Black hole collisions

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(CENTRA/IST & Olemiss)

PRL101:161101,2008; PRL103:131102,2009; arXiv:1006.3081

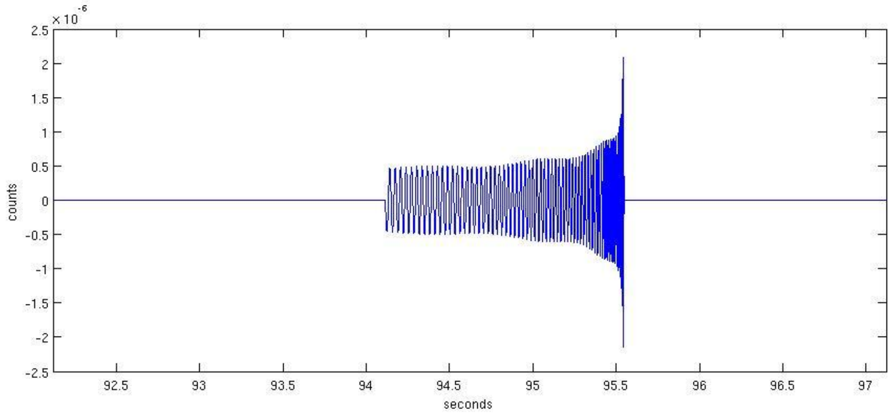
Why study dynamics

Gravitational-wave detection, GW astrophysics

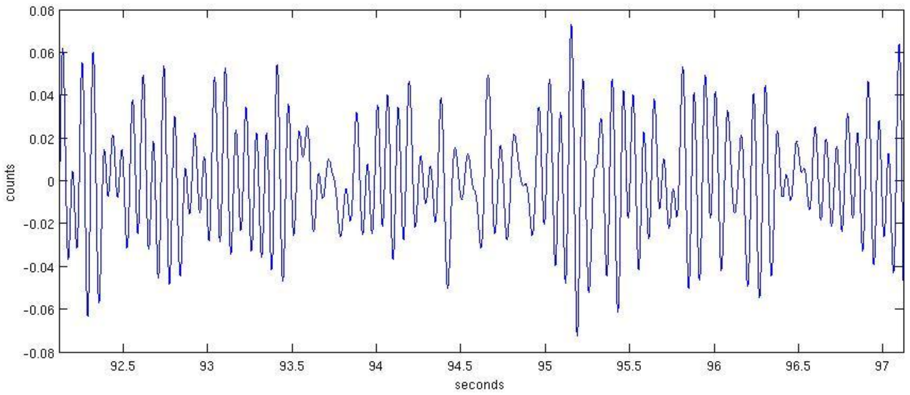
Mathematical physics

High-energy physics

Typical signal for coalescing binaries



Typical stretch of data



A typical problem!

Popular Science - July 1939

Hunts Needle in a Haystack

How long does it take to find a needle in a haystack? Jim Moran, Washington, D. C., publicity man, recently dropped a needle into a convenient pile of hay, hopped in after it, and began an intensive search for (a) some publicity and (b) the needle. Having found the former, Moran abandoned the needle hunt.

Why study dynamics

Gravitational-wave detection, GW astrophysics

Mathematical physics

High-energy physics

Cosmic Censorship: do horizons always form?

✷

Are black objects always stable? Phase diagrams...

✷

Universal limit on maximum luminosity c^5/G *(Dyson '63)*

✷

Critical behavior, resonant excitation of QNMs?

✷

Test analytical techniques, their predictions and power

(Penrose '74, D'Eath & Payne '93, Eardley & Giddings '02)

Why study dynamics

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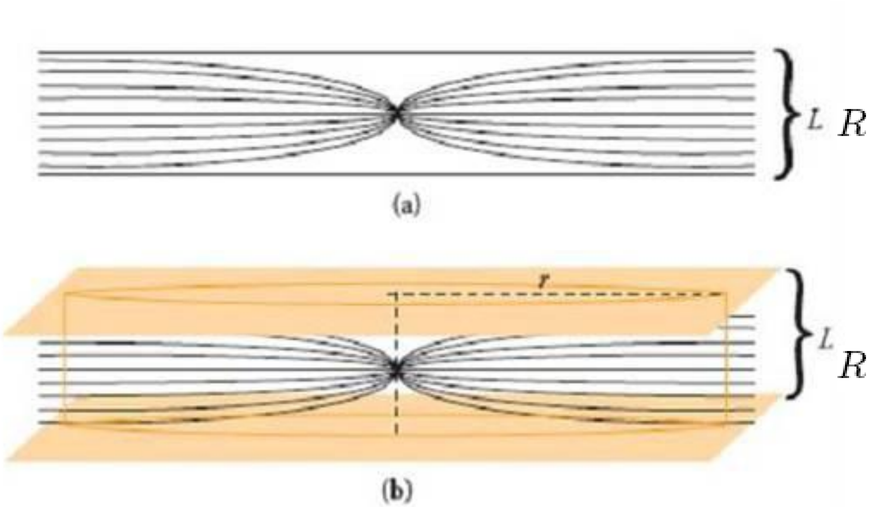
High-energy physics

Braneworlds



$$\mathcal{G} = \frac{G_{n+3}M}{r^{n+2}}$$

$$4\pi r^2 R^n \mathcal{G} = G_{n+3}M$$

$$\mathcal{G} = \frac{G_{n+3}M}{4\pi R^n r^2} = \frac{G_3 M}{r^2}$$

At large distances, one recovers Newton's gravity!

✸

Holography and HIC

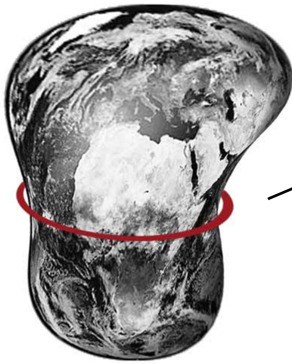
(Nastase '05; Amsel et al '08; Gubser et al '08)

Hoop Conjecture

(Thorne 1972)

"An imploding object forms a BH when, and only when, a circular hoop with circumference 2π the Schwarzschild radius of the object can be made that encloses the object in all directions."

