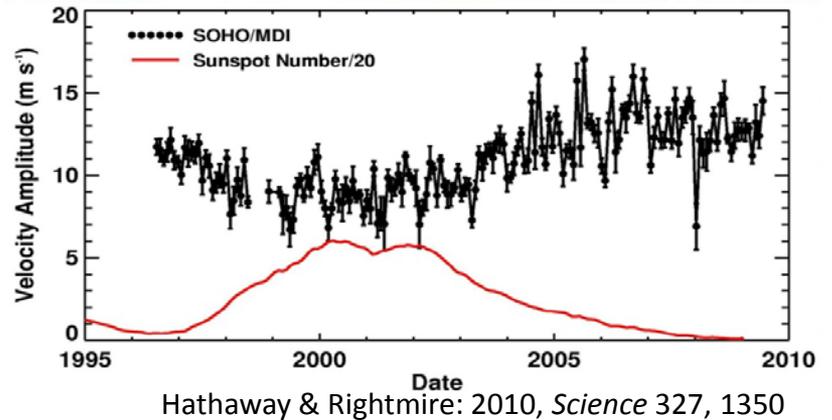


Probed by Helioseismology
for the entire convection zone

Meridional circulation (weak flow)



Theoretical and observational evidences
for MC variations from cycle to cycle



Magnetic features tracking (at the surface)

Modelling a flux transport dynamo

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \quad \text{Induction equation}$$

$$\bar{\mathbf{B}} = \mathbf{B}_\phi + \mathbf{B}_p \quad \mathbf{B}_p = \nabla \times (A_p \hat{e}_\phi) \quad \mathbf{u} = \frac{\Omega}{r \sin(\theta)} \hat{e}_\phi + v_p \quad \text{Axisymmetric approximation}$$

Evolution equations for the magnetic field

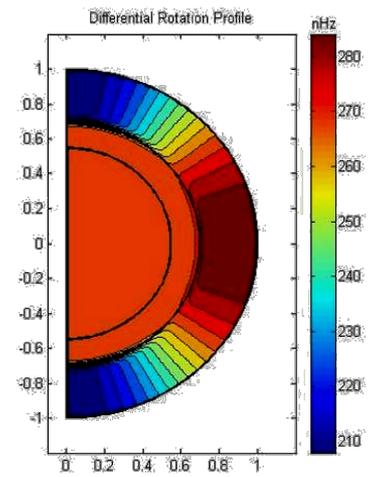
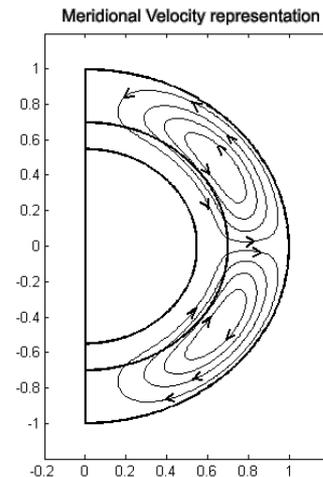
$$\frac{\partial B_\phi}{\partial t} = \eta \left(\nabla^2 - \frac{1}{\bar{r}^2} \right) B_\phi - \bar{r} v_p \cdot \nabla \left(\frac{B_\phi}{\bar{r}} \right) + \frac{1}{\bar{r}} \frac{\partial(\bar{r} B_\phi)}{\partial r} \frac{\partial \eta}{\partial r} - B_\phi \nabla \cdot \mathbf{v}_p + \bar{r} [\nabla \times (A_p \hat{e}_\phi)] \cdot \nabla \Omega$$

$$\frac{\partial A_p}{\partial t} = \eta \left(\nabla^2 - \frac{1}{\bar{r}} \right) A_p - \frac{v_p}{\bar{r}} \cdot \nabla (\bar{r} A_p) + \alpha B_\phi$$

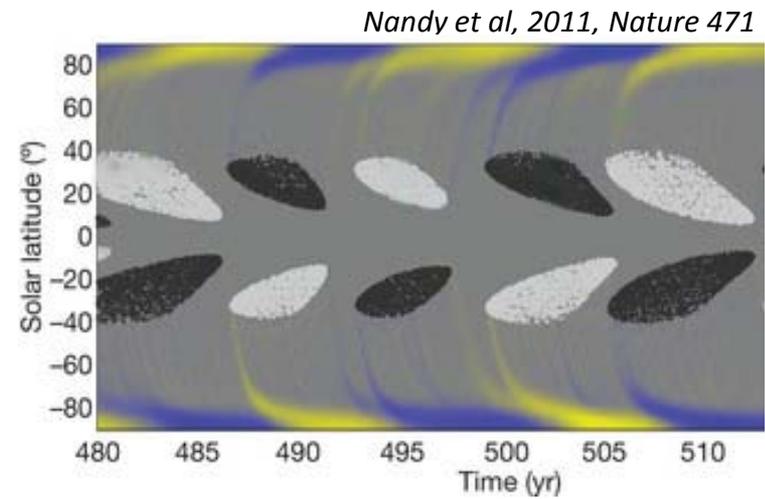
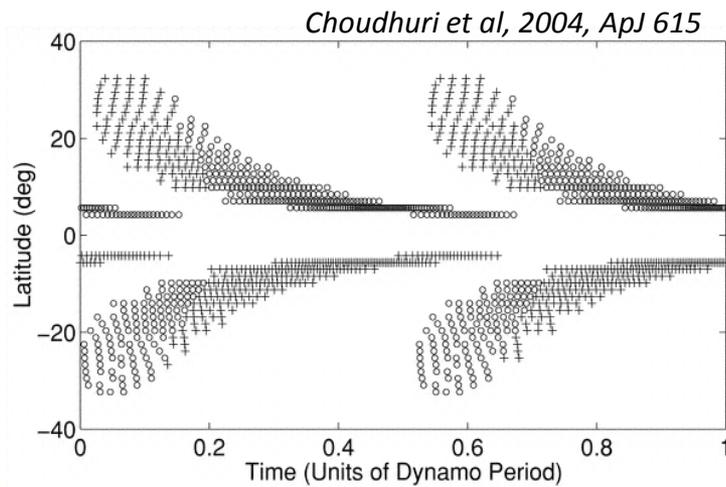
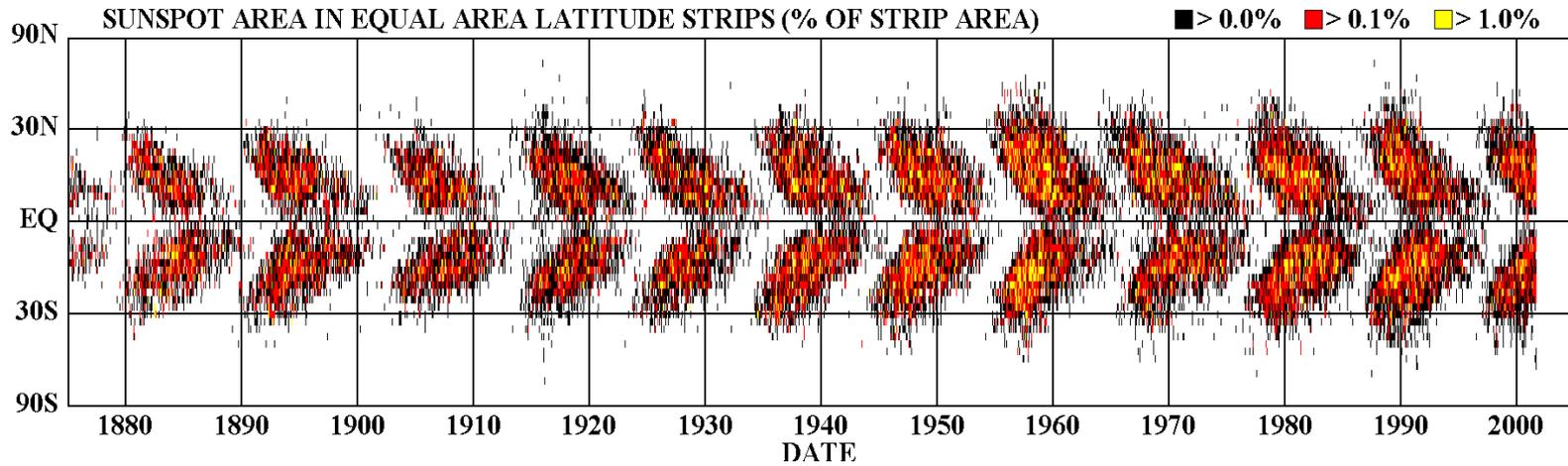
Equations solved in a
N x M meridional grid



