

# GRAVITY FROM SPACETIME THERMODYNAMICS

Goffredo Chirco



UNIVERSITEIT VAN AMSTERDAM

@ CENTRA - IST

Lisboa - June 25th



based on collaborations with: Liberati - SISSA, Eling, Sindoni, Oriti - AEI,  
Vitagliano - IST/CENTRA

# GRAVITY FROM SPACETIME THERMODYNAMICS ?

Goffredo Chirco



UNIVERSITEIT VAN AMSTERDAM

@ CENTRA - IST

Lisboa - June 25th



based on collaborations with: Liberati- SISSA, Eling, Sindoni, Oriti - AEI,  
Vitagliano - IST/CENTRA

# OUTLINE

---

- ▶ **introduction and motivation**
  - ▶ BH thermodynamics => local spacetime thermodynamics
  - ▶ GR from thermodynamics of local horizons
  - ▶ non-equilibrium & dissipation
- ▶ **from GR to higher order gravity theories**
  - ▶  $f(R)$  and generalized Brans-Dicke gravity
  - ▶ Higher curvature and Entanglement Entropy and Noether charge
- ▶ **summary: thermodynamical equilibrium as a general principle of gravitational dynamics?**

# A - the Einstein equation of state

---

## ► black hole thermodynamics

BH solutions can be described as dynamical systems in terms of a small number of parameters:  $M, J, Q_e$  (no hair theorem)

Israel 67, Christodoulou 71, Hawking 71, Bardeen, Carter 73

## ► mathematical analogy \to physical identity

from classical level

+

quantum level

0th  $\kappa$  constant along the horizon

1st  $dM = \frac{\kappa}{8\pi G} dA + \Omega dJ + \Phi dQ$

2nd  $dA \geq 0$

3rd  $\kappa = 0$  unattainability

$\Rightarrow$

QM+SM

Bekenstein 73

$$S = \frac{A}{4l_p^2}$$

QFT

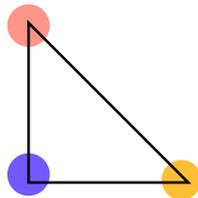
Hawking 75

$$T = \frac{\hbar\kappa}{2\pi} \Rightarrow S = \frac{A}{4G}$$

interplay

GR

QFT



THERMODYNAMICS

## ► how does GR know about T-k and S-A ?

Jacobson 95

# A - the Einstein equation of state

---

## ► black hole thermodynamics

BH solutions can be described as dynamical systems in terms of a small number of parameters:  $M, J, Q_e$  (no hair theorem)

Israel 67, Christodoulou 71, Hawking 71, Bardeen, Carter 73

## ► mathematical analogy \to physical identity

from classical level

+

quantum level

0th  $\kappa$  constant along the horizon

1st  $dM = \frac{\kappa}{8\pi G} dA + \Omega dJ + \Phi dQ$

2nd  $dA \geq 0$

3rd  $\kappa = 0$  unattainability

$\Rightarrow$

QM+SM

Bekenstein 73

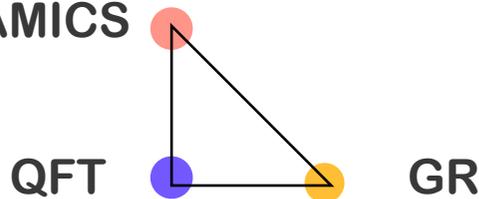
$$S = \frac{A}{l_p^2}$$

QFT

Hawking 75

$$T = \frac{\hbar\kappa}{2\pi} \Rightarrow S = \frac{A}{4G}$$

THERMODYNAMICS



semiclassical picture

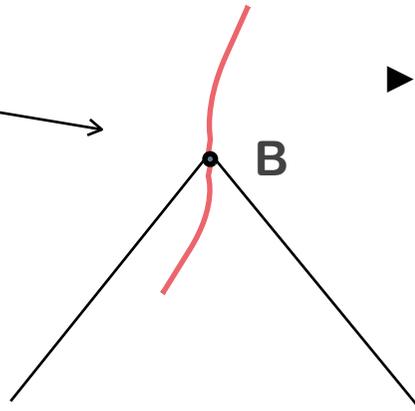
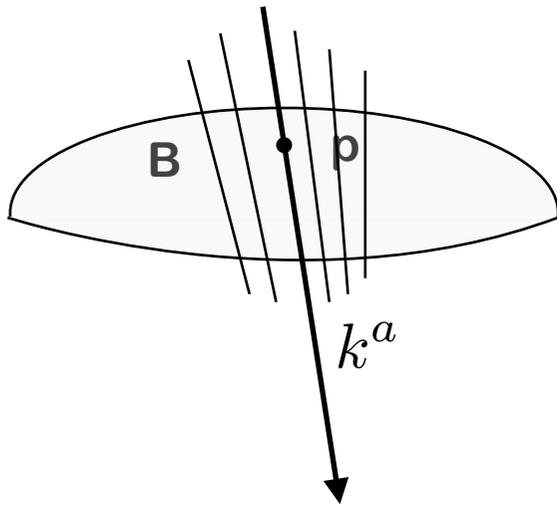
thermodynamics as a premise ?

# from BH to local horizon thermodynamics

## LOCAL CAUSAL HORIZON

- ▶ consider a local spacetime spacelike 2 surface patch  $B$  associated to an event  $p$

Jacobson 95, Padmanabhan 02



- ▶ **CAUSAL HORIZON:**  
boundary of the past of  $B$  = past directed null geodesic congruence normal to  $B$

- ▶ at  $p$  the 2 surface can be characterized by the kinematical d.o.f. of the bundle of null geodesics: expansion, shear...

not enough...