Large amount of energy in small region



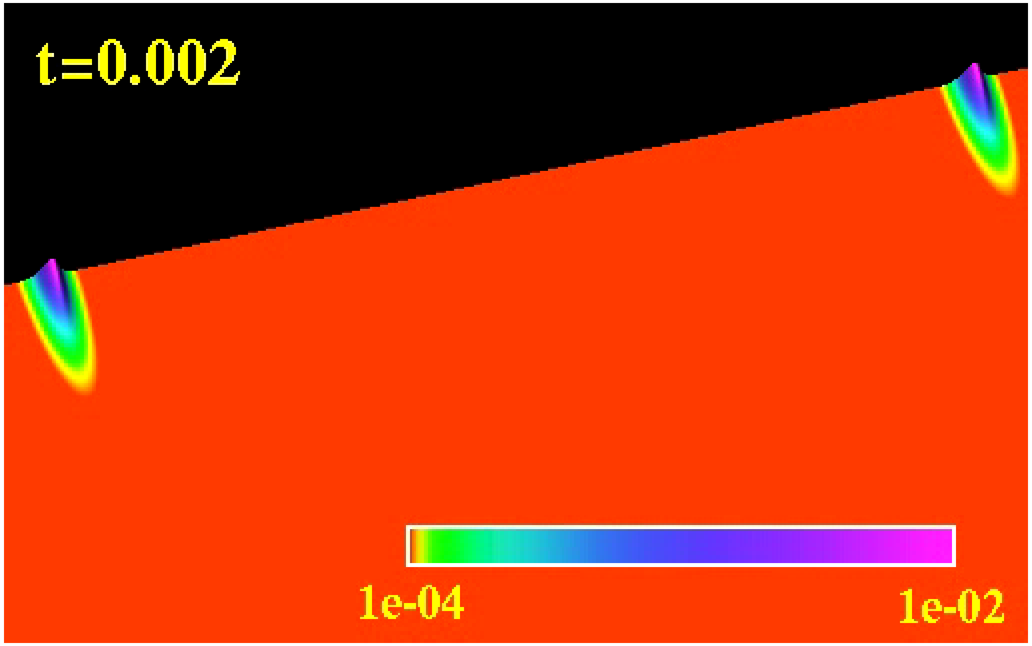
This is the hoop

$R=2GM/c^2$

Size of electron: 10^(-17) cm

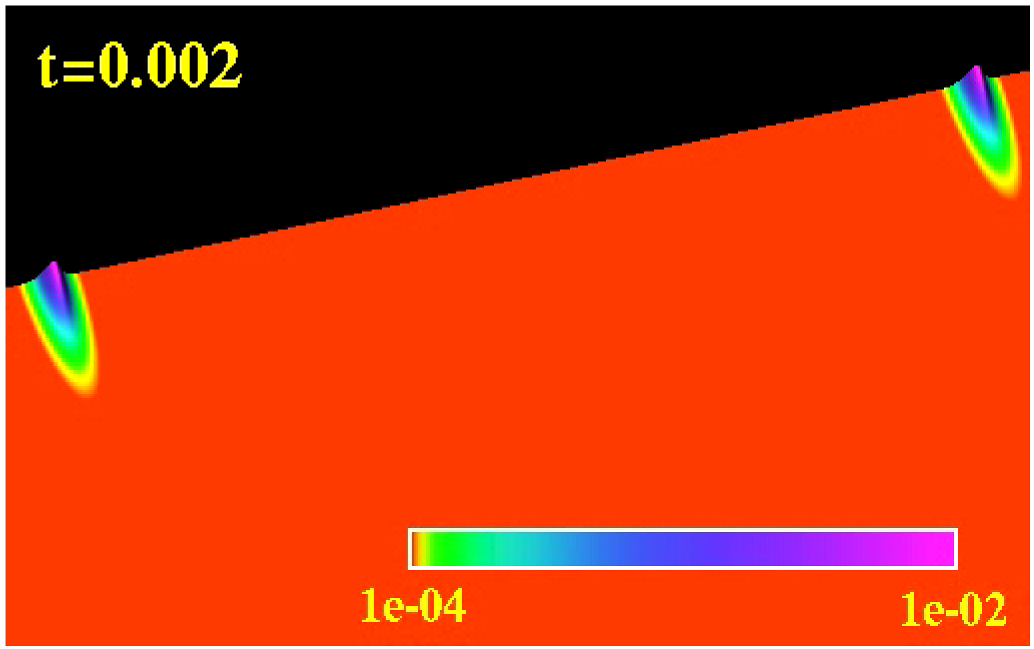
Schwarzschild radius: 10^(-55) cm

$$2M/R = 1/20 \Longrightarrow \gamma_{\text{crit}} \sim 10$$



(Choptuik & Pretorius, Phys.Rev.Lett. 104:111101,2010)

$$2M/R = 1/20 \Longrightarrow \gamma_{\text{crit}} \sim 10$$



(Choptuik & Pretorius, Phys.Rev.Lett. 104:111101,2010)

Black holes do form in high energy collisions

✷

Transplanckian scattering well described by BH collisions!

✷

How to go around and study BH collisions?

Perturbation theory

(*Regge & Wheeler, '57; DRPP '71; Cardoso & Lemos '02*)

Metric=Schwarzschild + small perturbation due to infalling particle

$$h_{\mu\nu} = \begin{bmatrix} H_0(r)f(r) & H_1(r) & 0 & 0 \\ H_1(r) & H_2(r)/f(r) & 0 & 0 \\ 0 & 0 & r^2K(r) & 0 \\ 0 & 0 & 0 & r^2K(r)\sin^2\theta \end{bmatrix} e^{-i\omega t} Y_{l0}(\theta)$$

$$\frac{d^2\Psi}{dr_*^2} + \left(w^2 - V\right)\Psi = \left(1 - \frac{2M}{r}\right) S(\omega, r)$$

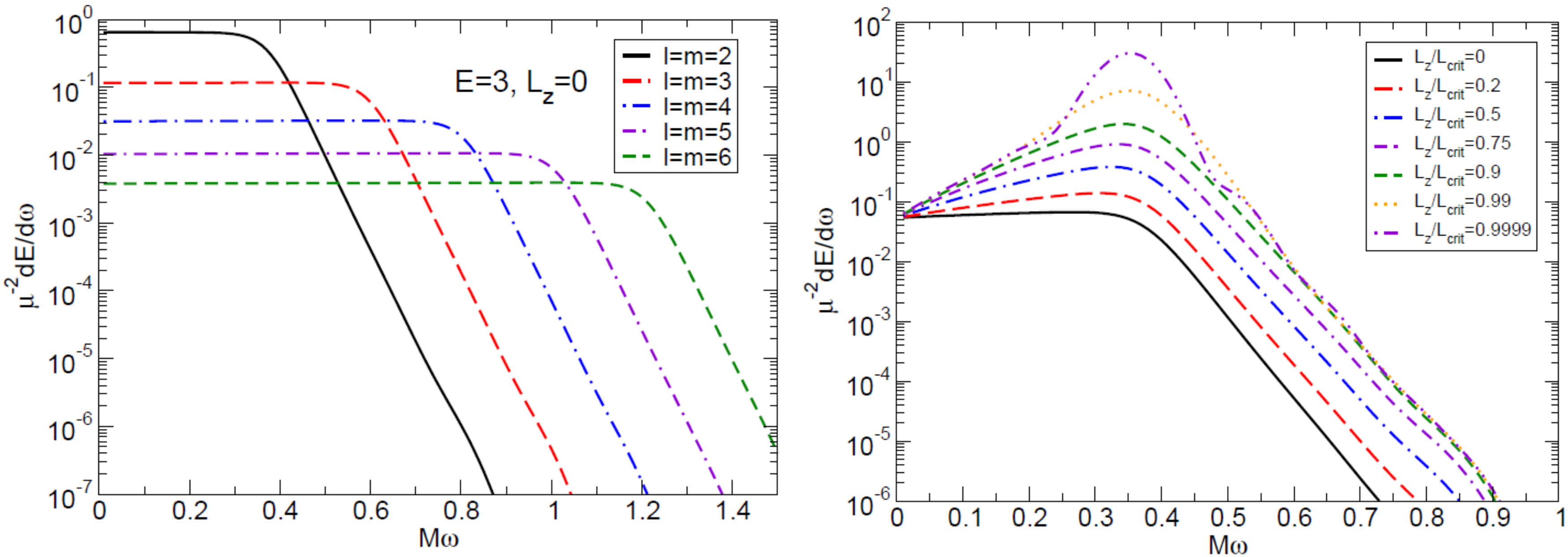
$$\frac{V}{1-2M/r} = \frac{2\lambda^2(\lambda+1)r^3 + 6\lambda^2 r^2 M + 18\lambda r M^2 + 18M^3}{r^3(3M+\lambda r)^2}$$