Parameterized
velocity fields



Example results from flux transport dynamo models



Modelling: Global Large-Eddy Simulation of the solar Convection Zone

Ghizaru, Charbonneau and Smolarkiewicz, ApJL 715, 2010

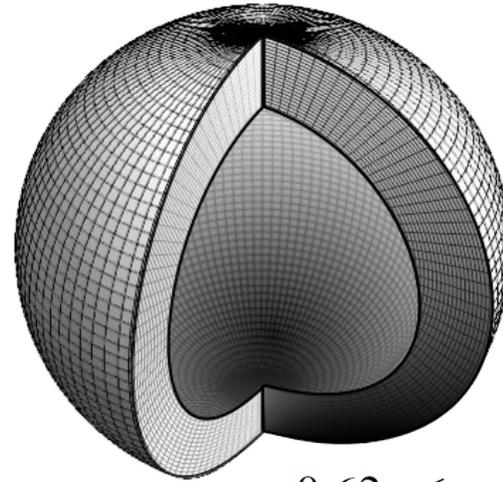
Anelastic form of the ideal MHD equations:

$$\frac{Du}{Dt} = -\nabla\pi' - \mathbf{g}\frac{\Theta'}{\Theta_o} + 2\mathbf{u} \times \boldsymbol{\Omega} + \frac{1}{\mu\rho_o} (\mathbf{B} \cdot \nabla) \mathbf{B},$$

$$\frac{D\Theta'}{Dt} = -\mathbf{u} \cdot \nabla\Theta_e + \mathcal{H} - \alpha\Theta',$$

$$\frac{D\mathbf{B}}{Dt} = (\mathbf{B} \cdot \nabla) \mathbf{u} - \mathbf{B}(\nabla \cdot \mathbf{u}).$$

$$\nabla \cdot (\rho_o \mathbf{u}) = 0, \quad \nabla \cdot \mathbf{B} = 0$$



Grid

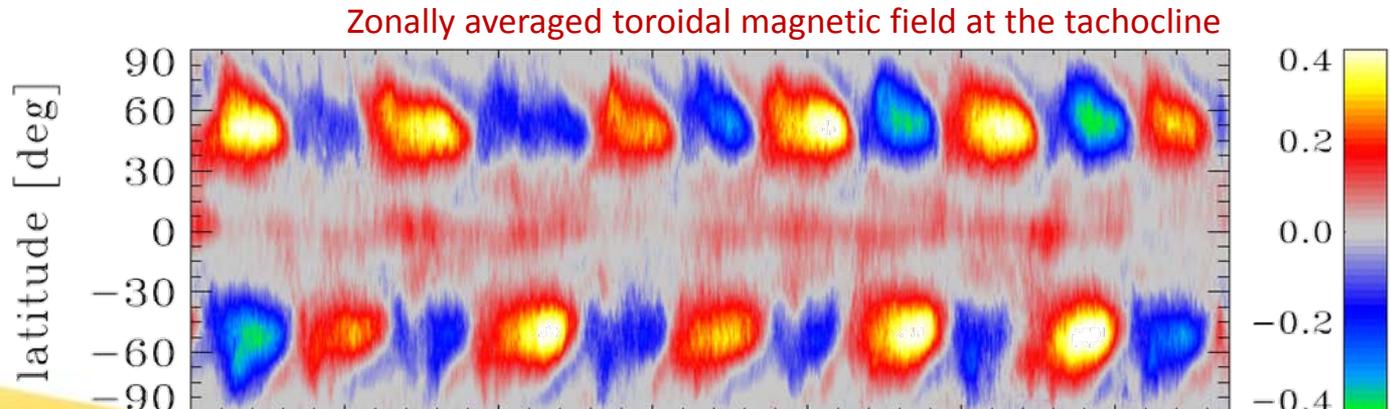
$$r = 47$$

$$\theta = 66$$

$$\phi = 128$$

$$0.62 \leq r/R_{\odot} \leq 0.96$$

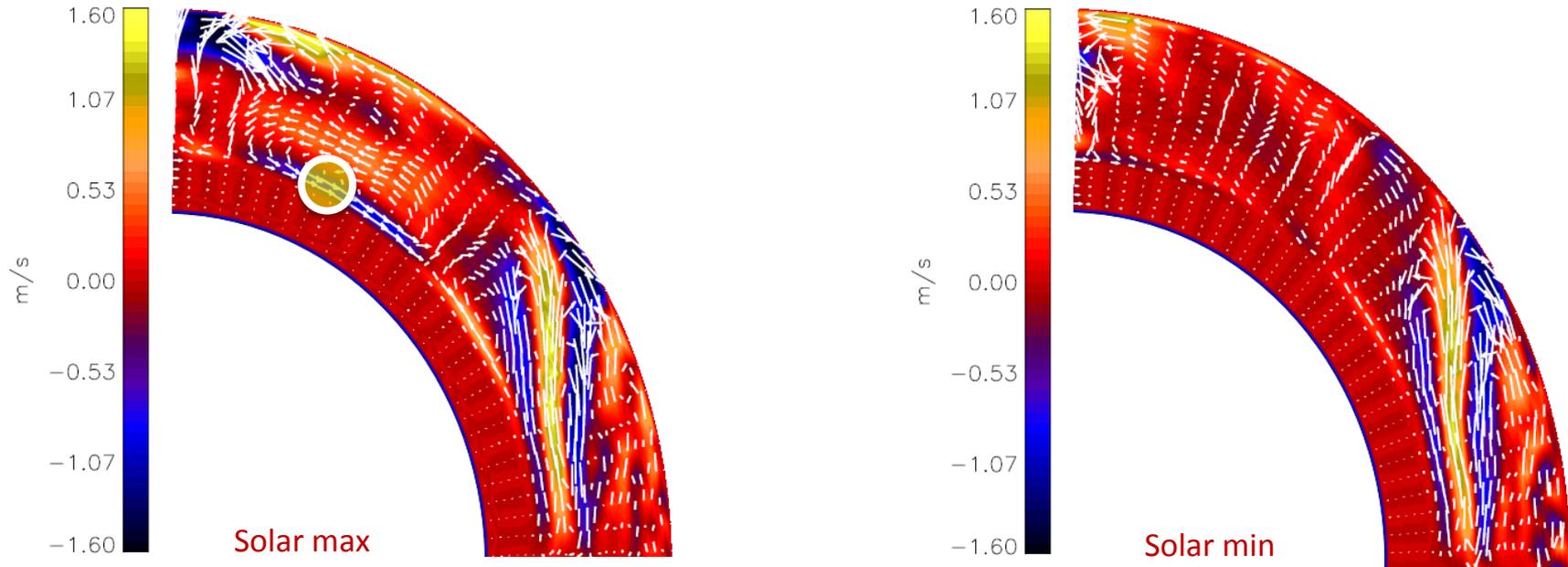
Magnetic Cycles!