- ▶ **QFT & THERMO** extended to local horizon physics

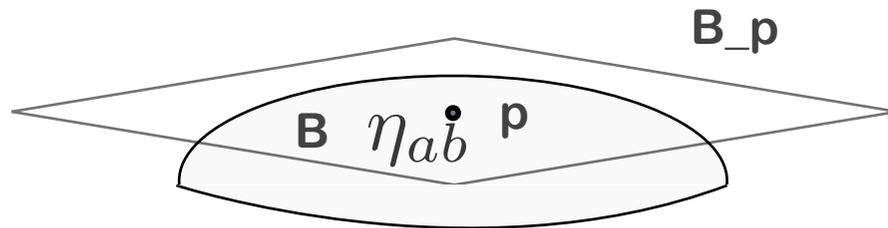
**basic ingredients:**

- stationarity :  $\exists$  time Killing vectors
- non inertial frames : Unruh effect
- horizon area-entropy relation

# local horizon thermodynamics

## STATIONARITY - LOCAL KILLING HORIZON

- ▶ introduce a **local inertial frame at p** (local Lorentz symmetry)



- ▶ consider an approximate Killing field  $\chi^a$  generating boosts orthogonal to  $B$

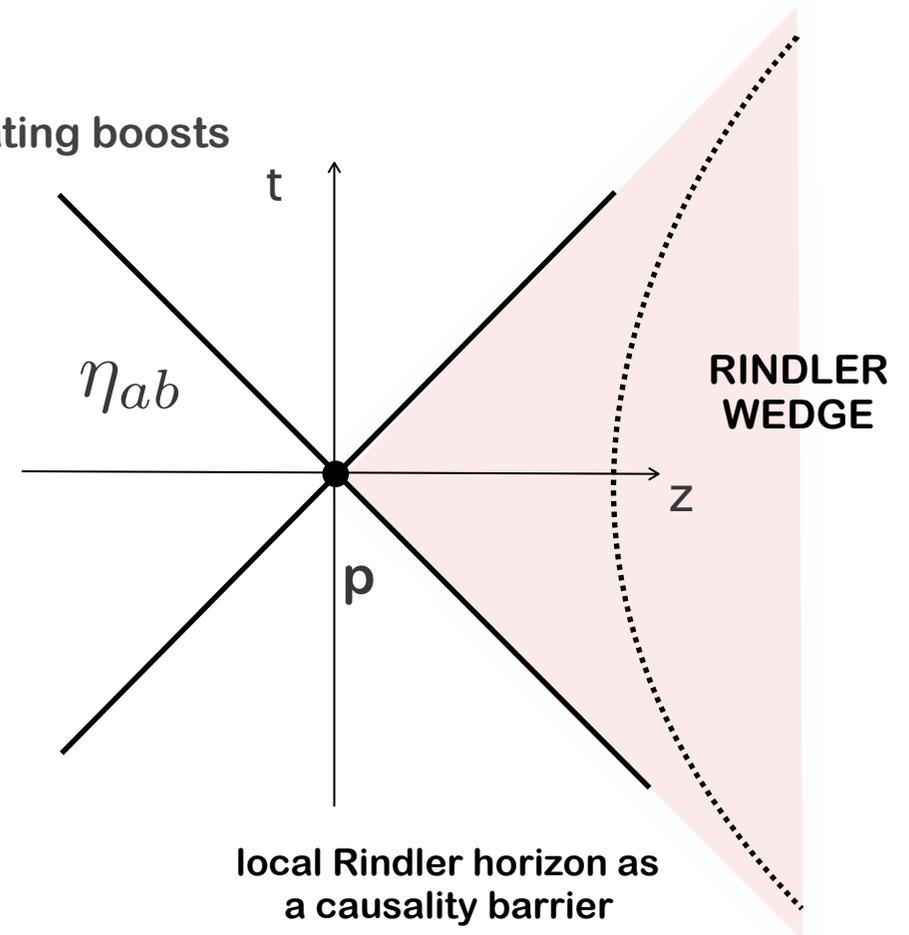
- ▶ Lorentz boost introduces a **local Rindler frame**

=> LOCAL RINDLER HORIZON

- ▶ arrange  $\chi^a$  so that:  $\chi^a = -\lambda k^a$

- + boost isometry => Killing horizon **stationarity**

uniformly accelerated frame....



# local horizon thermodynamics

---

## UNRUH EFFECT

- ▶ assume locally Minkowski vacuum
  - ▶ given local Lorentz symmetry + stability of the vacuum
- ⇒ when restricted to the Rindler wedge, the usual global Minkowski vacuum state  $|0\rangle$  in quantum field theory turns out to be equivalent to a **Gibbs thermal state** with an Unruh-Tolman temperature