$$S(\omega, r) = \frac{4i\mu\lambda\sqrt{4\ell+2}}{\omega(3M+\lambda r)^2} e^{-i\omega r_*}\,, \quad \lambda \equiv \frac{(\ell-1)(\ell+2)}{2}$$

Velocity		Radiated energy	Area theorem
0	Point particle	$E_{\rm rad} = 0.010\dfrac{\mu^2}{M}$	$E_{\rm rad} < \dfrac{\mu^2}{M}$
	Equal mass	$E_{\rm rad} = 0.00057M$	$E_{\rm rad} < 0.29M$
1	Point particle	$E_{\rm rad} = 0.26\dfrac{(\mu\gamma)^2}{M}$	$E_{\rm rad} < \dfrac{(\mu\gamma)^2}{M}$
	Equal mass	$E_{\rm rad} = 0.13M$	$E_{\rm rad} < M$

dE/dω flat at sufficiently low ω, multipole $E_l \propto \frac{1}{l^2}$

Roughly 65% of maximum possible at γ=3



(Berti et al, 2010)

Generic D

(Kodama & Ishibashi, '03; Berti, Cardoso & Kipapa, to appear)

D		Radiated energy	Area theorem
5	Point particle	$E_{\rm rad} = 0.016 \frac{\mu^2}{M}$	$E_{\rm rad} < \frac{\mu^2}{M}$
	Equal mass	$E_{\rm rad} = 0.001 M$	$E_{\rm rad} < 0.21 M$
6	Point particle	$E_{\rm rad} = 0.020 \frac{\mu^2}{M}$	$E_{\rm rad} < \frac{\mu^2}{M}$
	Equal mass	$E_{\rm rad} = 0.0012 M$	$E_{\rm rad} < 0.16 M$

v=1: $dE/d\omega \propto \omega^{D-4}$ *(Berti, Cavaglia & Gualtieri, '03)*

ZFL

(Weinberg '64; Smarr '77)

Take two free particles, changing abruptly at t=0

$$T^{\mu\nu} = \sum_{i=1,2} \frac{P_i^\mu P_i^\nu}{E_i} \delta^3(x - v_i t)\theta(-t) + \frac{P_i^{'\mu} P_i^{'\nu}}{E_i'} \delta^3(x - v_i' t)\theta(-t)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,, \qquad h_{\mu\nu} = 4\int \frac{T_{\mu\nu}(t_r, x') - 1/2\eta_{\mu\nu}(t_r, x')}{|x' - x|} d^3x'$$

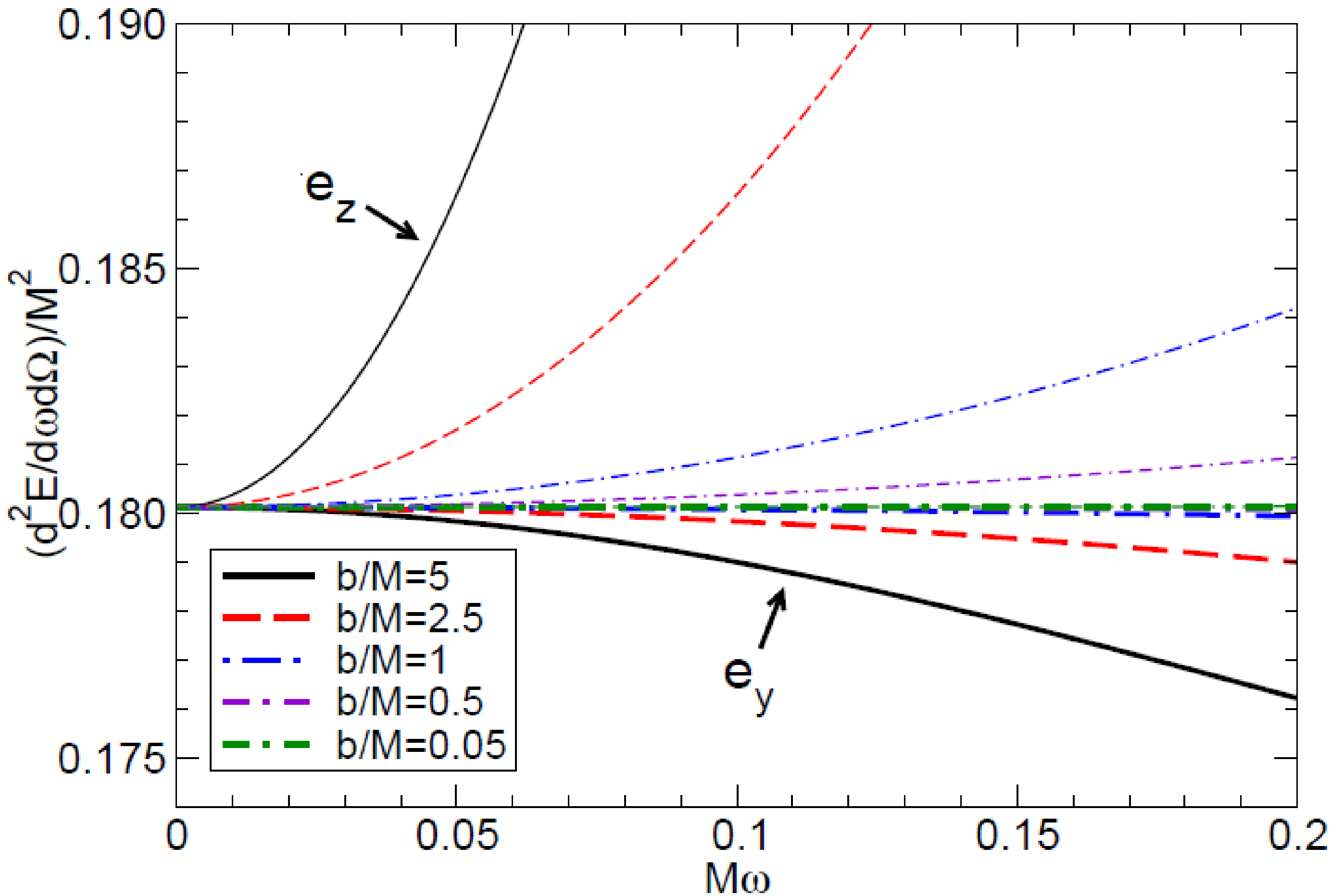
$$\frac{dE}{d\omega d\Omega} = \frac{M^2\gamma^2 v^4}{\pi^2} \frac{\sin^4\theta}{(1 - v^2\cos^2\theta)^2}$$

