Inverse scattering construction of dipole black rings

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Introduction / The big picture

Context: Gravity in higher dimensions is much richer than in 4D. Novel features include:

1. more than 1 independent rotation plane;
2. existence of black holes (BHs) with non-spherical topology of the horizon;
3. non-uniqueness – a BH is not uniquely specified by its conserved charges;
4. possibility of non-conserved dipole charges, when considering gauge fields.

While 1 is obvious, novelties 2 and 3 are realized by the Emparan-Reall black ring [1], whose horizon has $S^1 \times S^2$ topology. Feature 4 is explicitly displayed by Emparan's dipole ring solutions [2]. The presence of dipoles promotes the *a priori* discrete BH non-uniqueness to a continuous non-uniqueness.

For D=5 vector-coupled black rings, (magnetic) dipoles arise from integration of the magnetic components of the two-form field strength over a 2-sphere linking the ring's horizon.

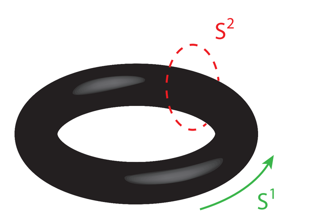


Fig.1 Topology of a 5D black ring.

Considerable amount of guesswork has usually been involved in the discovery of many higher-dimensional black holes (e.g., black ring [1], dipole ring [2]). Whenever solution-generating techniques are available, much quicker progress in the construction of new black hole solutions has been possible.

Main problem: A technically challenging problem is the construction of the most general asymptotically flat black ring in a simple 5D supergravity theory — a five-parameter solution carrying mass, two angular momenta, electric charge and magnetic dipole charge. Until recently the main stumbling block was the systematic generation of the dipole charge and the most general known solutions possessed only 3 parameters.

It was conjectured in [3] that a (non-supersymmetric) black ring should exist in minimal supergravity with all five charges (M, J_ψ, J_ϕ, Q, q) independent. The same conjecture should hold for D=5 Einstein-Maxwell-dilaton, a venue very similar to minimal supergravity, specially in the case of Kaluza-Klein dilaton coupling.

Motivation: In addition to the desire of having a complete knowledge of the spectrum of black ring solutions, the search for such conjectured solutions is motivated by the possibility of a microscopic account of their entropy. In fact, string theory realizations of charged 5D black holes were essential in deriving a microscopic interpretation of the Bekenstein-Hawking entropy formula.

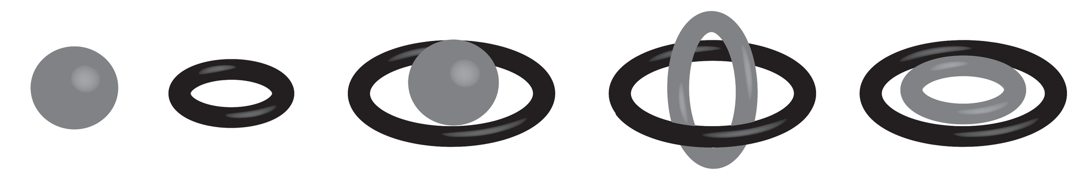


Fig.2 Some exactly known asymptotically flat, 5D vacuum black hole systems (from left to right): Myers-Perry black hole, singly- or doubly-spinning black ring, black Saturn, bicycling ring, di-ring.

Goal: We will show how to systematically generate charged black ring solutions in Einstein-Maxwell-dilaton (EMd) theory with a specific dilaton coupling,

$$S = \frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\left(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}e^{-a\phi}F_{\mu\nu}F^{\mu\nu}\right)\,, \qquad a = \frac{2\sqrt{2}}{\sqrt{3}}\,. \quad (1)$$

This theory can be obtained by dimensionally reducing 6D vacuum gravity on a circle. It can also be thought of as the NS sector of low energy string theory in Einstein frame.

Methods

Inverse scattering method (ISM): Over the past decade, solution-generating techniques have allowed for the ´tailoring` of higher-dimensional black holes. The ISM [4] has proved extremely useful to construct solutions of vacuum gravity in cases when sufficient symmetry is available. More specifically:

1. The ISM can be applied to the construction of cohomogeneity-2 solutions of D-dimensional vacuum gravity displaying D−2 commuting isometries;
2. This class of solutions can be asymptotically flat only for D≤5. For D≥6 one necessarily has Kaluza-Klein (KK) asymptotics [5].