Unruh 76

- ▶  $H$  operator generating Lorentz boost on the quantum fields noninertial frame translations in hyperbolic angle

$$\beta^{-1} \rightarrow T = \frac{1}{2\pi}$$

RINDLER WEDGE thermal system

$$\rho = \exp(-\beta H)/Z \quad \left| \begin{array}{l} \langle E \rangle = \text{Tr}(\rho H) \\ S = -\text{Tr}(\rho \ln \rho) \end{array} \right.$$

for  $\delta\rho \ll \rho$

$$\Rightarrow \delta S = \beta \delta \langle E \rangle \quad \checkmark$$

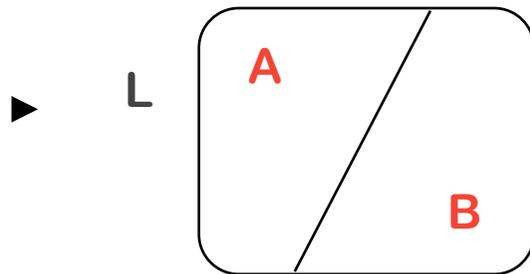
**EQUILIBRIUM**

# local horizon thermodynamics

---

## HORIZON AREA - ENTROPY RELATION

some holographical assumption



$$\rho_L = |\Psi\rangle\langle\Psi| \quad \text{pure}$$

$$\rho_A = \text{tr}_B \rho \quad \text{mixed}$$

missing information - von Neumann entropy

**entanglement entropy**

$$S(\rho_A) = -\text{tr} \rho_A \ln \rho_A$$

+ Rindler horizon  $\Rightarrow$  entanglement entropy = thermal entropy

▶ the entanglement entropy scales with the area but is infinite. need UV regulation

$$S = \alpha A \quad \Rightarrow \quad S = f(\text{geometry})$$

we get horizon thermodynamics without any help from GR....

**MEM**

the entropy density depends on the nature of the quantum fields and their interactions and can be some complicate function of the position in spacetime

# GR from local horizon thermodynamics

---

## Einstein equation of state: ANALOGY

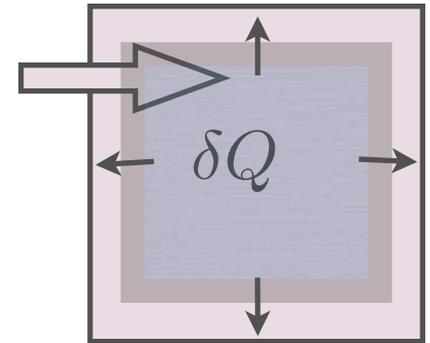
$$S(E, V) \Rightarrow dS = (\partial S / \partial E) dE + (\partial S / \partial V) dV$$
$$\delta Q = dE + p dV$$

equilibrium  
entropy balance

$$\delta Q = T dS \Rightarrow$$

$$T^{-1} = (\partial S / \partial E)$$
$$p = T (\partial S / \partial V)$$

equation of state



**IDEA**

geometric entropy  
functional

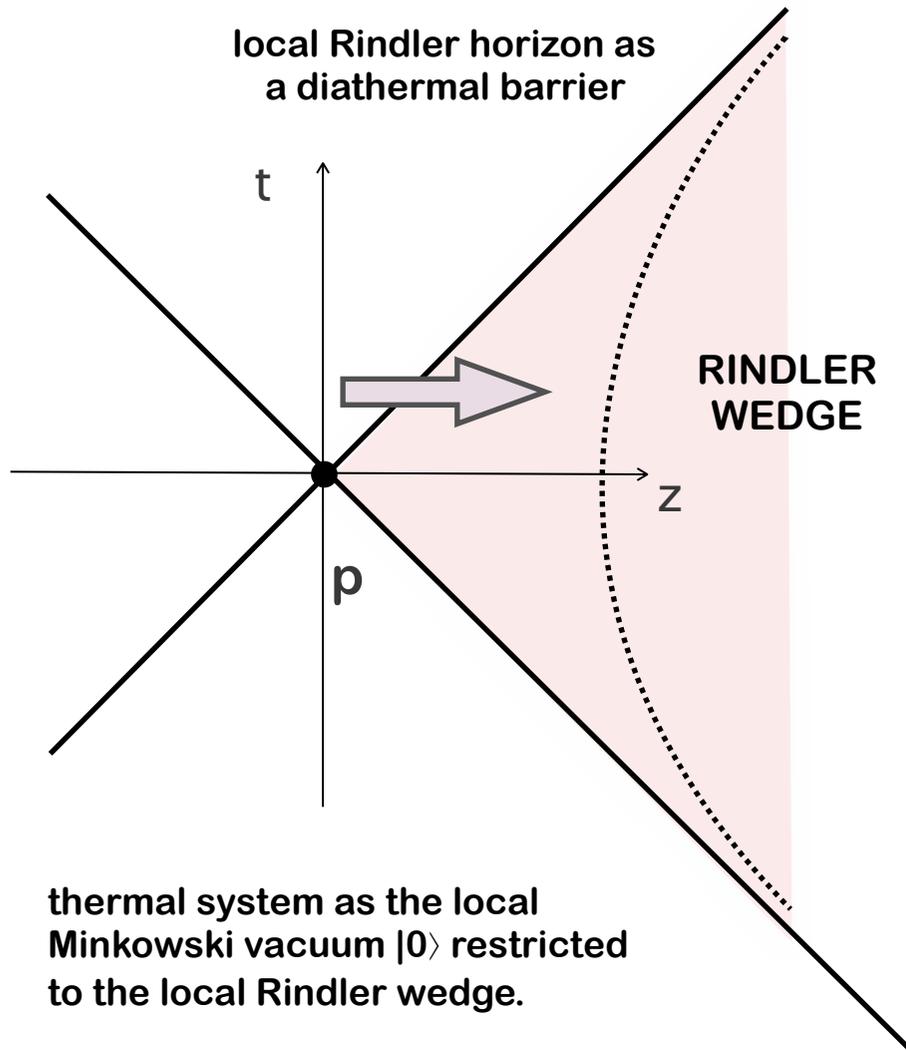
+

local matter-energy  
perturbations

$\Rightarrow$

Einstein eq. as  
equation of state

# GR from local horizon thermodynamics



- ▶ for the horizon system the heat flow is the energy current of the matter as measured with respect to the boost Hamiltonian for which the state is thermal