Radiation isotropic in the UR limit, multipole structure $E_l \propto \frac{1}{l^2}$

Functional relation $E_{rad}(\gamma)$, flat spectrum

Roughly 65% of maximum possible at γ=3

With cutoff M $\omega\approx$0.4 we get 25% efficiency for conversion of gws



(M. Lemos, MSc (2010); Berti et al 2010)

Trapped surface formation

(Penrose '74, Eardley & Giddings '02)

Superpose two Aichelburg-Sexl metrics, find future trapped surface

Upper limit on gravitational radiation: 29% M

Perturb superposed A-S metric, correction: 16% M

(D'Eath & Payne '90s)

Numerical evolution

GR: "Space and time exist together as Spacetime"

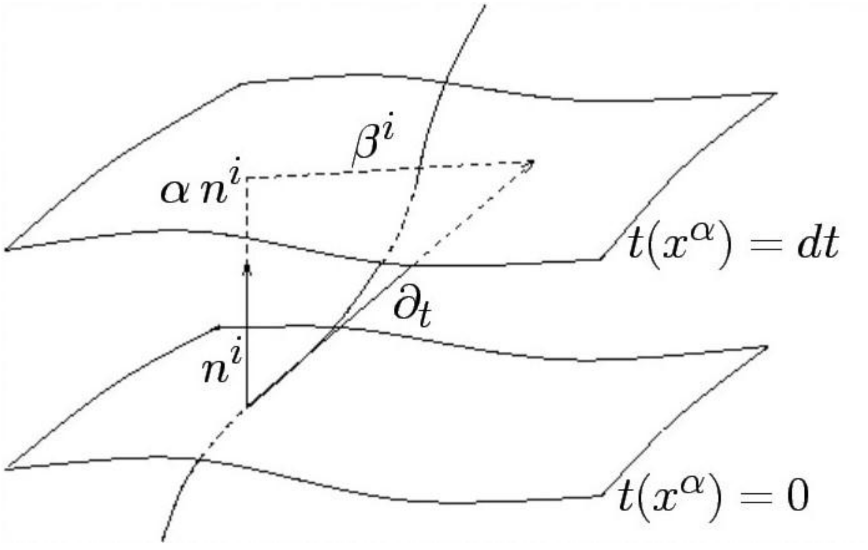
Numerical relativity: reverse this process!

ADM 3+1 decomposition

Arnowitt, Deser, Misner (1962); York (1979); Choquet-Bruh)at, York (1980)

3-metric	γ_{ij}
lapse	α
shift	β^i

lapse, shift $\Rightarrow$ Gauge



Time projection $R_{\alpha\beta}n^{\alpha}n^{\beta}=0 \quad \Rightarrow \boxed{R+K^{2}-K_{ij}K^{ij}=0}$

Hamiltonian constraint

Mixed projection $\perp^{\mu}{}_{\alpha}\, R_{\mu\beta}n^{\beta}=0 \quad \Rightarrow \boxed{D^{i}K-D_{j}K^{ij}=0}$

Momentum constraints

Spatial projection $\perp^{\mu}{}_{\alpha}\perp^{\nu}{}_{\beta}\, R_{\mu\nu}=0$

$$\Rightarrow \boxed{(\partial_{t}-L_{\beta})K_{ij}=-D_{i}D_{j}\alpha+\alpha[R_{ij}-2K_{im}K^{m}{}_{j}+K_{ij}K]}$$

Evolution equations

Numerical simulations

LEAN code (*Sperhake '07)*

Based on the Cactus computational toolkit

BSSN formulation (ADM-like, but strongly hyperbolic)

Puncture initial data (*Brandt & Brügmann 1996)*

Elliptic solver: TwoPunctures (*Ansorg 2005)*

Mesh refinement: Carpet (*Schnetter '04)*

Numerically very challenging!

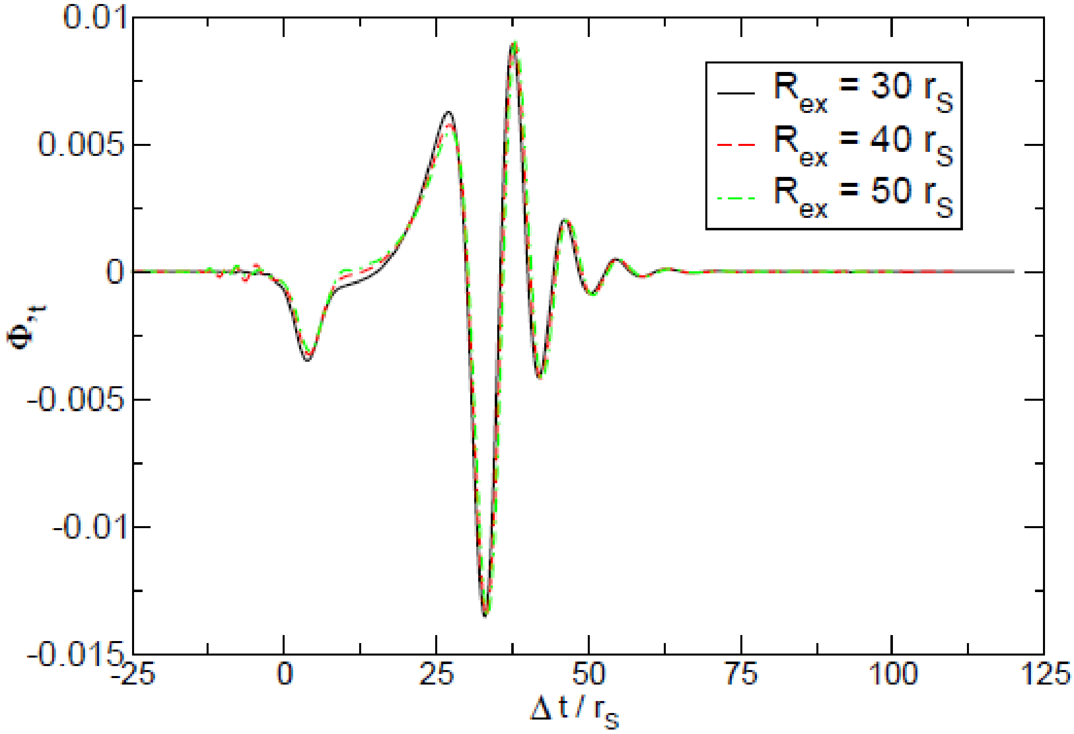
Length scales: $M_{ADM} \propto \gamma M_0$

Horizon Lorentz-contracted "Pancake"