The ISM relies on the integrability properties of a $GL(D-2,\boldsymbol{R})$ non-linear sigma model on the corresponding two-dimensional surface. Inverse-scattering techniques are particularly well suited to generate black rings in 5D since these solutions admit three commuting Killing vectors (along time and two angular directions). Due to 2 it would seem that the applicability of the ISM for the construction of asymptotically flat (AF) solutions is limited to dimensions smaller than six… but we show otherwise in the following section.

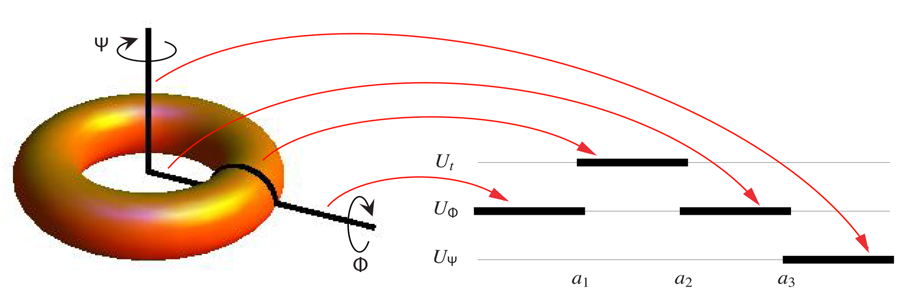


Fig.3 Mapping between the fixed point sets of the isometries (on the left) and the rod diagram (on the right)

Review of the ISM: Consider stationary, axisymmetric solutions of Einstein's equations in vacuum, $R_{\mu\nu}=0$, and assume the existence of D-2 commuting Killing vector fields. Then the metric may be expressed in canonical form [5, 6], which is block diagonal and cohomogeneity-2:

$$ds^2 = \sum_{i,j=0}^{D-3} G_{ij}(\rho,z)dx^i dx^j + e^{2\nu(\rho,z)}\left[d\rho^2 + dz^2\right]\,, \qquad \det G = -\rho^2\,, \quad (2)$$

The vacuum Einstein equations then separate into two groups:

$$\partial_\rho U + \partial_z V = 0\,, \qquad \text{where} \quad U \equiv \rho(\partial_\rho G)G^{-1}\,, \quad V \equiv \rho(\partial_z G)G^{-1}\,.$$

$$\partial_\rho \nu = -\frac{1}{2\rho} + \frac{1}{8\rho}\mathrm{Tr}(U^2 - V^2)\,, \qquad \partial_z \nu = \frac{1}{4\rho}\mathrm{Tr}(UV)\,.$$

The integrability condition $\partial_\rho\partial_z\nu = \partial_z\partial_\rho\nu$ is automatically satisfied, so we need only care about G_{ij}.

Constructing static (diagonal) solutions within this class is straightforward [5]. Writing the Killing part of the metric in terms of ´potentials` $U_i(\rho,z)$,

$$G = \mathrm{diag}\{-e^{2U_0}, e^{2U_1}, e^{2U_2}, \dots\}\,,$$

the problem reduces to finding D-2 solutions U_i of the Laplace equation in an auxiliary cylindrically symmetric 3D flat space. Such Newtonian potentials are of course known to be determined by rod-like sources along the axis of rotation (in the auxiliary 3D space). In particular, for constant density rods the potentials are entirely specified by the location of the rod endpoints a_k, which appear in combinations known as solitons and anti-solitons:

$$\mu_k = \sqrt{\rho^2 + (z-a_k)^2} - (z - a_k)\,, \qquad \overline{\mu_k} = -\sqrt{\rho^2 + (z-a_k)^2} - (z - a_k)\,.$$

Constraint (2) translates into the restriction that the sources must add up to an infinite rod. In conclusion, <u>static vacuum solutions of the Einstein equations with D-2 commuting isometries are completely determined by rod diagrams</u>, as in Fig.3.