$$\delta Q = \delta \langle E \rangle = \int T_{ab} \chi^a d\Sigma^b$$

energy crossing the horizon =  
**vacuum perturbation**

for  $\delta\rho \ll \rho \Rightarrow \delta S = \beta \delta \langle E \rangle \Rightarrow T\delta Q = \delta(\alpha A)$

- ▶ then the assumption of entropy balance (Clausius law) provides a **local matter/geometry constitutive relation**

## GR from local horizon thermodynamics

► assume  $\alpha$  constant  $\Rightarrow \delta S = \alpha \delta A \quad \delta A = \int_H \tilde{\epsilon} \theta d\lambda$

► **equilibrium** = Rindler horizon bifurcation surface

$$\delta S = \alpha \delta A = 0 \quad \Rightarrow \quad \theta_p = 0$$

► **perturbation** via heat flux

$$\theta \approx \theta_p + \lambda \left. \frac{d\theta}{d\lambda} \right|_p + \mathcal{O}(\lambda^2) \quad \text{by Raychaudhuri}$$

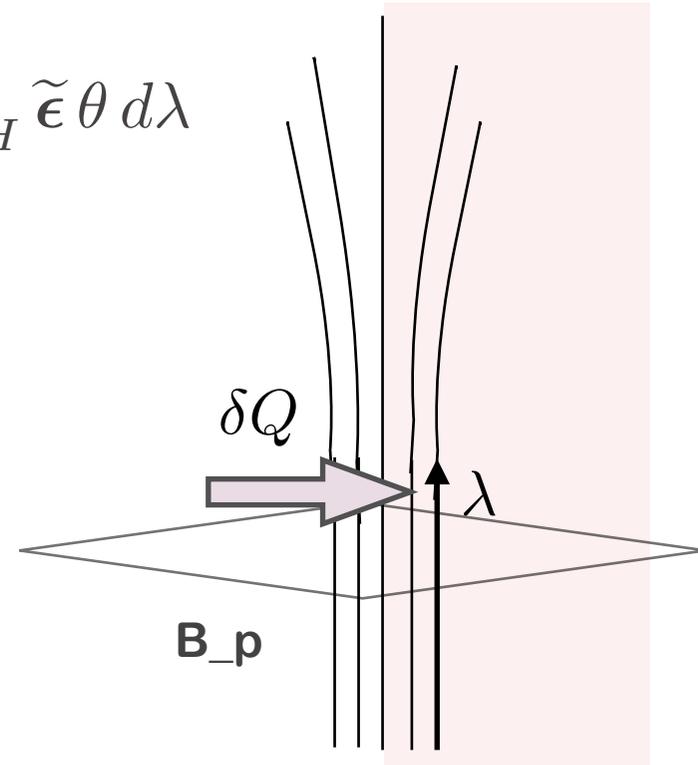
$$\Rightarrow dS = \alpha \int_H \tilde{\epsilon} d\lambda [\theta - \lambda(1/2 \theta^2 + \|\sigma\|^2 + R_{ab} l^a l^b)]_p$$

► **equilibrium recovery** via entropy balance law  $\delta Q = T dS$

$$T dS = \alpha \frac{\kappa \hbar}{2\pi} \int_H \tilde{\epsilon} d\lambda \left[ \underbrace{\theta - \lambda(1/2 \theta^2 + \|\sigma\|^2)}_{=0} + \underbrace{R_{ab} l^a l^b} \right]_p =$$

$$= \int_H \tilde{\epsilon} d\lambda (-\lambda \kappa) \underbrace{T_{ab} l^a l^b} = \delta Q$$

$\delta A$  has local and non-local contributions



# GR from local horizon thermodynamics

LOCAL LEVEL:

for all null  $k^a$   $\frac{2\pi}{\hbar\alpha} T_{ab} = R_{ab} + \Phi g_{ab}$  local constitutive relation

to define  $\Phi = -\frac{1}{2}R - \Lambda$  • local energy conservation  $\nabla^b T_{ab} = 0$

• Bianchi identity  $\nabla^b R_{ab} = \frac{1}{2}\nabla_a R$

$$\Rightarrow 8\pi G T_{ab} = R_{ab} - \frac{1}{2}R g_{ab} - \Lambda g_{ab}$$

Jacobson 95

$$\text{if only } \alpha = \frac{1}{4G}$$

UV cutoff  $\Leftrightarrow$  G  
INDUCED GRAVITY

the equation is then extended to the whole spacetime via EP

NON-LOCAL  
LEVEL:

$$dS_i = -\alpha \int_H \tilde{\epsilon} d\lambda \lambda \|\sigma^2\|$$

How to get rid of the  
non local terms ?