Mergers extremely violent

Substantial amounts of unphysical "junk" radiation

Rest

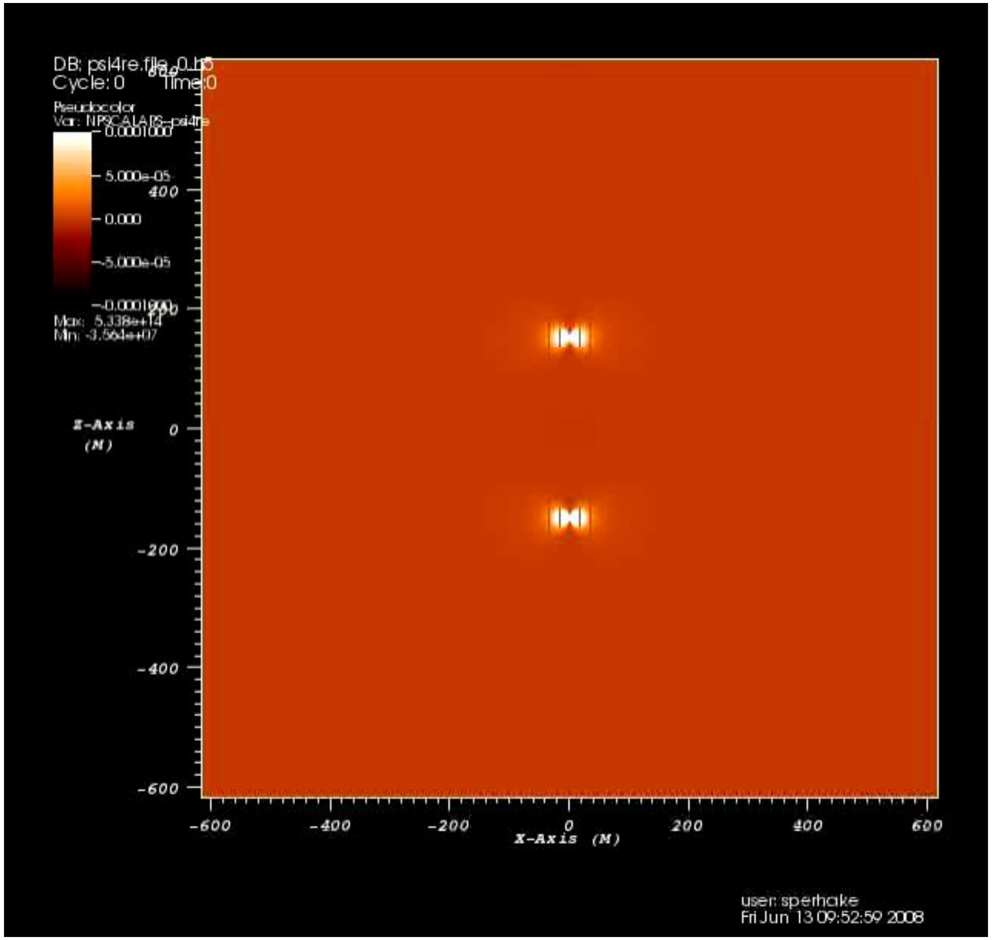


$$E_{\rm rad} = 0.00057M$$

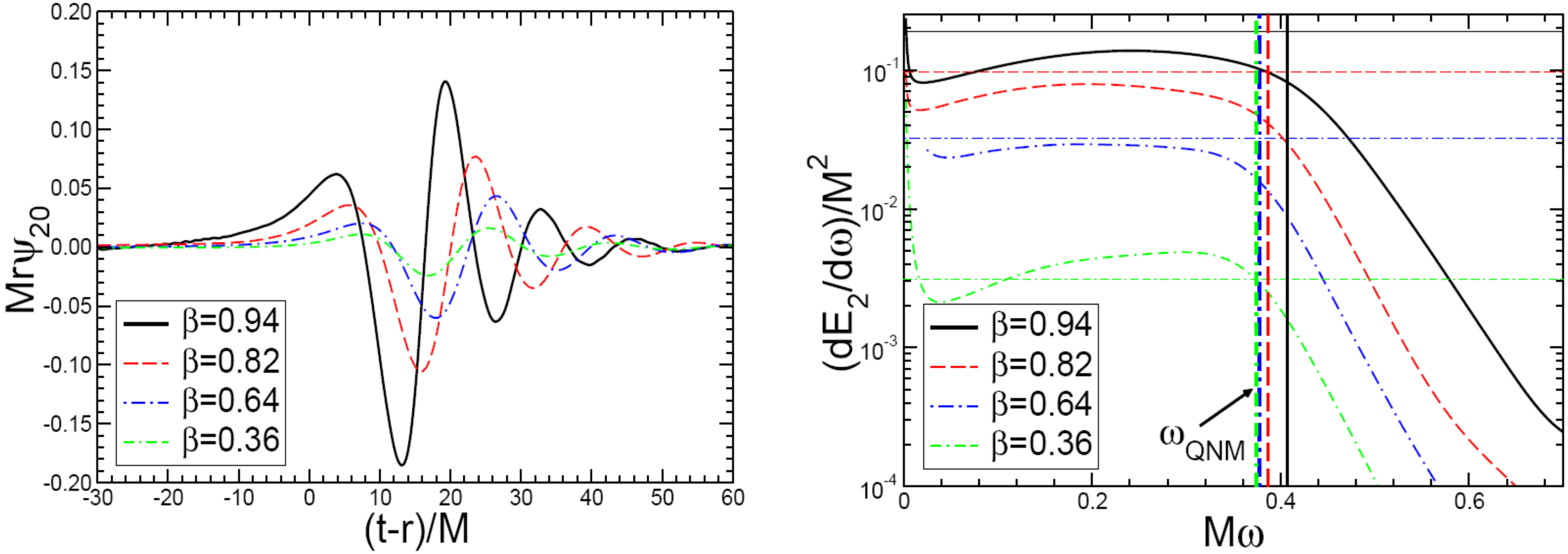
(Witek et al, arXiv:1006.3081 [gr-qc])