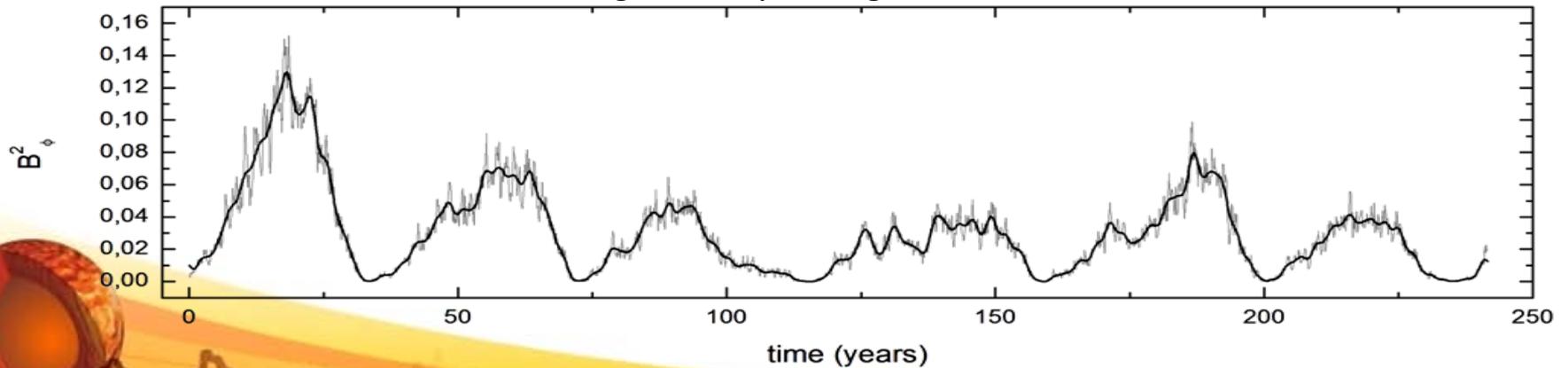
Analysis of the 3D simulation

Meridional circulation in the north hemisphere

Racine et al 2011, Apj 735

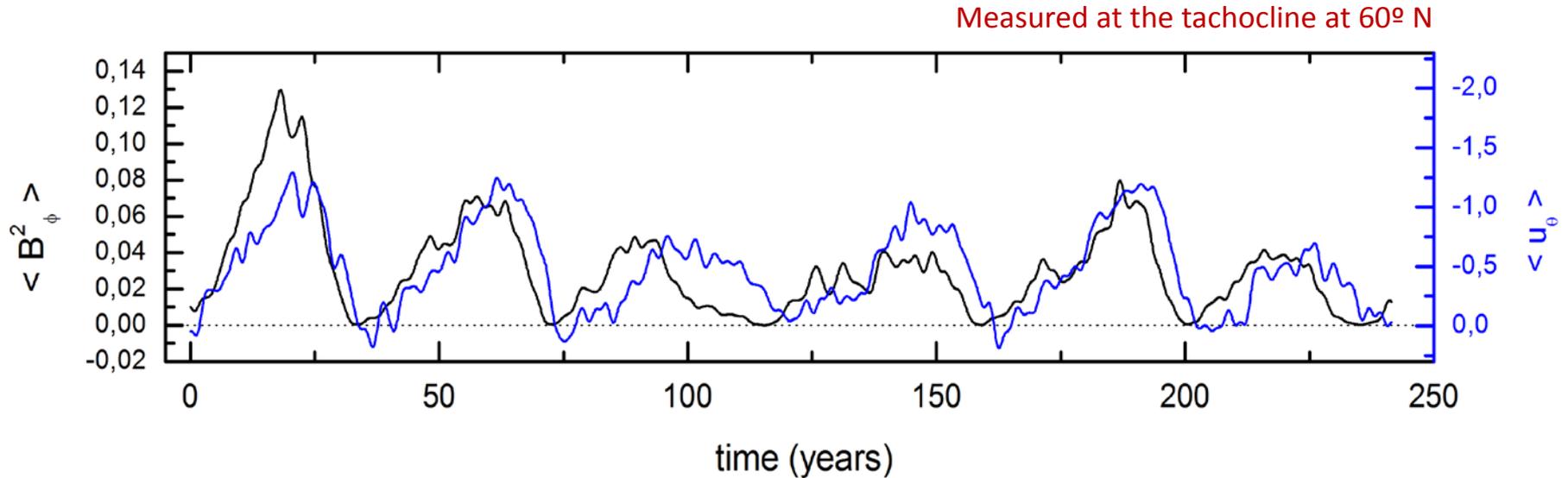


Longitudinally averaged toroidal field at the tachocline at 60° N



Analysis of the 3D simulation

Chosen quantities: *Toroidal field, B_ϕ and meridional flow u_θ at the tachocline*



- ❑ Meridional flow varies in phase with B_ϕ
- ❑ Meridional flow lags behind B_ϕ by 2~3 months

Kinematic approximation fails!

Does this has any influence in the long term evolution of the dynamo?



LODM: Low Order Dynamo Model

Equations for the magnetic field evolution

$$\frac{dB_\phi}{dt} = \left(c_1 - \frac{v_p(t)}{\ell_0} \right) B_\phi + c_2 A_p - c_3 B_\phi^3$$

$$\frac{dA_p}{dt} = \left(c_1 - \frac{v_p(t)}{\ell_0} \right) A_p + \alpha B_\phi$$

Equations for the meridional flow evolution

$$v_p(t) = v_0 + v(t)$$

$$\frac{dv(t)}{dt} = \underbrace{a B_\phi A_p}_{\text{Lorentz}} - \underbrace{b v(t)}_{\text{Drag}}$$

Structural coefficients

$$c_1 = \frac{\eta}{\ell_0^2} - \frac{\eta}{R^2} \quad \text{Mag. diffusivity}$$

$$c_2 = \frac{R\Omega}{\ell_0^2} \quad \text{Diff. rotation}$$

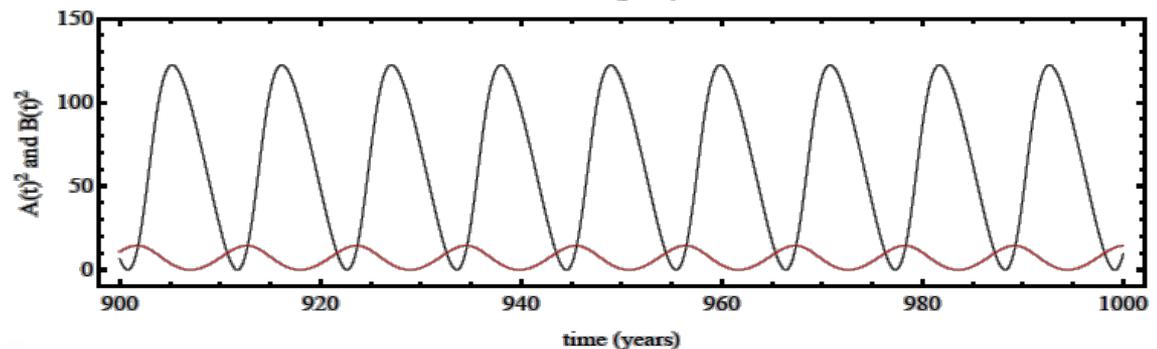
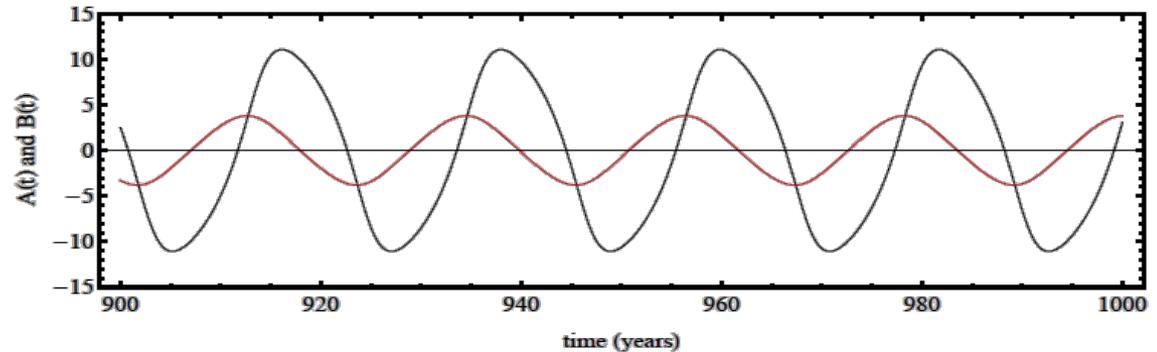
$$c_3 = \frac{\gamma}{8\pi\rho} \quad \text{Mag. buoyancy}$$

LODM main characteristics:

- Dynamo action
- Polarity change
- Phase shift between **A** and **B**
- Structural coefficients can be “calibrated” from observations

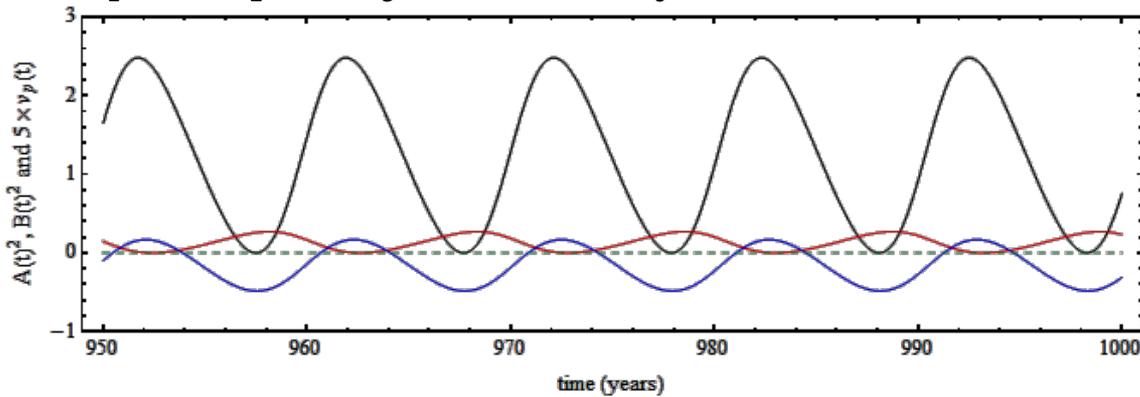
Reference solution without feedback:

$$c_1 = -0.01, c_2 = 0.95, c_3 = 0.002, \alpha = -0.1, v_0 = -0.1$$



Solution with feedback:

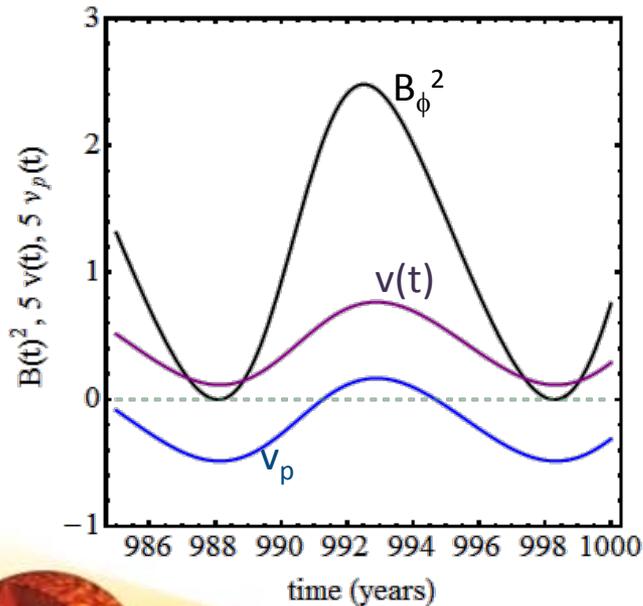
$$c_1 = -0.01, c_2 = 0.95, c_3 = 0.002, \alpha = -0.1, v_0 = -0.1, \quad a = 0.1, b = 0.05$$



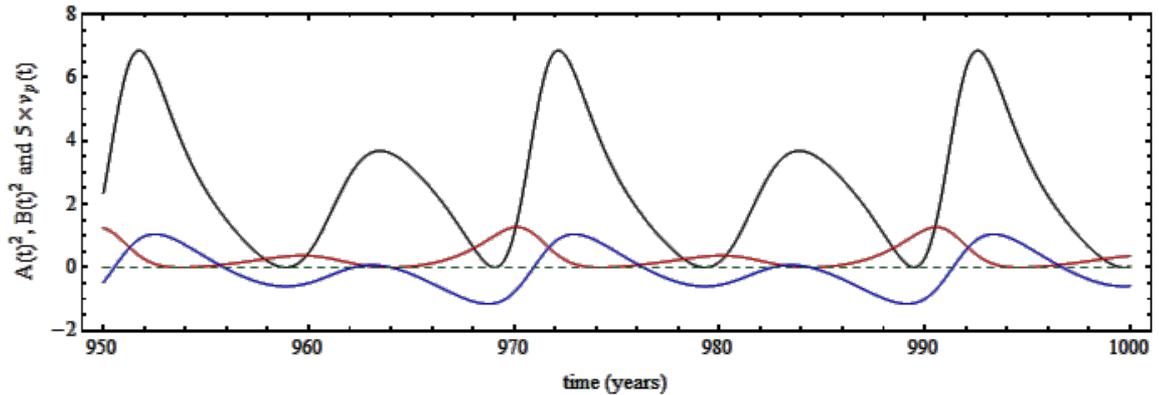
B_ϕ^2 in black
 A_p^2 in red
 v_p in blue

$$v_p = v_0 + v(t)$$

kinetic
 +
feedback

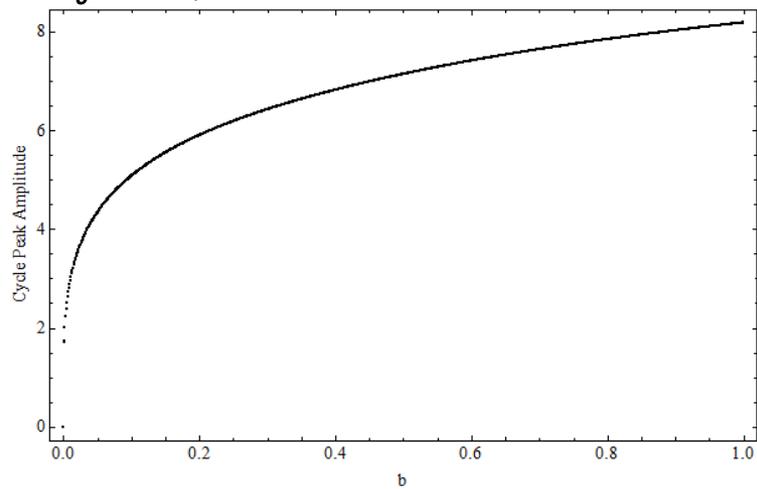


Solution with feedback:
 $c_1 = -0.01, c_2 = 0.95, c_3 = 0.002, \alpha = -0.1, v_0 = -0.1, \quad a = 0.1, b = 0.25$