## B - the non-equilibrium regime

---

### non-equilibrium horizon thermodynamics

- ▶ **impossible!** it is associated to an arbitrary kinematical d.o.f. of the null congruence, associated to the arbitrary choice of B
- ▶ the non-local entropy term can be written as an **internal entropy production** term

=>

$$dS = \underline{dS_{eq}} + \underline{dS_i}$$

LOCAL = EQUILIBRIUM

NON-LOCAL = NON-EQUILIBRIUM

### GENERALIZED CLAUSIUS EQUATION

Eling 2006, Chirco 2009

- ▶ to recover the eom for GR one needs a generalization of the entropy balance relation where the non-local entropy term is interpreted as **unmatched heat**

$$dS = \frac{\delta Q}{T} + \delta N$$

**irreversible processes**

=> microscopic level ?

## non-equilibrium horizon thermodynamics

---

- ▶ the unmatched heat for GR coincides with the expression for the tidal heating dissipation in BH (Hawking-Hartle)

Chirco 2009

**internal entropy production** term must be associated to some horizon viscosity

**membrane paradigm** : horizon congruence dynamics is well described by 2+1 viscous fluid equations

Damour 79, Thorne 86

$$dS_i = \frac{2\eta}{T} \hat{\sigma}_{\mu\nu} \hat{\sigma}^{\mu\nu} + \frac{\xi_B}{T} \hat{\theta}^2$$

$$T dS_i = 2\eta \int_H \tilde{\epsilon} dv \|\sigma^2\| = \frac{1}{8\pi G} \int_H \tilde{\epsilon} dv \|\sigma^2\|$$

- ▶ spacetime viscosity will be related to the UV cut-off scale of the theory, through the entropy density

=> introduce a horizon viscosity coefficient

$$\eta = \frac{T\alpha}{2} = \frac{\hbar\alpha}{4\pi} = \frac{1}{16\pi G}$$

- > as for BH, the shear viscosity to entropy density ratio for the Rindler horizon system saturate the **Kotvun Son Starinets bound**

$$\eta/\alpha = \frac{1}{4\pi}$$

## focussing on the irreversible sector

---

- ▶ in AdS/CFT the KSS bound is interpreted as a universal lower bound for all strongly coupled field theories with gravity dual

Policastro 2001, Starinets, Kotvun 2003

### INTERESTING...

KSS ratio seem to be rooted in gravitational physics, but the Rindler wedge is a subregion of Minkowski space time: **NO GRAVITY AT ALL**

- ▶ interpretation of dissipative effects as a consequence of an underlying fluctuating behaviour of spacetime at the UV cut-off scale (Candelas-Sciama and AdS/CFT)

> **ENTANGLEMENT VISCOSITY**

Chirco 2010

if one could calculate  $\eta$  directly from the fluctuations of the matter fields in the thermal vacuum, that would **characterize the KSS bound as a fundamental property of quantum entanglement and its associated holography**

# entanglement viscosity

---

**PROBLEM** how to relate a phenomenological transport coefficient  $\eta$  from a **fluid-wise** description (membrane) of the **horizon** to the quantum **vacuum state on the bulk ??**

**MAIN IDEA**  
(AdS/CFT)

- ▶ on large spatial and time scale the thermal vacuum can be **effectively** described by **hydrodynamics**
- ▶ calculate the hydrodynamics transport coefficient from microscopic theory using KUBO FORMULA **involving the Green's function of the energy momentum tensor for the matter fields in the wedge**

Kotvun 03, Son 07, Starinets 09

▶ no holographic duality like AdS/CFT in Rindler wedge ... **but PRE-HOLOGRAPHY**

**area scaling behaviour of entanglement entropy** : quantum degrees of freedom of the wedge seem to be packed on the stretched horizon surface

>

**TRY a lower dimensional description of the vacuum fields associated to the horizon**

## lower dimensional description of bulk fields

---

### WHAT WE WANT:

dual lower dimensional description of the vacuum state in terms of a strongly coupled thermal CFT effectively living on a (D-1) Minkowski (horizon membrane)

### RECIPT

- ▶ start with the canonical energy momentum tensor for the Rindler wedge

$$T_{(R)\nu}^{\mu} = \frac{\partial L_R}{\partial(\partial_{\mu})\psi} \partial_{\nu}\psi - \delta_{\nu}^{\mu} L_R \quad \text{where } L_R = \sqrt{-g} L_M$$

**ANSATZ:** on large scales, the holographic vacuum state is described by a conserved lower dimensional SET

$$\langle \hat{T}_{\mu\nu}^{(D-1)} \rangle = Z^{-1} \text{Tr}(\rho \hat{T}_{\mu\nu}^{(D-1)}) = \langle 0 | \hat{T}_{\mu\nu}^{(D-1)} | 0 \rangle$$

Minkowski vacuum expectation value

thermal average at Tolman-Unruh temperature  $T_{(R)\nu}^{\mu} = \kappa \xi T_{(M)\nu}^{\mu}$

- ▶ **DIMENSIONAL REDUCTION**

$$\langle \hat{T}_{\mu\nu}^{(D-1)} \rangle = \int_{l_c}^{\infty} d\xi \langle \hat{T}_{\mu\nu}^{(R)} \rangle = \int_{l_c}^{\infty} d\xi \kappa \xi \langle \hat{T}_{\mu\nu}^{(M)} \rangle$$

## Kubo like formula for the horizon viscosity

II

apply the formalism of viscous hydrodynamics and calculate the shear viscosity through a **Green-Kubo approach** in terms of the effective lower dimensional SET