High energy head-ons

β=0.93



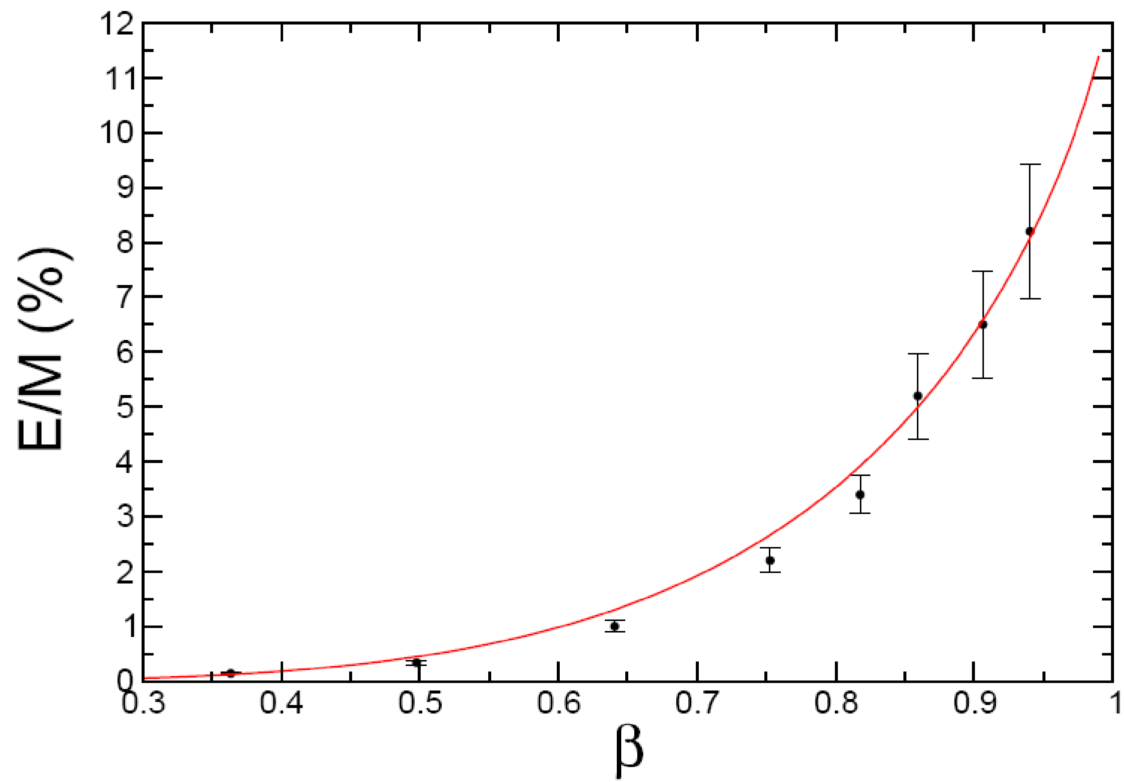
(*Sperhake et al, Phys.Rev.Lett. 101:161101, 2008*)



Waveform is almost just ringdown

Spectrum is flat, in good agreement with ZFL

Cutoff frequency at the lowest quasinormal frequency

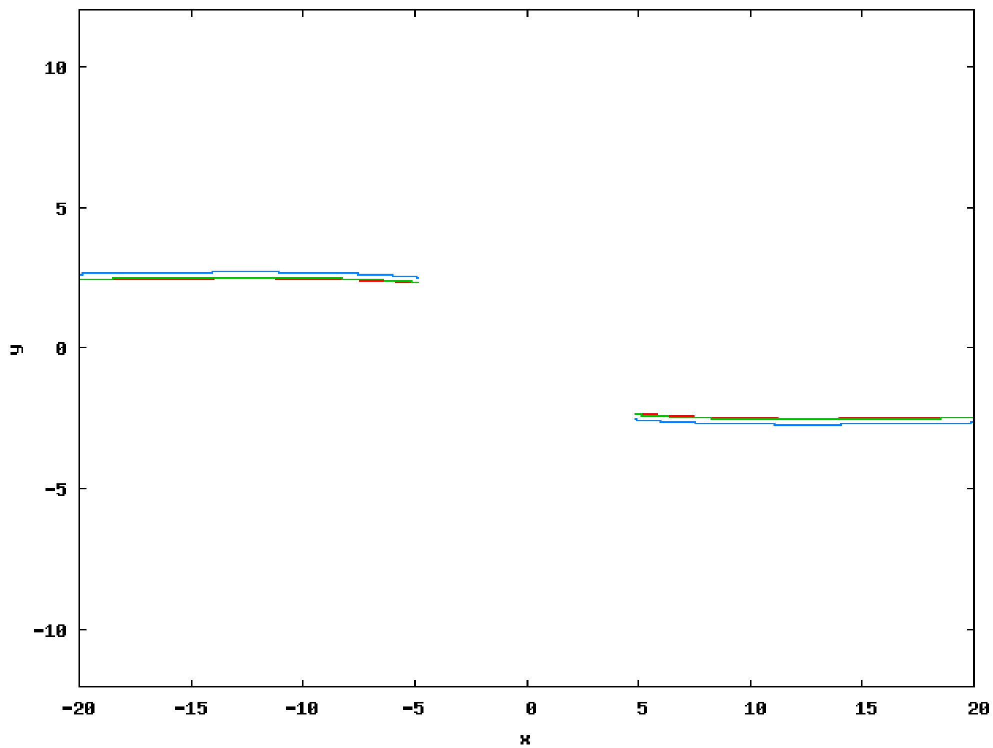


14%

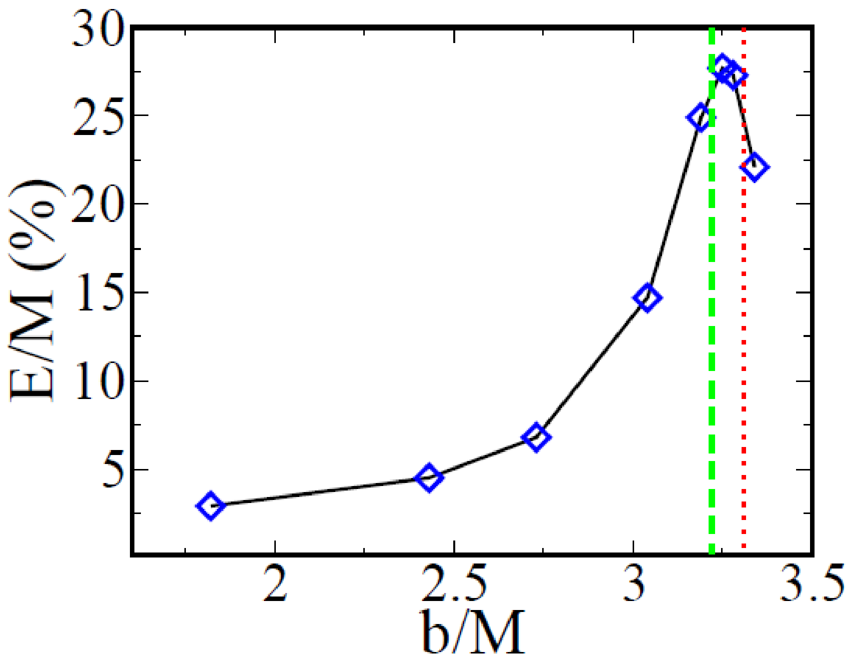
$$\frac{E}{M} = E_\infty \left(\frac{1 + 2\gamma^2}{2\gamma^2} + \frac{(1 - 4\gamma^2)\log\left(\gamma + \sqrt{\gamma^2 - 1}\right)}{2\gamma^3\sqrt{\gamma^2 - 1}} \right)$$