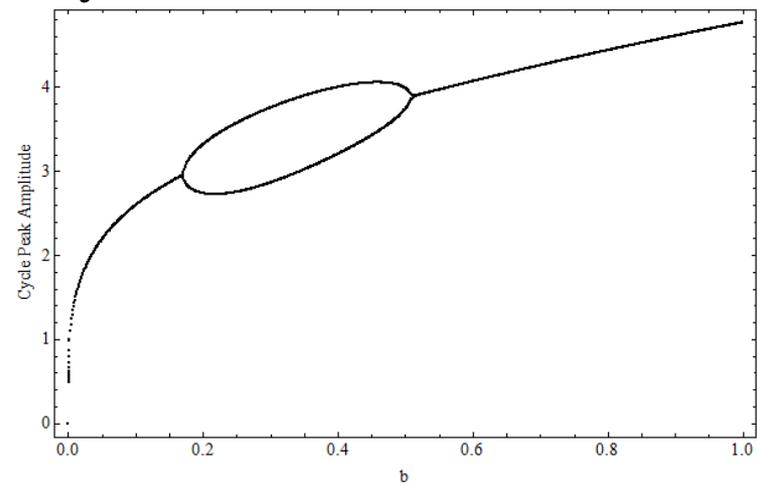


Bifurcation Maps

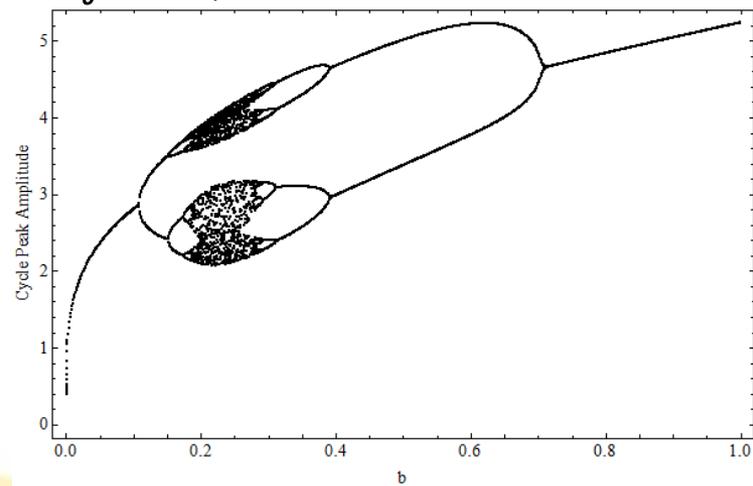
$V_0 = -0.1, a = 0.01$



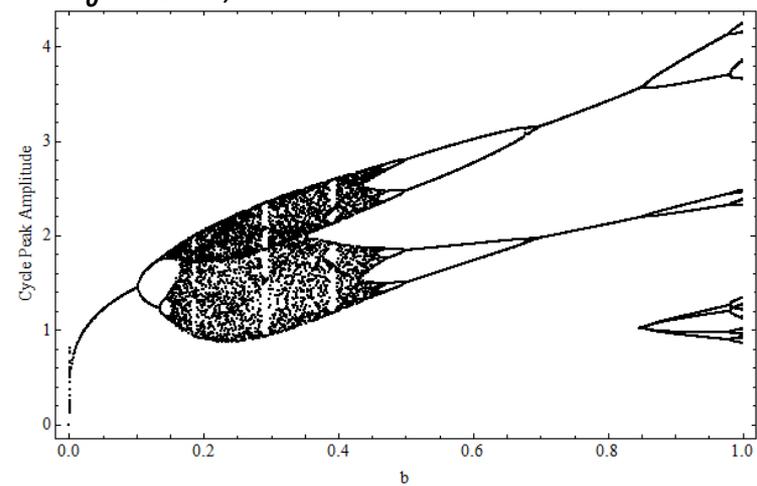
$V_0 = -0.1, a = 0.05$



$V_0 = -0.13, a = 0.05$

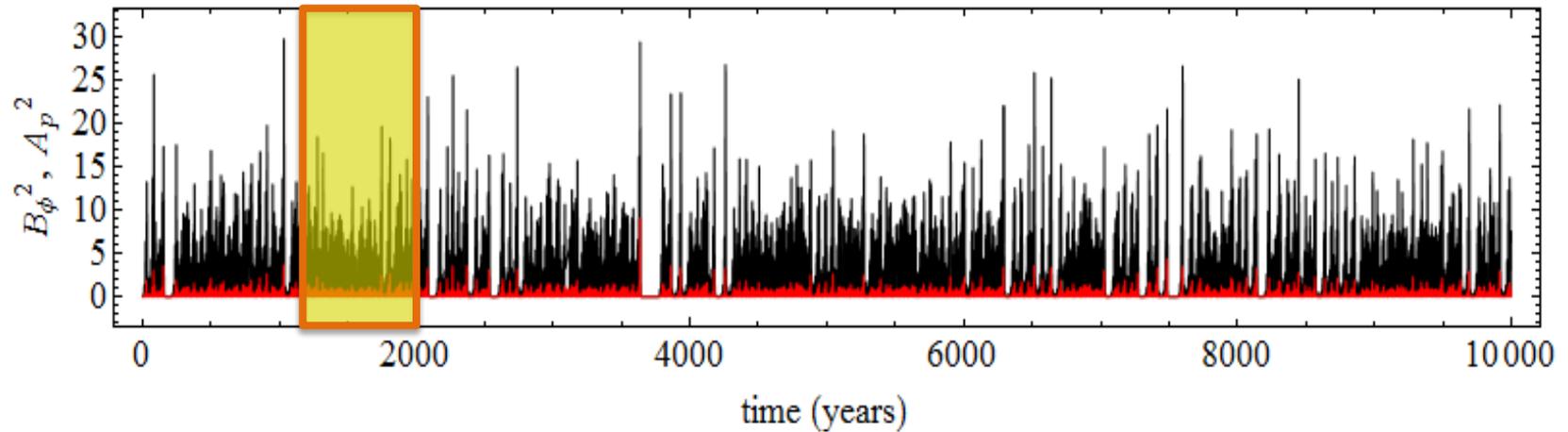


$V_0 = -0.12, a = 0.2$

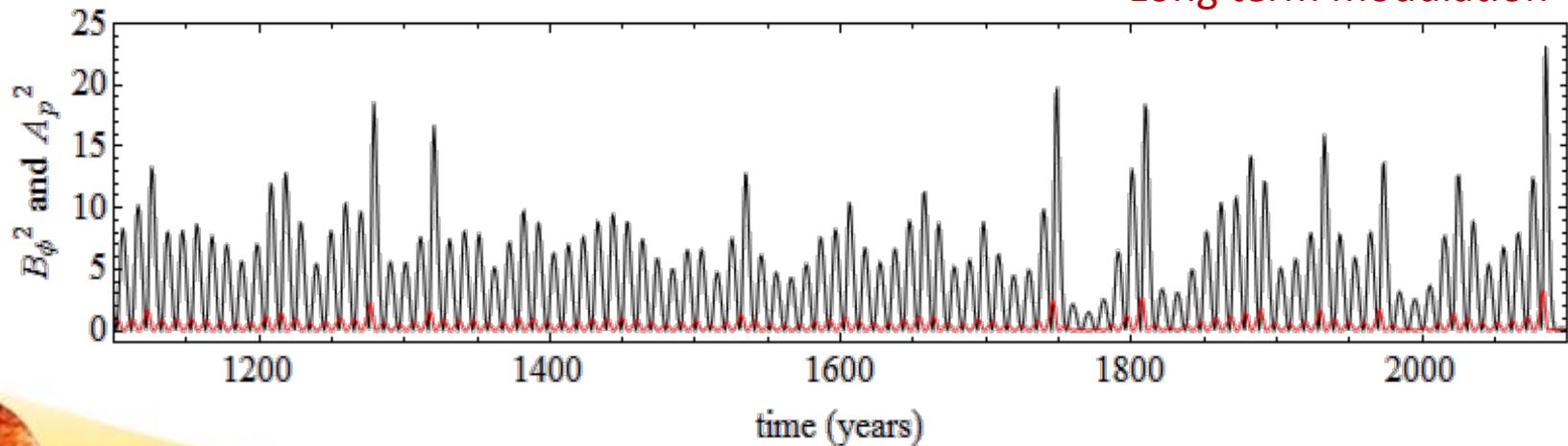


Stochastic Fluctuations

$\tau = 1$ year, $\mathbf{a} \in [0.01, 0.04]$, $\mathbf{b} = 0.05$

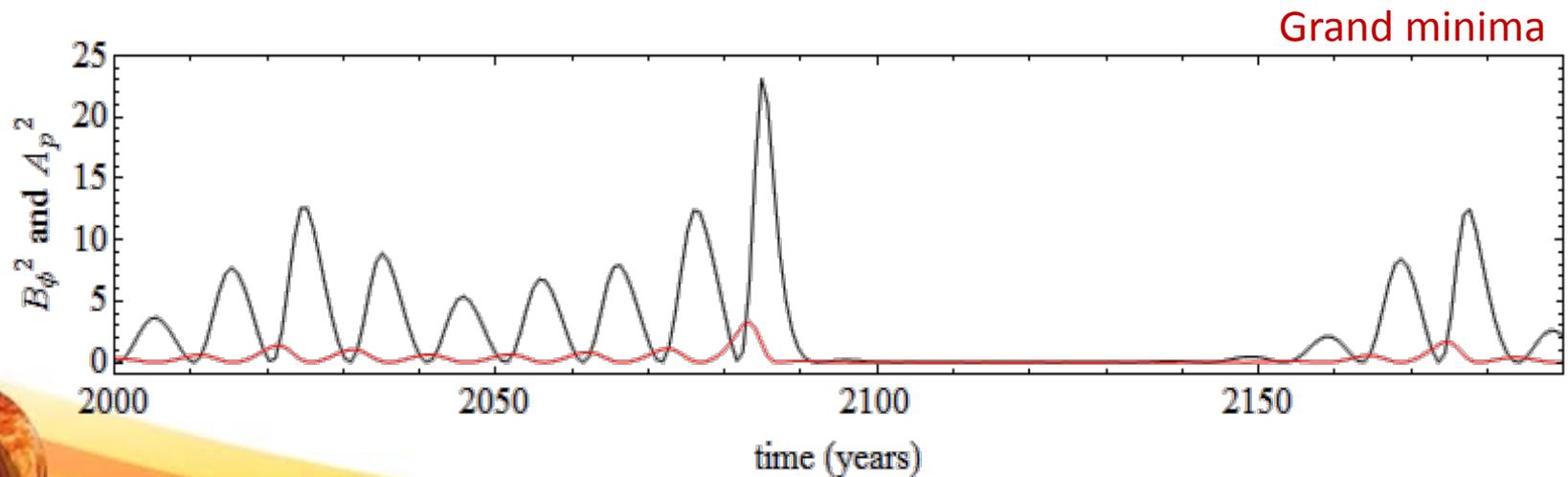
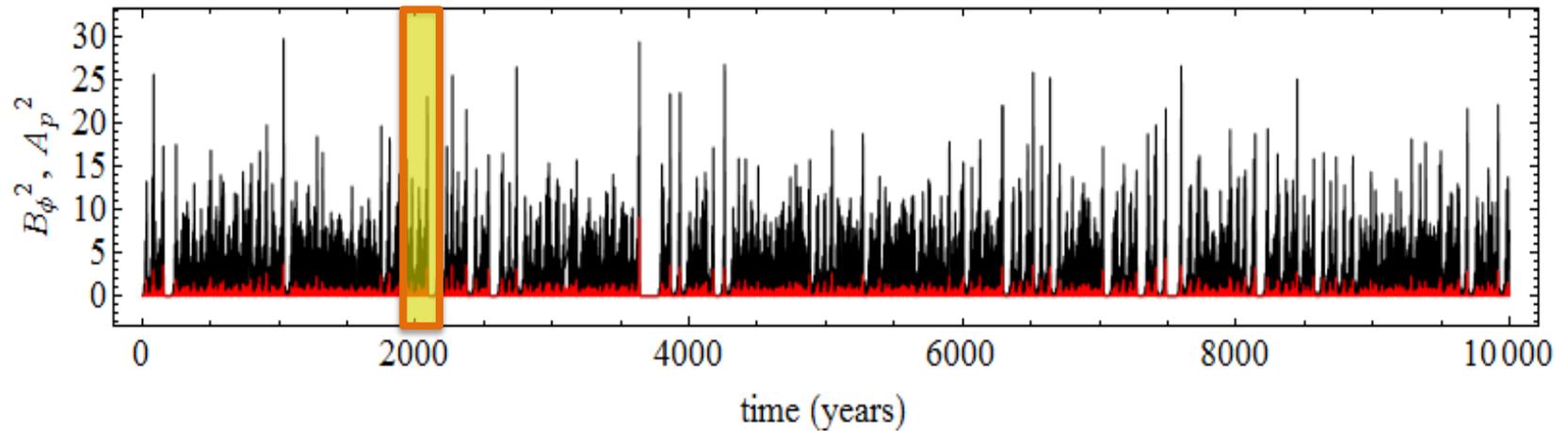