- ▶ consider a metric perturbation  $h_{\mu\nu}$  associated to the bulk vacuum perturbation  $\delta\langle E\rangle$  as source for the (D-1) field theory operator  $\hat{T}_{\mu\nu}^{D-1}$
- ▶ assuming the perturbation is small, from **linear response theory**, one can calculate the change of the expectation value of  $\hat{T}_{\mu\nu}^{D-1}$

$$\langle \delta\hat{T}_{\mu\nu}^{D-1}(k^0, \vec{k}) \rangle = G_R(k^0, \vec{k}) h_{\mu\nu}(k^0, \vec{k})$$

where  $G_R$  is the retarded 2-point thermal Green's function of  $\hat{T}_{\mu\nu}^{D-1}$

$$G_R(k^0, \vec{k}) = \int d\tau d^{D-2}x e^{ik^0\tau} e^{-i\vec{k}\vec{x}} \langle [\hat{T}_{\mu\nu}^{D-1}(\tau, \vec{x}) \hat{T}_{\mu\nu}^{D-1}(0, \vec{0})] \rangle$$

- ▶ in the limit  $(k^0, \vec{k}) \rightarrow 0$   $\langle \hat{T}_{xy}^{D-1}, \hat{T}_{xy}^{D-1} \rangle (k^0, \vec{k} \rightarrow 0) = i\eta k^0 - P + \mathcal{O}(\omega^2)$

from which one gets the **quantitative** expression for the shear viscosity

$$\eta = \lim_{k^0 \rightarrow 0} \frac{1}{k^0} G_R^{xy,xy}(k^0, 0)$$

# KSS bound for the Rindler horizon

- ▶ in our particular case we have

$$\eta = \lim_{k^0 \rightarrow 0} \frac{1}{k^0} \int_{l_c}^{\infty} \xi' \int_{l_c}^{\infty} \xi \int d\tau d^{D-2}x e^{ik^0 \tau} \theta(\tau) \kappa^2 \xi \xi' \langle [T_{xy}^D(\tau, x, y, \xi), T_{xy}^D(0, \xi')] \rangle$$

where  $\langle [T_{xy}^D(\tau, x, y, \xi), T_{xy}^D(0, \xi')] \rangle$

is the **Minkowski 2-point correlator of the bulk field theory**

Chirco 2010

**THEN**

for a free minimally coupled scalar field in 4D Rindler spacetime is

$$\eta = \frac{1}{1440\pi^2 l_c^2}$$

**AREA SCALING  
ENTANGLEMENT  
VISCOSITY**

- ▶ from the thermal description of the dimensionally reduced vacuum fields

$$\epsilon = \frac{\pi^2 T^4}{30} = \frac{1}{480\pi^2 \xi^4} \quad \text{--->} \quad \epsilon_r = \frac{\kappa}{960\pi^2 l_c^2}$$

$$s = \frac{2\pi^2}{45} T^3 = \frac{1}{180\pi \xi^3} \quad \text{--->} \quad s = \frac{1}{360\pi l_c^2}$$

**all area scaling quantities**

4D - Planckian form :  
massless free scalar field =  $\epsilon = 3P$   
ultrarelativistic boson gas

$$\text{>} \quad \eta/s = \frac{1}{4\pi}$$

**KSS bound satisfied by just entanglement quantities !!!**

## KSS bound for the Rindler horizon

---

### NON equilibrium thermodynamical description:

- ▶ propagation of purely gravitational dof associate with macro dissipative effects
- ▶ a microscopic description for the macro shear viscosity in terms of the fluctuations of the Rindler wedge thermal state in a finite temperature QFT

### KSS ratio from entanglement:

local Rindler horizon system

NO GRAVITY

NO HOLOGRAPHIC DUALITY LIKE ADS/CFT

**GOAL**

KSS ratio may be a fundamental holographic property of spacetime and quantum entanglement

support for the hypothesis that semi-classical gravity on macroscopic scales is **induced as an effective theory** of some lower dimensional strongly coupled quantum system with a large number of degrees of freedom

Does it work for generalized gravity theories ?

**interestingly successful enough to try to go from GR to higher order gravity theories...**

Is there a general principle for gravitational dynamics ?

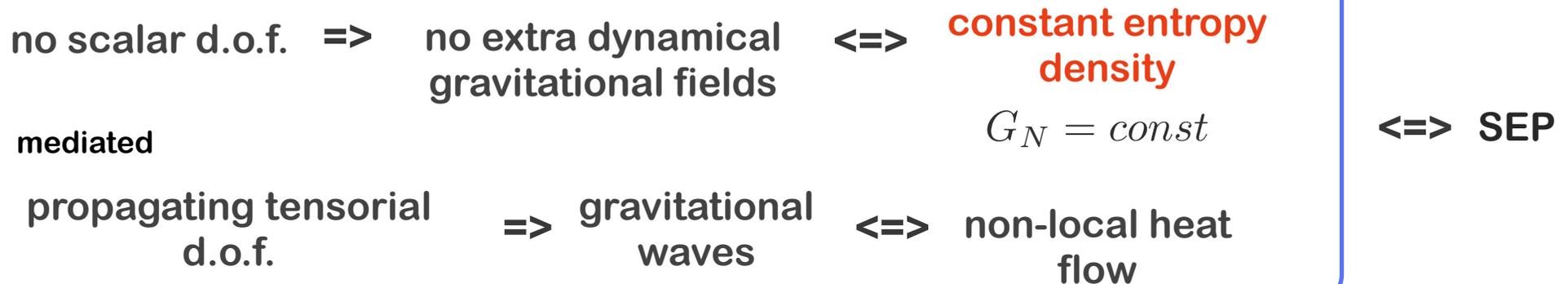
Discriminant factor for gravity theory: a metatheory of gravity ?