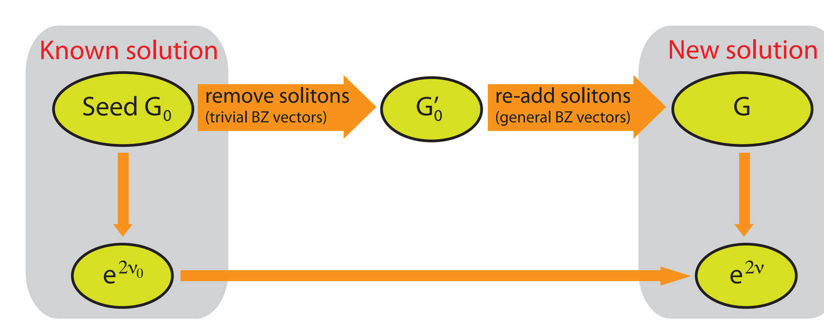
This rod structure classification can be generalized to the stationary (not necessarily diagonal) case [6]. The main difference is that the rods acquire non-trivial directions, which for timelike rods yield information about the angular velocities of event horizons. However, constructing non-static solutions is not as straightforward…

The inverse scattering method, developed by Belinsky-Zakharov (BZ) in [4] for stationary solutions, consists in replacing the original (non-linear) equation for $G(\rho,z)$ by a pair of linear equations for a generating matrix $\Psi(\lambda,\rho,z)$ such that $\Psi(0,\rho,z)=G(\rho,z)$. New solutions can be obtained by ´dressing` the generating matrix Ψ_0 of a known seed solution G_0.

The power of the BZ approach stems from the following observation: <u>If the seed is diagonal and the dressing procedure is restricted to a certain class (solitonic transformations), then the whole scheme is purely algebraic!</u> The only input needed are the set of solitons a_k and a set of (constant) BZ vectors $m_0^{(k)}$ used in the transformations. If the BZ vectors mix time and spatial Killing directions, then this algorithm yields a rotating version of the original static solution.

Some relevant points:

1. Generically, after a solitonic transformation (2) is no longer obeyed (if D>4). This issue can be circumvented by removing solitons with trivial BZ vectors and then re-adding them with more general BZ vectors [7];
2. The seed solution need not be regular;
3. One might need to impose additional constraints to generate a regular solution.



Our approach / Results

From 6D vacuum gravity to 5D EMd: The action (1) can be obtained from 6D vacuum gravity by performing a dimensional reduction on S^1 according to the KK ansatz:

$$ds_6^2 = e^{\frac{\phi}{\sqrt{6}}}ds_5^2 + e^{-\frac{\sqrt{3}\phi}{\sqrt{2}}}(dw + A)^2\,.$$

The sixth dimension is parameterized by w, while A is the vector potential from which the field strength is derived, $F=dA$.

Our approach is now clear: <u>We can generate charged black ring solutions within the theory considered by applying the ISM in 6D and then reducing on a circle down to 5D.</u> Thus, we exploit $GL(4,\boldsymbol{R})$ integrability structure of the theory inherited from 6D vacuum gravity.

Inverse scattering construction of dipole black rings: At this stage we only need to identify a seed solution and the solitonic transformations in order to generate charged black rings of the EMd theory under consideration.

The seed solution adopted in [8] is defined by the rod diagram shown in Fig.4. Although it was motivated by the knowledge of the 6D uplift of the previously known dipole ring [2], it may be used as a starting point for the construction of more general black rings [9,10].

<u>The novel ingredient in our construction is the finite rod along the KK direction w, which allows the addition of dipole charge.</u> The dashed rod in Fig.4 has negative mass density and is included to facilitate adding the S^1 angular momentum to the static seed.

Fig.4 Rod diagram of the seed metric used in [8-10] to generate dipole black rings.

The singly-spinning dipole ring can be generated by the following steps [8].

1. Perform two 1-soliton transformations on the seed G_0 to obtain G_0': remove an anti-soliton at $z=a_0$ with trivial BZ vector (1,0,0,0) and remove a soliton at $z=a_4$ with trivial BZ vector (0,0,0,1);
2. Perform a 2-soliton transformation on G_0' to obtain G: re-add anti-soliton at $z=a_0$ with BZ vector $(1,0,c_1,0)$ and re-add soliton at $z=a_4$ with BZ vector $(0,c_2,0,1)$;
3. Obtain the conformal factor $e^{2\nu}$ from the knowledge of G, G_0 and $e^{2\nu_0}$.
4. Tune the BZ parameters c_1 and c_2 to guarantee regularity of the solution at ($\rho=0$, $z=a_0$) and ($\rho=0$, $z=a_4$);
5. The result (G, $e^{2\nu}$), with the above-mentioned constraints imposed, is the 6D uplift of Emparan's dipole ring.