Long term modulation

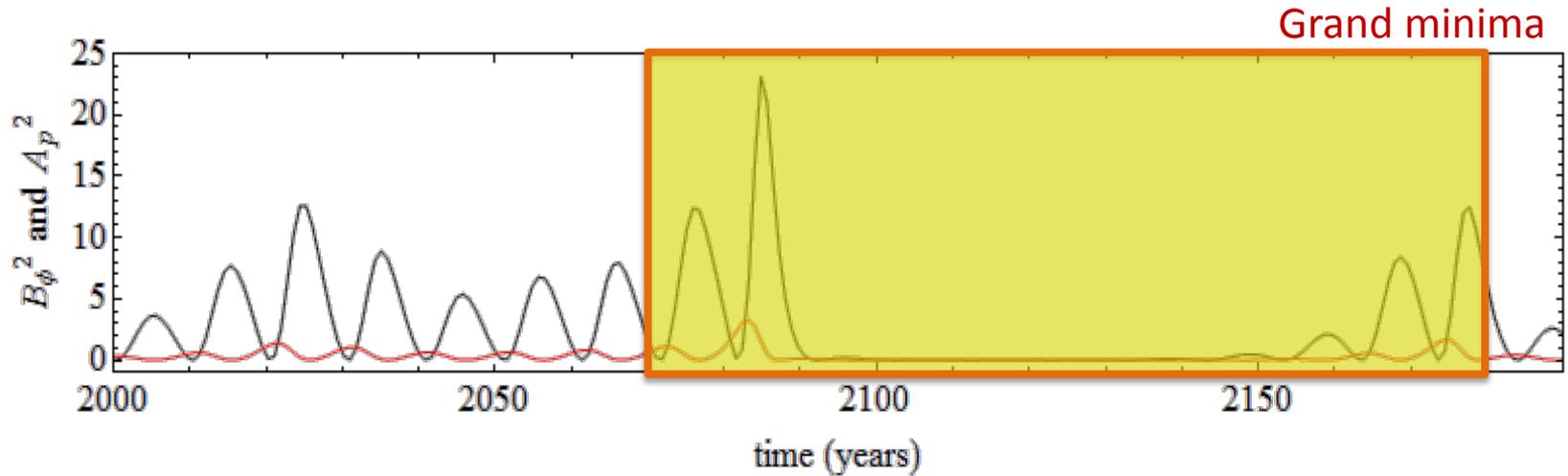


Stochastic Fluctuations

$\tau = 1$ year, $\mathbf{a} \in [0.01, 0.04]$, $\mathbf{b} = 0.05$



“Zooming in” into grand minima



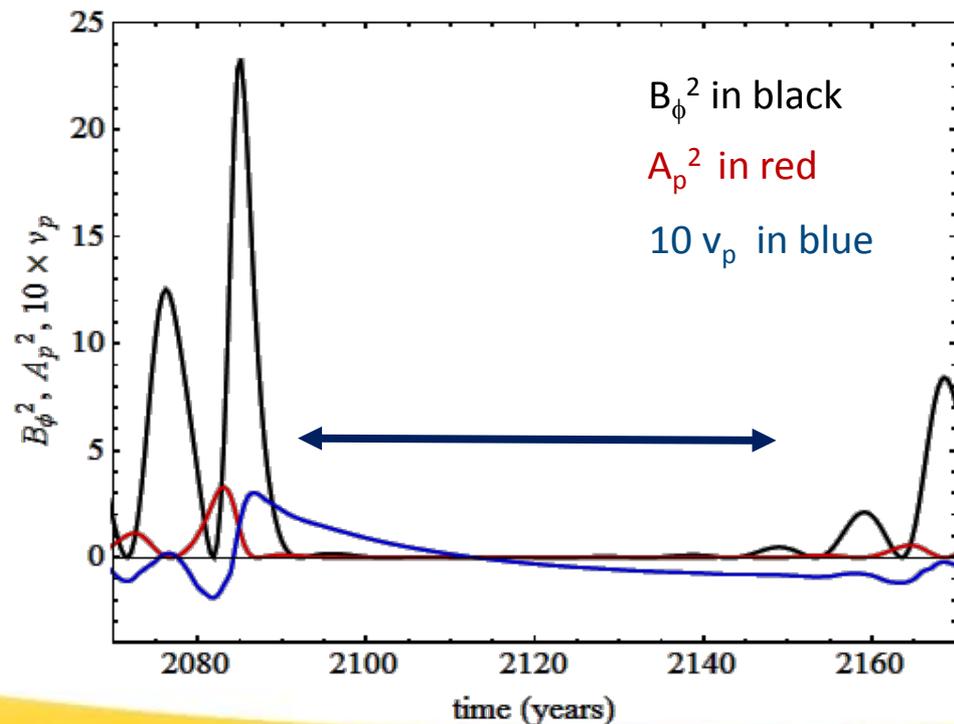
$$\frac{dv(t)}{dt} = a B_\phi A_p - b v(t)$$

Abrupt entrance into minimum

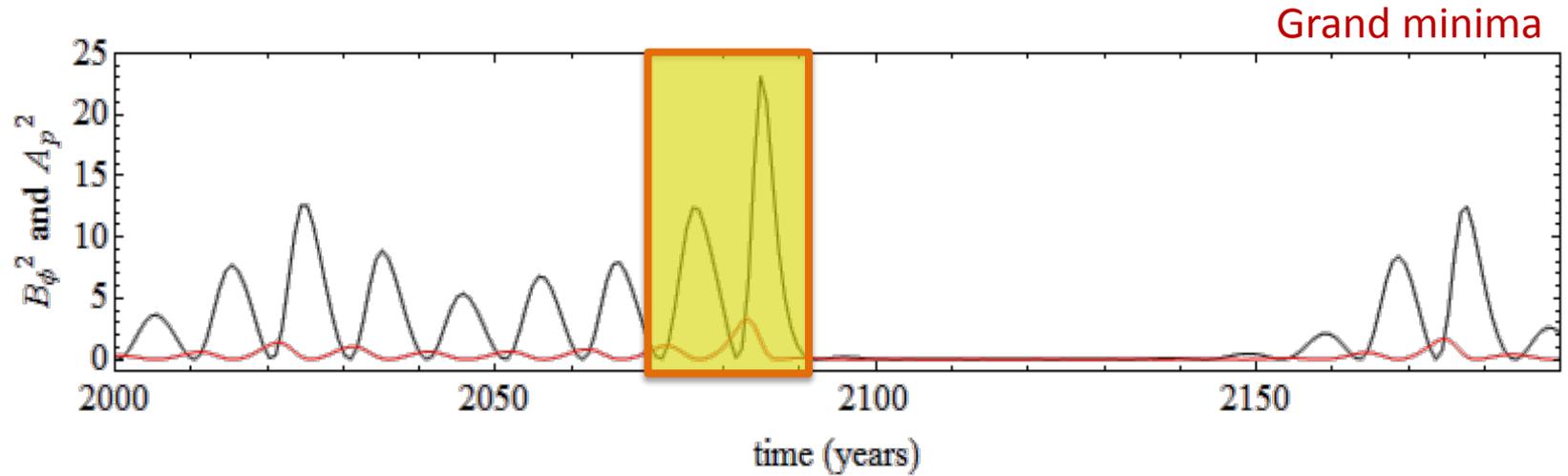
Gradual build up of mag. activity

Length of grand minima $\propto v_p$ and b .