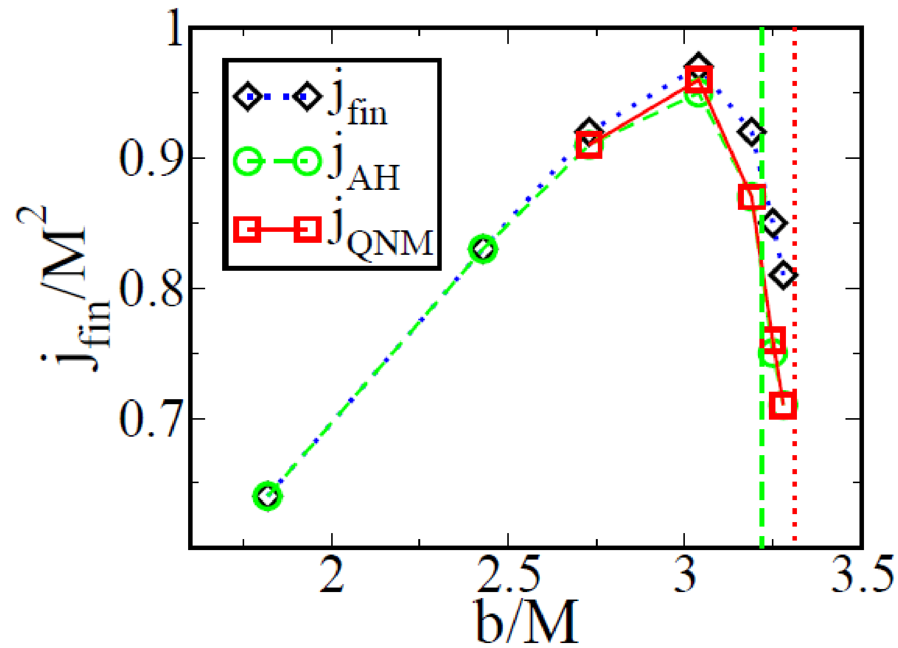
Grazing collisions

Plunge, zoom-whirl and scattering



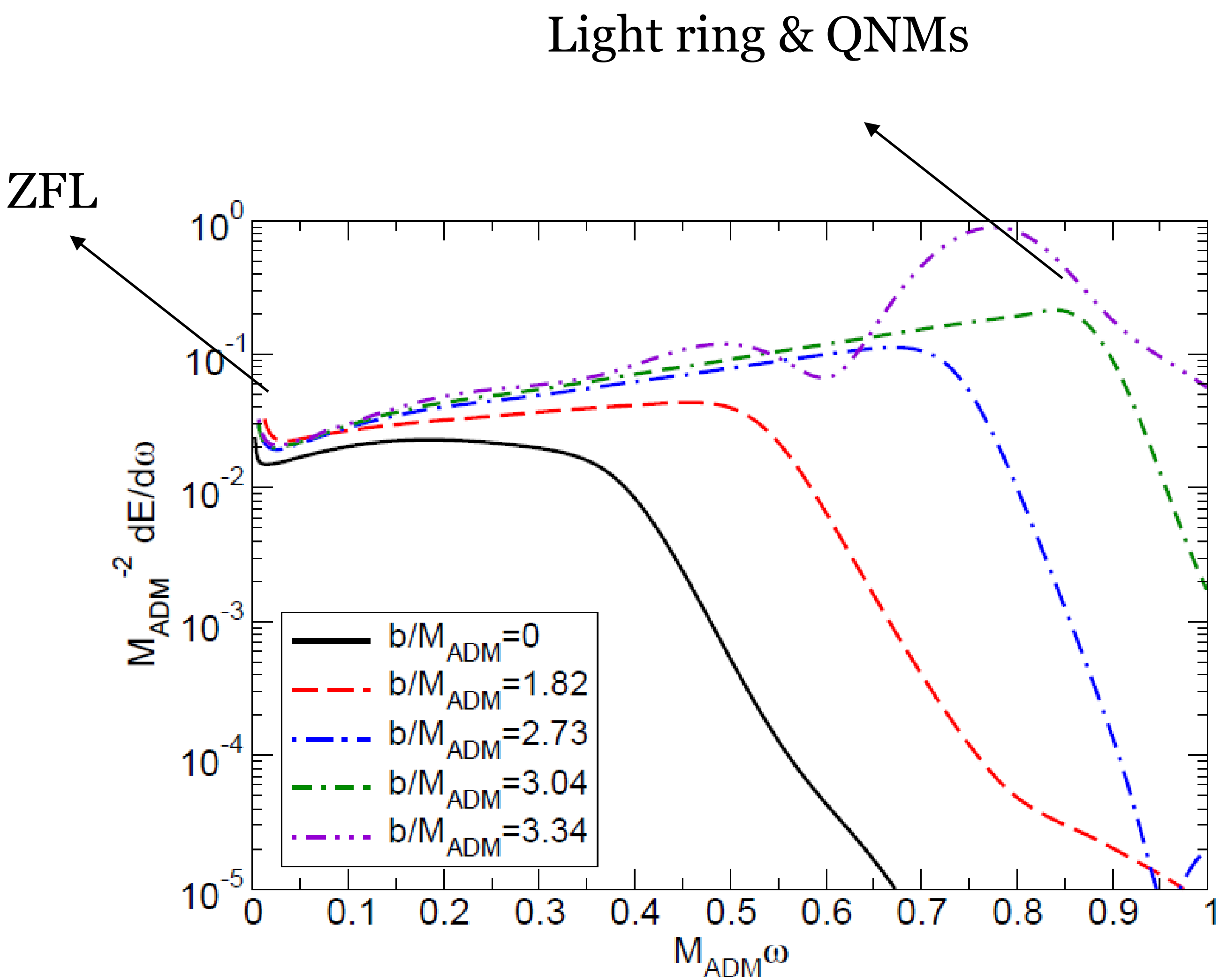
(Sperhake et al, Phys.Rev.Lett. 103:131102, 2008)





More than 25% CM energy radiated for v=0.75 c!

Final BH rapidly spinning



Light ring & QNMs
ZFL
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
M_ADM^-2 dE/dω
M_ADM ω
b/M_ADM=0
b/M_ADM=1.82
b/M_ADM=2.73
b/M_ADM=3.04
b/M_ADM=3.34