The general solution thus obtained features a conical singularity along the disk bounded by the black ring. However, one might impose a further condition to ´balance` the ring and remove this pathology.

A simple counting reveals we are in the presence of a 3 parameter solution, which may be identified with mass, one angular momentum and dipole charge:

$$\underbrace{\#a_i}_{5} + \underbrace{\#c_j}_{2} - \underbrace{(\text{translational invariance in z})}_{1} - \underbrace{(\text{regularity conditions})}_{2} - \underbrace{(\text{balance condition})}_{1} = 3$$

More general black rings: To generate more general black rings one needs to introduce rotation along the S^2 component. This may be accomplished with more complicated solitonic transformations [9,10]. Alternatively, one may start with a different seed, already including rotation along the S^1 [11].

Starting from the seed of Fig.4 one can generate a more general black ring by replacing the previous steps 1 and 2 by the following:

i. Remove solitons at a_0, a_1 and a_4 with trivial BZ vectors (0,0,1,0), (0,1,0,0) and (0,0,0,1), respectively, and remove an anti-soliton at a_2 with trivial BZ vector (0,1,0,0);
ii. Perform a 4-soliton transformation that adds back solitons at a_0, a_1 and a_4 with BZ vectors $(c_1,0,1,0)$, $(0,1,b_1,0)$ and $(0,c_2,0,1)$, and adds back the anti-soliton at a_2 with BZ vector $(b_2,1,0,b_3)$;

This procedure introduces 3 additional parameters, b_i.

In [10] we have focused on the simpler case $b_3=0$, but even so the analysis of the novel solution is quite involved. Expressing the metrics generated through the ISM in a simpler form, i.e. converting from Weyl coordinates (ρ,z) to C-metric coordinates (x,y), is typically a tough task. Nevertheless, <u>the full analysis can be performed in Weyl coordinates.</u> This includes finding the regularity and balance conditions, imposing asymptotic flatness (of the 5D solution), and computing all the physical charges.

<u>The solution thus obtained supports both electric charge and magnetic dipole charge, in addition to two angular momenta. Nevertheless, it has only 4 free parameters so these charges are not all independent</u> (see Fig.5, top left panel). The solution presented in [10] appears to be irregular at one of the poles of the S^2. However, we checked that the more general solution with $b_3\neq0$ is regular, so the former might be viewed as a singular limit of a family of regular solutions.

Finally, we confirmed that all physical charges of our solution reproduce those of the dipole ring [2] in the singly-spinning limit ($b_1=b_2=0$) and of the doubly-spinning ring [12] in the neutral limit ($a_4-a_2=b_2=0$). It is interesting that these two solutions can be simultaneously generated with a 3-soliton transformation (by setting $b_2=b_3=0$ in our construction).