Higher values of $b \Rightarrow$ shorter minima



“Zooming in” into grand minima

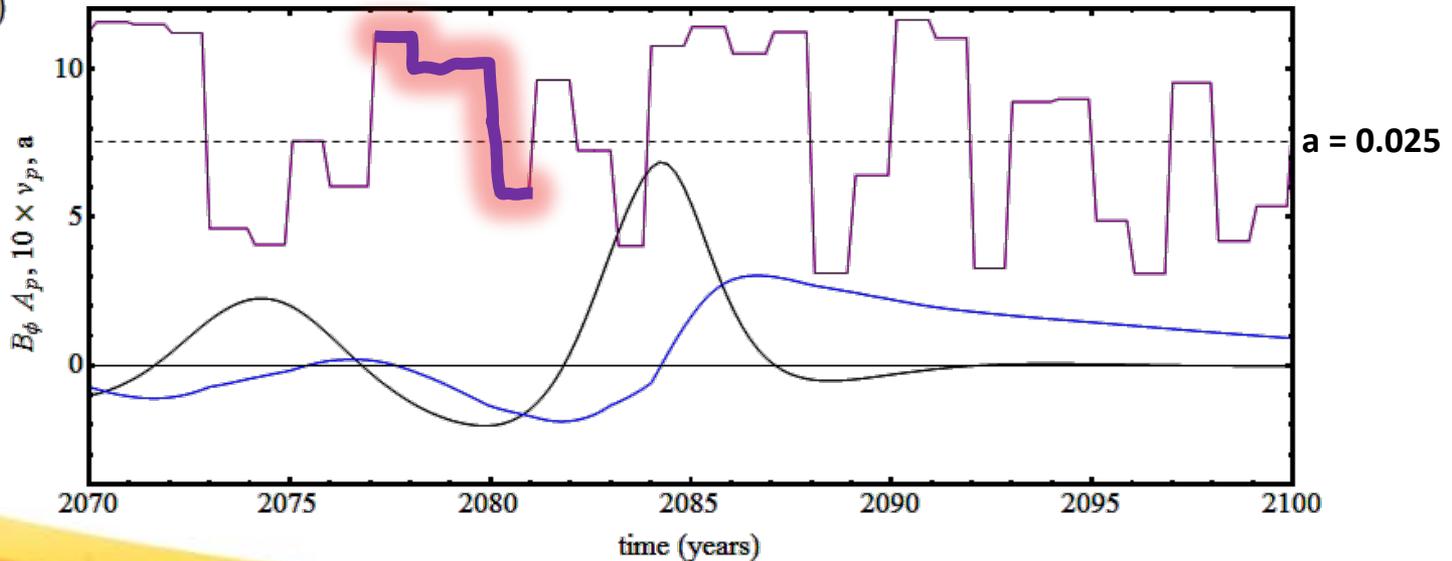


$$\frac{dv(t)}{dt} = a B_\phi A_p - b v(t)$$

$$300 \times a \in [0.01, 0.04]$$

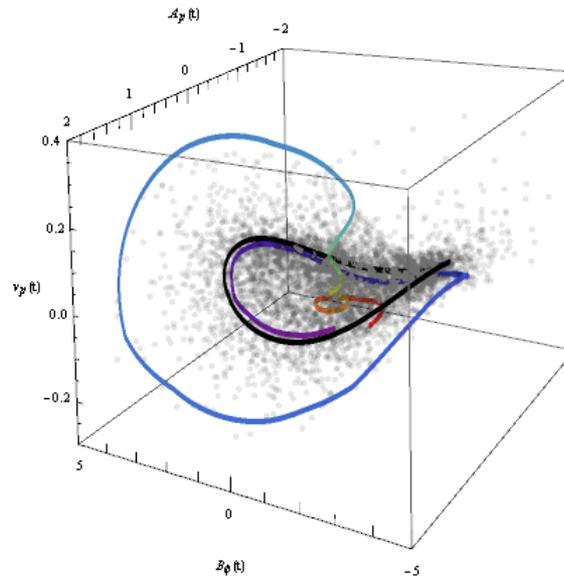
$$10 v_p$$

$$B(t) \quad A(t)$$



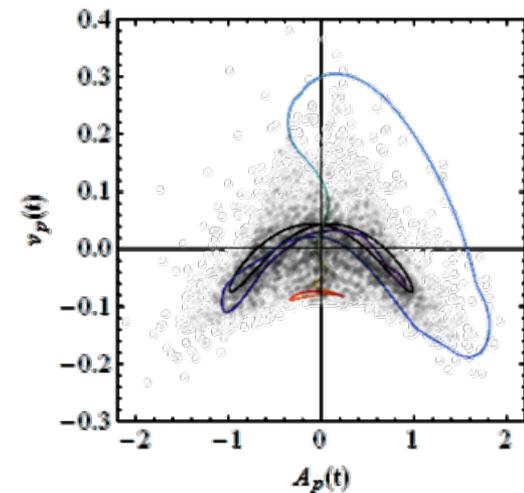
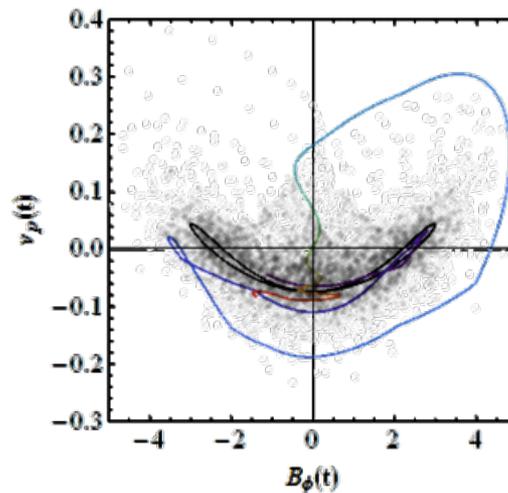
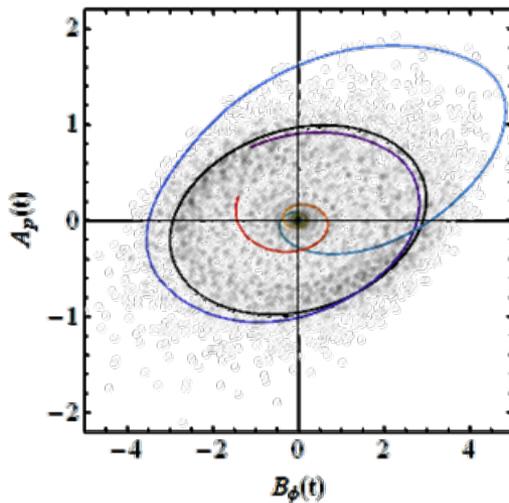
Looking into the phase space

5000 < time < 10000
 $a = 0.025$ in black



$a \in [0.01, 0.04]$ in gray

Minimum
2060 < time < 2160



Main point: the Lorentz force feedback is important for the long term dynamics

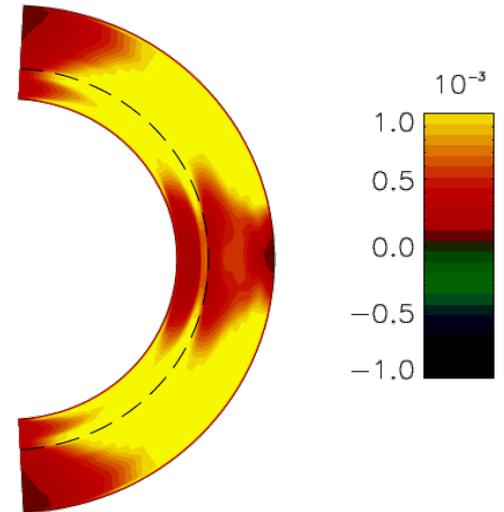
Back to the 3D simulation

Lorentz force

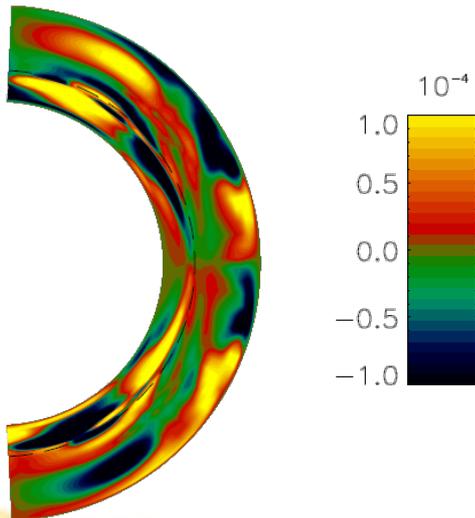
$$\mathbf{F} = \mathbf{j} \times \mathbf{B} = \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B}$$

- Compute \mathbf{F} at every grid point
- Average over all longitudes (ϕ)

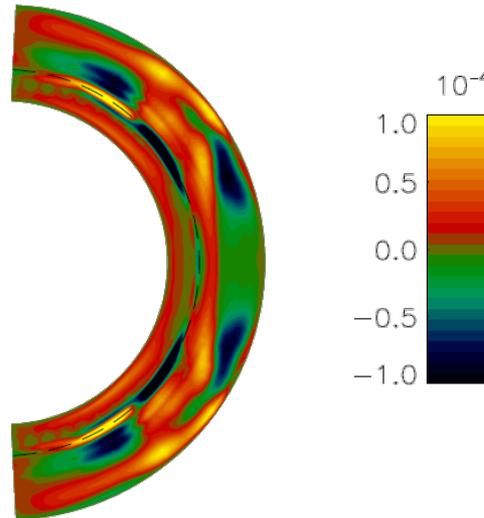
Time averaged $\langle \mathbf{F} \rangle$