## extension to generalized gravity theories

---

GR is effectively purely geometrical. very special.

**RATIO** the entropy functional may be constrained by the different formulations of the **equivalence principle**



=>

changing the assumed entropy functional would change the associated gravitational field equation

How does the thermodynamical derivation work in the case ?

# extension to generalized Brans-Dicke gravity

---

## ENTROPY

- promote the entropy density to be an independent field

$$\alpha \rightarrow \phi(x) \quad \Rightarrow \quad S = \int d^4x \sqrt{h} \phi(x)$$

- again, consider the variation of S along the horizon geodesic bundle

$$\delta S = \int \sqrt{h} \left( \phi \theta + \frac{d\phi}{d\lambda} \right) d\lambda d^2x$$

- Taylor expand around p ( $\lambda = 0$ )

$$\delta S = \int \sqrt{h} \left[ \left( \phi \theta + \frac{d\phi}{d\lambda} \right) + \lambda \left( \theta \frac{d\phi}{d\lambda} + \phi \frac{d\theta}{d\lambda} + \frac{d^2\phi}{d\lambda^2} + \phi \theta^2 + \theta \frac{d\phi}{d\lambda} \right) \right]_p$$

Raychaudhuri

- equilibrium**  $\mathcal{O}(\lambda^0)$   $\Rightarrow \theta_p = -\phi^{-1} \frac{d\phi}{d\lambda}$       the kinematical d.o.f.  $\theta_p$  is linked to the derivative of the spacetime scalar field
- at  $\mathcal{O}(\lambda)$

$$\delta S = \int \sqrt{h} d\lambda d^2x \lambda \left( -\phi R_{\mu\nu} k^\mu k^\nu + k^\mu k^\nu \nabla_\mu \nabla_\nu \phi - \frac{3}{2} \phi \theta^2 - \phi \sigma_{\mu\nu} \sigma^{\mu\nu} \right)$$

## extension to generalized Brans-Dicke gravity

### EXTRA ENTROPY

- ▶ **new non-equilibrium contributions?**

$$\delta S = \int \sqrt{h} d\lambda d^2 x \lambda (-\phi R_{\mu\nu} k^\mu k^\nu + k^\mu k^\nu \nabla_\mu \nabla_\nu \phi - \frac{3}{2} \phi \theta^2 - \phi \sigma_{\mu\nu} \sigma^{\mu\nu})$$

$$dS_i = \frac{2\eta}{T} \hat{\sigma}_{\mu\nu} \hat{\sigma}^{\mu\nu} + \frac{\xi_B}{T} \hat{\theta}^2$$

the additional scalar d.o.f. may appear as a new gravitational channel for dissipating energy

- ▶ **not correct:** given the equilibrium condition

$$\frac{3}{2} \phi \theta^2 = \frac{3}{2\phi} k^\mu k^\nu \nabla_\mu \phi \nabla_\nu \phi \quad \Rightarrow$$

unlike the shear term, the bulk term is LOCAL

- ▶ after the  $k^\mu$  are peeled of, local terms at p are **frame independent**

**PUZZLE !!**

Jacobson, Padmanabhan

these terms exist for any observer in the local spacetime patch

$\Rightarrow$

they will end up describing the dynamics of the global spacetime

## extension to generalized Brans-Dicke gravity

---

### HEAT

- ▶ principle of **background independence**  $\Rightarrow \phi(x)$  must be varied like other fields
- ▶ must contribute to the total Lagrangian

$$L_{matt}(g_{\mu\nu}, \psi) + L_{scalar}(g_{\mu\nu}, \phi) \Rightarrow \delta Q \sim k^\mu k^\nu (T_{\mu\nu}^M + T_{\mu\nu}^\phi)$$

most general contribution to the heat flux:

assume the action constructed out of first derivatives of the scalar field

$k^\mu k^\nu$  projection  $\Rightarrow$  no relevant contribution from interaction terms

$$\Rightarrow \delta Q_{scalar} \sim \frac{\Omega(\phi)}{\phi} k^\mu k^\nu \nabla_\mu \phi \nabla_\nu \phi$$

- ▶ restricting to  $\Omega(\phi) \equiv \Omega$  constant, for simplicity:

$$\Rightarrow \frac{\delta Q}{T} = - \int d^4x \sqrt{h} \lambda \left( 2\pi T_{\mu\nu}^M k^\mu k^\nu + \left( \frac{\Omega}{\phi} \right) k^\mu k^\nu \nabla_\mu \phi \nabla_\nu \phi \right)$$

## extension to generalized Brans-Dicke gravity

---

### ENTROPY BALANCE

- ▶ at local level:

$$\phi R_{\mu\nu} - \nabla_{\mu} \nabla_{\nu} \phi + \left( \frac{3/2 - \Omega}{\phi} \right) \nabla_{\mu} \phi \nabla_{\nu} \phi + \Phi g_{\mu\nu} = 2\pi T^M{}_{\mu\nu}$$

$\Rightarrow$  by defining the Dicke constant  $\omega = \Omega - 3/2$  one obtains the constitutive relations capturing any Brans-Dicke theory

- ▶  $\Phi$  come from local matter-energy conservation + Bianchi identity + ....