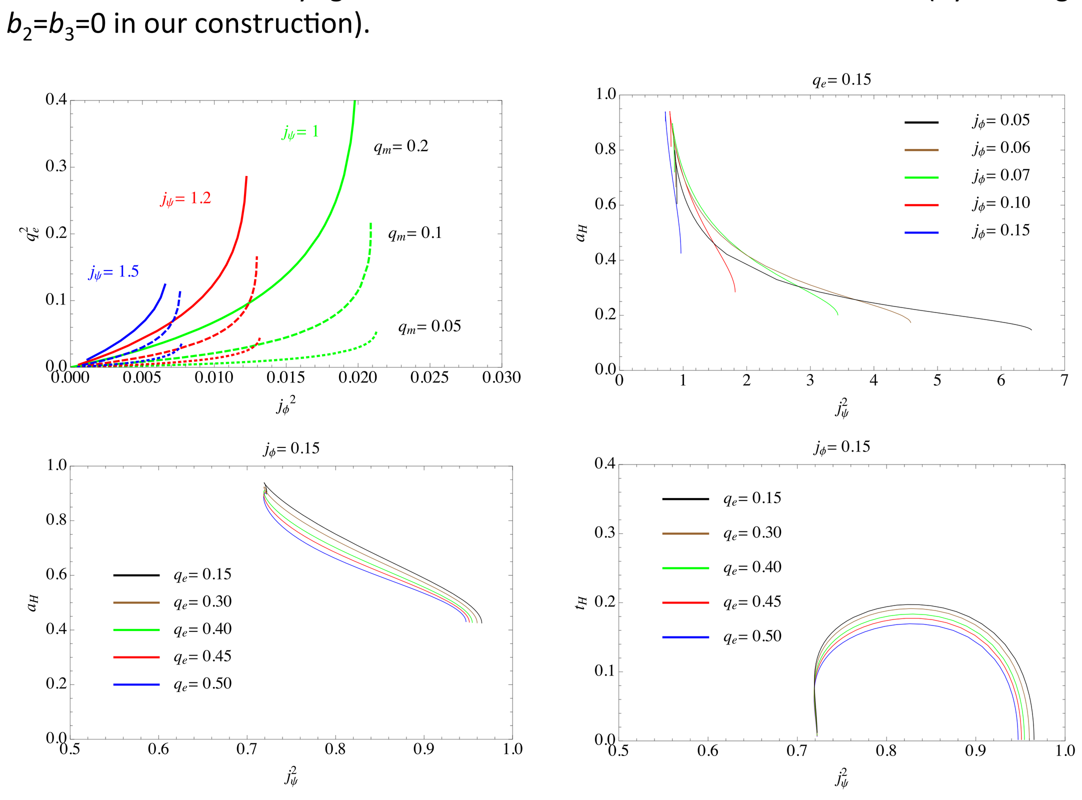


Fig.5 Phase diagrams for various values of fixed charges. The quantities q_e, q_m, j_ϕ, j_ψ, a_H and t_H are dimensionless combinations of the electric charge, magnetic dipole, S^2 angular momentum, S^1 angular momentum, event horizon area and temperature, respectively.

Conclusions / Outlook

We have presented a framework, based on the inverse scattering method for 6D vacuum gravity, suited for the generation of charged, doubly spinning black rings in 5D Einstein-Maxwell-dilaton theory with a particular dilaton coupling. This approach was the breakthrough needed to systematically construct the much sought-after family of most general (i.e., with 5 free parameters) black rings in a minimal extension of 5D general relativity. There has been very recent progress on this subject from various groups.

There is an obvious interest in obtaining solutions for other values of the dilaton coupling. In such cases the approach presented herein cannot be employed, so different techniques must be developed. On the other hand, focusing on problems that can be tackled with this scheme, it would be interesting to generate and investigate the physics of charged concentric black hole systems, within the theory considered.

References

[1] R. Emparan and H. S. Reall, "A Rotating black ring solution in five-dimensions," Phys. Rev. Lett. 88, 101101 (2002).
[2] R. Emparan, "Rotating circular strings, and infinite nonuniqueness of black rings," JHEP 0403, 064 (2004).
[3] H. Elvang, R. Emparan and P. Figueras, "Non-supersymmetric black rings as thermally excited super-tubes," JHEP 0502, 031 (2005).
[4] V. A. Belinski, V. E. Zakharov, "Stationary Gravitational Solitons with Axial Symmetry," Sov. Phys. JETP 50 1 ,1979.
[5] R. Emparan, H. S. Reall, "Generalized Weyl solutions," Phys. Rev. D65, 084025 (2002).
[6] T. Harmark, "Stationary and axisymmetric solutions of higher-dimensional general relativity," Phys. Rev. D70, 124002 (2004).
[7] A. A. Pomeransky, "Complete integrability of higher-dimensional Einstein equations with additional symmetry, and rotating black holes,"Phys. Rev. D73, 044004 (2006).
[8] J. V. Rocha, M. J. Rodriguez and A. Virmani, "Inverse Scattering Construction of a Dipole Black Ring," JHEP 1111, 008 (2011).
[9] Y. Chen, K. Hong and E. Teo, "A doubly rotating black ring with dipole charge," arXiv:1204.5785 [hep-th].
[10] J. V. Rocha, M. J. Rodriguez and O. Varela, "An electrically charged doubly spinning dipole black ring," arxiv:1205.0527 [hep-th].
[11] A. L. Feldman and A. A. Pomeransky, "Charged black rings in supergravity with a single non-zero gauge field," arxiv:1206.1026 [hep-th].
[12] A. A. Pomeransky and R. A. Sen'kov, "Black ring with two angular momenta," hep-th/0612005.

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