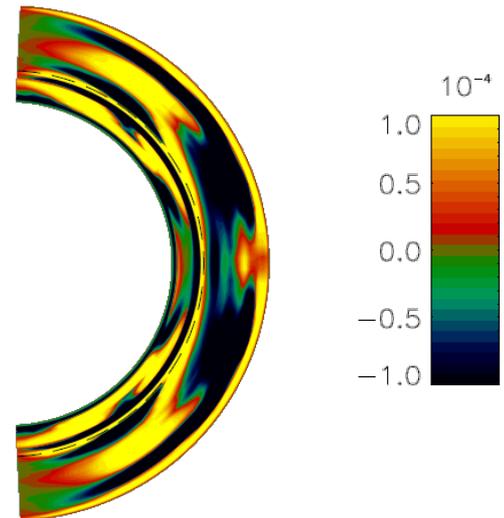
(C) $\langle F_y \rangle$



(B) $\langle F_x \rangle$



(D) $\langle F_z \rangle$



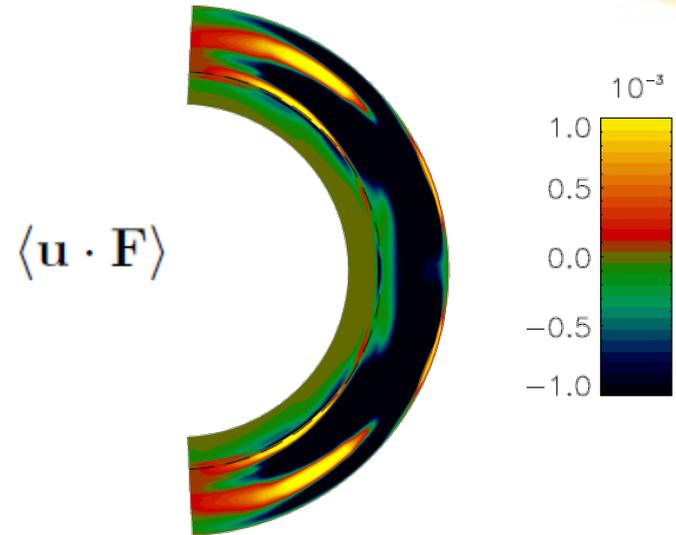
Energy balance

Magnetic energy evolution

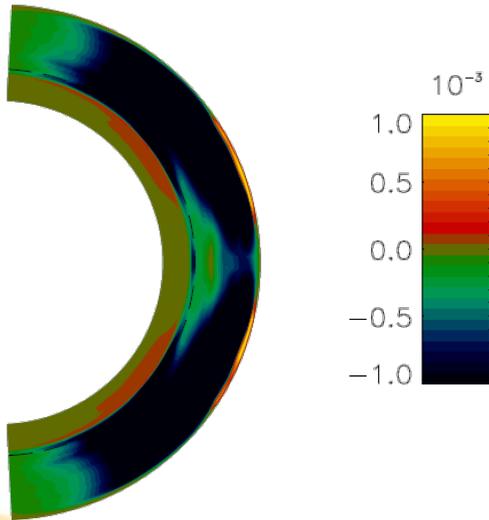
$$\frac{\partial \epsilon_B}{\partial t} = - \int_V \mathbf{u} \cdot \mathbf{F} dV - \int_V \frac{j^2}{\sigma} dV$$

Magnetic Fields gets energy from differential rotation and puts energy into the meridional flow...

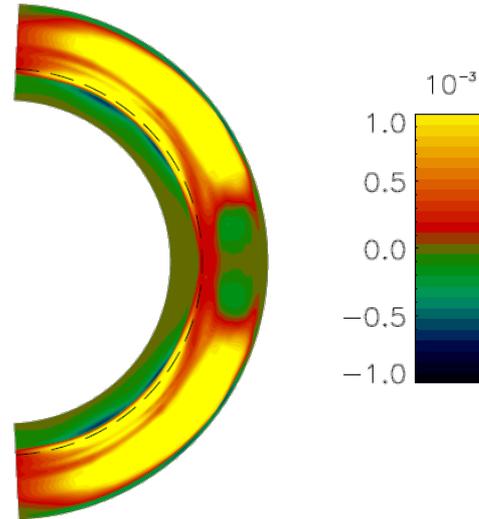
Time averaged $\langle \mathbf{u} \cdot \mathbf{F} \rangle$



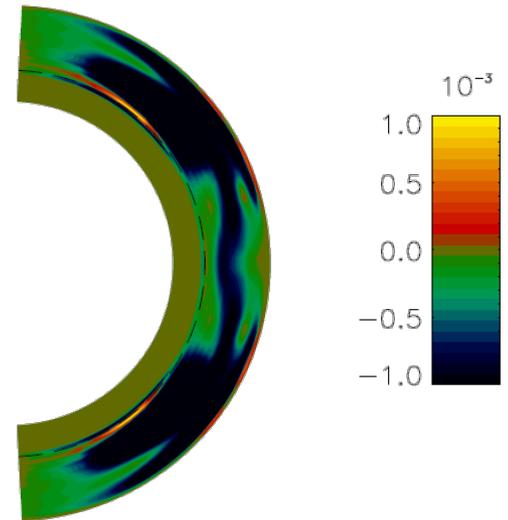
(B) $\langle u \cdot F_x \rangle$



(C) $\langle v \cdot F_y \rangle$



(D) $\langle w \cdot F_z \rangle$



Mean field enough?

“Mean field” type decomposition

$$\mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{B}'$$

$$\mathbf{F} = \frac{1}{\mu_0} (\nabla \times (\langle \mathbf{B} \rangle + \mathbf{B}')) \times (\langle \mathbf{B} \rangle + \mathbf{B}')$$

$$\mathbf{F} = \langle \mathbf{F} \rangle + \langle \mathbf{F}'' \rangle$$

$$\langle \mathbf{u} \cdot \mathbf{F} \rangle = \langle \mathbf{u} \cdot \mathbf{F} \rangle + \langle \mathbf{u} \cdot \mathbf{F}'' \rangle$$

$$\langle \mathbf{B} \rangle = \langle \mathbf{B} \rangle_\phi$$

$$\langle \mathbf{B}' \rangle = 0$$

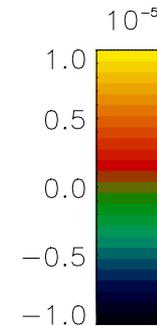
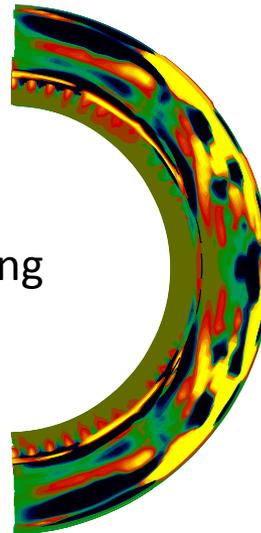
$$\langle \mathbf{B}' \mathbf{B}' \rangle \neq 0$$

Preliminary conclusion:

For the meridional flow, the small scales have a larger impact in modelling the solar magnetic field.

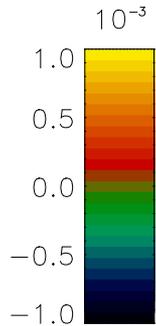
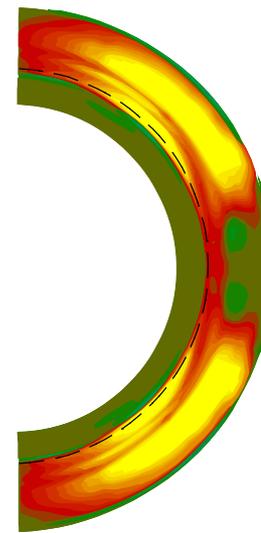
Large scale

$\langle C \rangle < v.FI_y >$



Small scale

$\langle C \rangle < v.Fs_y >$



Conclusions (...some preliminary!!)

- ✓ Meridional flow changes with time (observed in the solar surface and in global MHD sims)
- ✓ Flux transport dynamo models working in the kinematic disregard this important info
- ✓ The Lorentz force feedback of the field into the flow generates important long term dynamic behavior...
- ✓ ...that coupled with stochastic fluctuations can even produce grand minima episodes
- ✓ Magnetic field takes energy from solar differential rotation and puts energy into the flows in the meridional plane
- ✓ The biggest contribution to this phenomena comes from the small scale field fluctuations

- ✓ **Main conclusion:** the results obtained through kinematic models must be taken lightly . These models should include a Lorentz force feedback term in order to be used for prediction purposes or long term studies of solar activity.

*Passos, Charbonneau, Beaudoin, Sol.Phys, 2011/12
(in preparation)*