$$\begin{aligned} \nabla_{\nu} \Phi &= \nabla_{\nu} \left( \square \phi - \frac{1}{2} \phi R + \frac{\omega}{2\phi} \nabla_{\mu} \phi \nabla^{\mu} \phi \right) + \\ &+ \left( \frac{1}{2} R + \frac{\omega}{2\phi^2} \nabla_{\mu} \phi \nabla^{\mu} \phi + \frac{\omega}{\phi} \square \phi \right) \nabla_{\nu} \phi \end{aligned}$$

... plus an **integrability condition** on the last term  $(\dots) \nabla_{\nu} \phi = \nabla_{\nu} V(\phi)$

then 
$$\frac{dV}{d\phi} = R + \frac{\omega}{\phi^2} \nabla_{\mu} \phi \nabla^{\mu} \phi + 2 \frac{\omega}{\phi} \square \phi$$

$$\Rightarrow \Phi = \square \phi - \frac{1}{2} \phi R + \frac{\omega}{2\phi} \nabla_{\mu} \phi \nabla^{\mu} \phi + \frac{1}{2} V(\phi)$$

## extension to generalized Brans-Dicke gravity

---

... therefore: from  $S = \int d^4x \sqrt{h} \phi(x)$

$$\Rightarrow I_{gen} = \frac{1}{4\pi} \int \sqrt{-g} d^4x \left[ (\phi R - \frac{\omega}{\phi} \nabla_\mu \phi \nabla^\mu \phi + V(\phi)) + L_{matt} \right]$$

- ▶ the entropy functional holds for the general Brans-Dicke theory
- ▶ **IMPORTANT:** the information about the dynamics of the scalar d.o.f. is encoded in the **integrability condition** together with the trace of the metric field equation

$\omega = 0 \Rightarrow$  metric  $F(R)$

$$3\Box\phi + 2V(\phi) - \phi \frac{dV}{d\phi} = (2\pi) T^M{}_\mu{}^\mu \Rightarrow \text{propagating scalar sourced by matter}$$

$\omega = -3/2 \Rightarrow$  Palatini  $F(R)$

$$2V(\phi) - \phi \frac{dV}{d\phi} = (2\pi) T^M{}_\mu{}^\mu \Rightarrow \text{scalar and matter in algebraic relation: non propagating scalar}$$

## summary

---

- ▶ the thermodynamical derivation works for the simplest cases of scalar tensor higher order gravity, naturally providing the dynamics for the extra dynamical scalar d.o.f.
- ▶ **but...  $f(R)$  is** still special since they are trivially related to GR coupled to a scalar field by a field dependent conformal rescaling of the metric: no new lesson

### MAIN POINT

can we capture higher curvature corrections to GR with the local thermodynamic reasoning ?

- NO** no clear thermodynamical interpretation for higher curvature corrections to entanglement and high: thermal derivation from entanglement is limited to theories where  **$S \sim A$**

Fursaev, Solodukhin

- YES** **Noether charge entropy** functional naturally brings the extra information about curvature corrections (e.g. Lovelock gravity): it works!

Parikh 98, Padmanabhan, Jacobson 2011

## summary

---

For stationary horizons, Wald entropy for  $L = R + aR^2 + \dots$  is

$$S_{bh} = \frac{A}{4\hbar G_N} + \text{curvature terms} = \frac{2\pi}{\hbar} \int_{\Sigma} Q^{ab}[\chi] N_{ab} dA$$

$$Q^{ab}[\chi] = W^{abc} \chi_c + X^{abcd} \nabla_c \chi_a$$

Noether potential for the  
horizon generating Killing flow

For  $L = L[g_{ab}, R_{abcd}]$ , can choose  $X^{abcd} = \frac{\partial L}{\partial R_{abcd}}$

but... and  $W^{abc} = 2\nabla_d X^{abcd}$

Cardoso 99

- ▶ not complete thermodynamical interpretation : problems with 2nd Law
- ▶ the causal structure of higher curvature theories is generally not the metric light cone
- ▶ Noether charge is associated to a gravitational Lagrangian: **Entropy loses its statistical interpretation**

it seems that the **local** thermodynamical derivation can only capture the leading order area term in the entropy....

## thermodynamical equilibrium as a general principle ?

---



in fact the local character makes the derivation very general: can we consider the entropy balance equation as a general principle for deriving the gravitational dynamics ??

### further investigations :

- ▶ confirm the validity of the approach at the GR level in a **different formalism**  $\Rightarrow$  Plebanski formulation of GR in terms of self dual forms
    - > **BF theory with constraints**
  - ▶ extend the approach with more classical geometric variables: **new degrees of freedom**  $\Rightarrow$ 
    - metric affine theories
    - non zero torsion: Einstein-Cartan
    - > **Poincare' gauge gravity**
- $\Rightarrow$  **characterize possible continuous limits in an emergent geometry perspective**

# thermodynamical equilibrium as a general principle ?

---

## interesting relations and hints :

- ▶ **fluid/gravity duality in flat spacetime**

Bredberg 2011, Compere 2011, Chirco 2011

- ▶ **holographic entanglement entropy and spacetime reconstruction**

Raamsdonk 2012, Takayanagi 2012

⇒ **provide a dynamical principle for the geometry  
'emerging' from some holographic quantum theory**

## intrinsic local character:

**just a failure or telling something deep on the  
holographic principle ...?**

**obrigado !**