Cosmic Censor: as strong as ever

Production Cross-section: $b/M=2.5/v$

Peak luminosity: Close to Dyson limit c^5/G

Maximum spin: >0.95

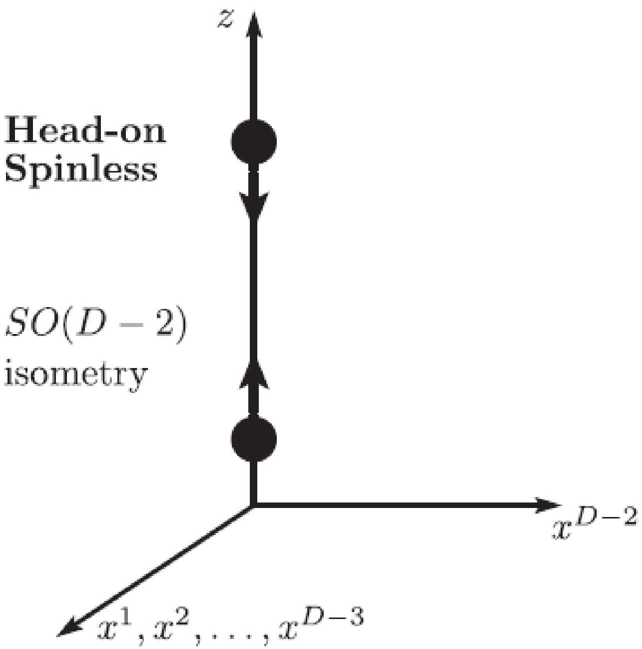
Radiated energy: $>35\%$ CM

Junk: ~2 Erad, interesting topic for further study

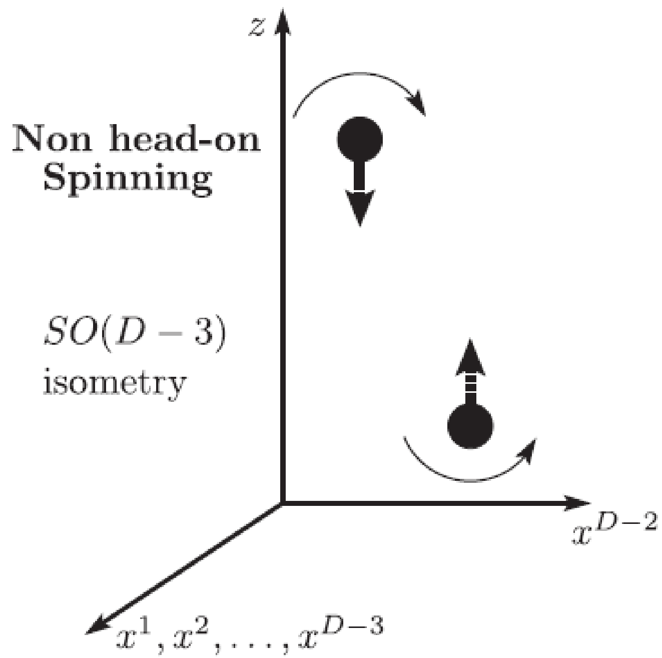
Radiation: Almost just ringdown, relation with ZFL...

Other spacetimes

Axial symmetry



Head-on in D>4



Grazing in D>5

Can be reduced to effective 3+1

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu + e^{2\phi} d\Omega_{D-4}^2 \\
\mu &= 0, 1, 2, 3
\end{aligned}
$$

D-dimensional Einstein equations imply

$$
e^{2\phi} \left[(D-4) \partial^\alpha \phi \partial_\alpha \phi + \nabla^\alpha \partial_\alpha \phi \right] = D - 5
$$

$$
R_{\mu\nu} = (D-4) \left[\nabla_\nu \partial_\mu \phi + \partial_\mu \phi \partial_\nu \phi \right]
$$

Effective 3+1 system with sources

$$(\partial_t - \mathcal{L}_\beta)\,\gamma_{ij} = -2\alpha K_{ij}$$

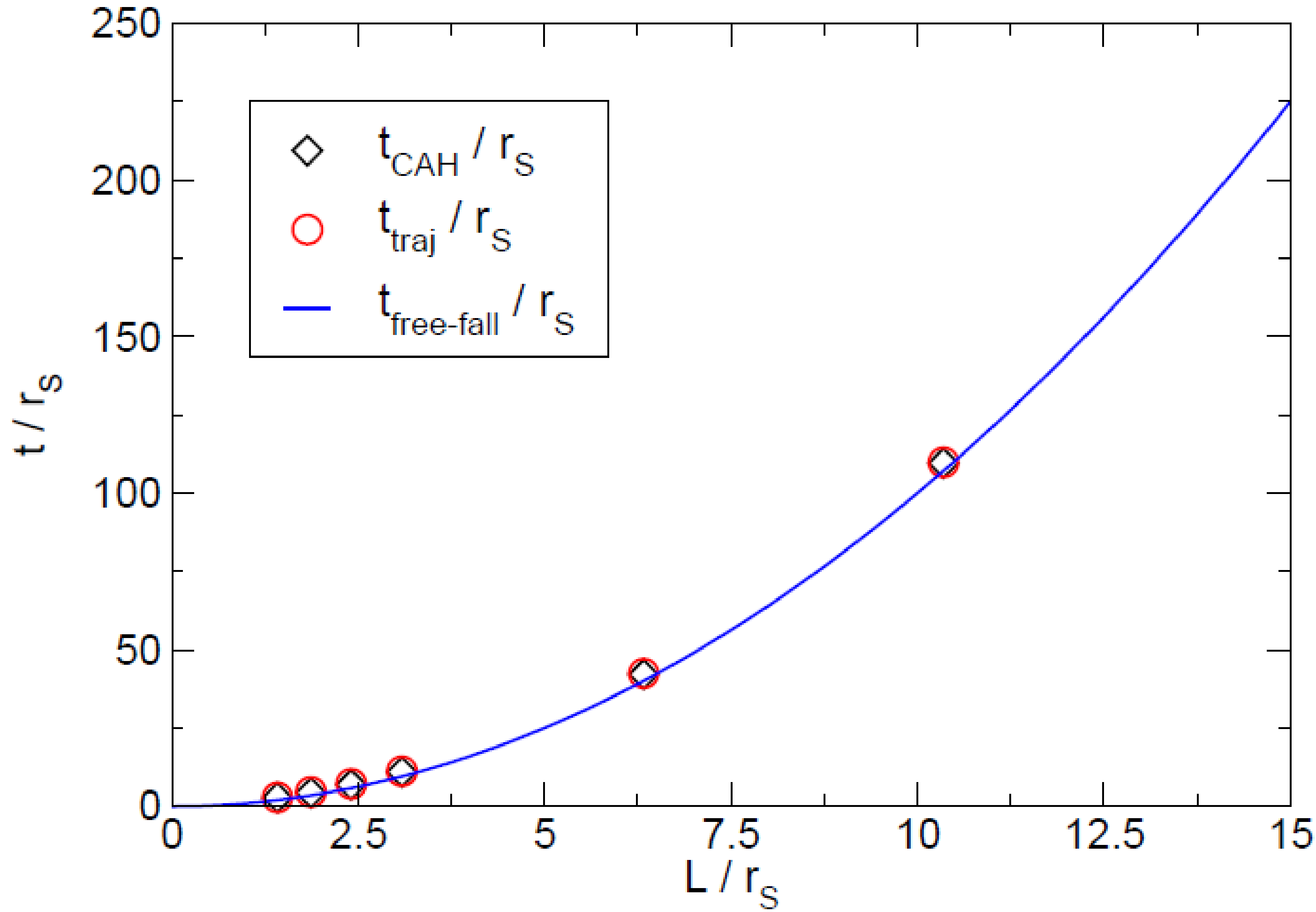
$$(\partial_t - \mathcal{L}_\beta)\,K_{ij} = -D_i\partial_j\alpha + \alpha\left[{}^{(3)}R_{ij} + KK_{ij} - 2K_{ik}K^k_j\right]$$

$$-\alpha(D-4)\left(D_i\partial_j\phi - K_{ij}K_\phi + \partial_i\phi\partial_j\phi\right)$$

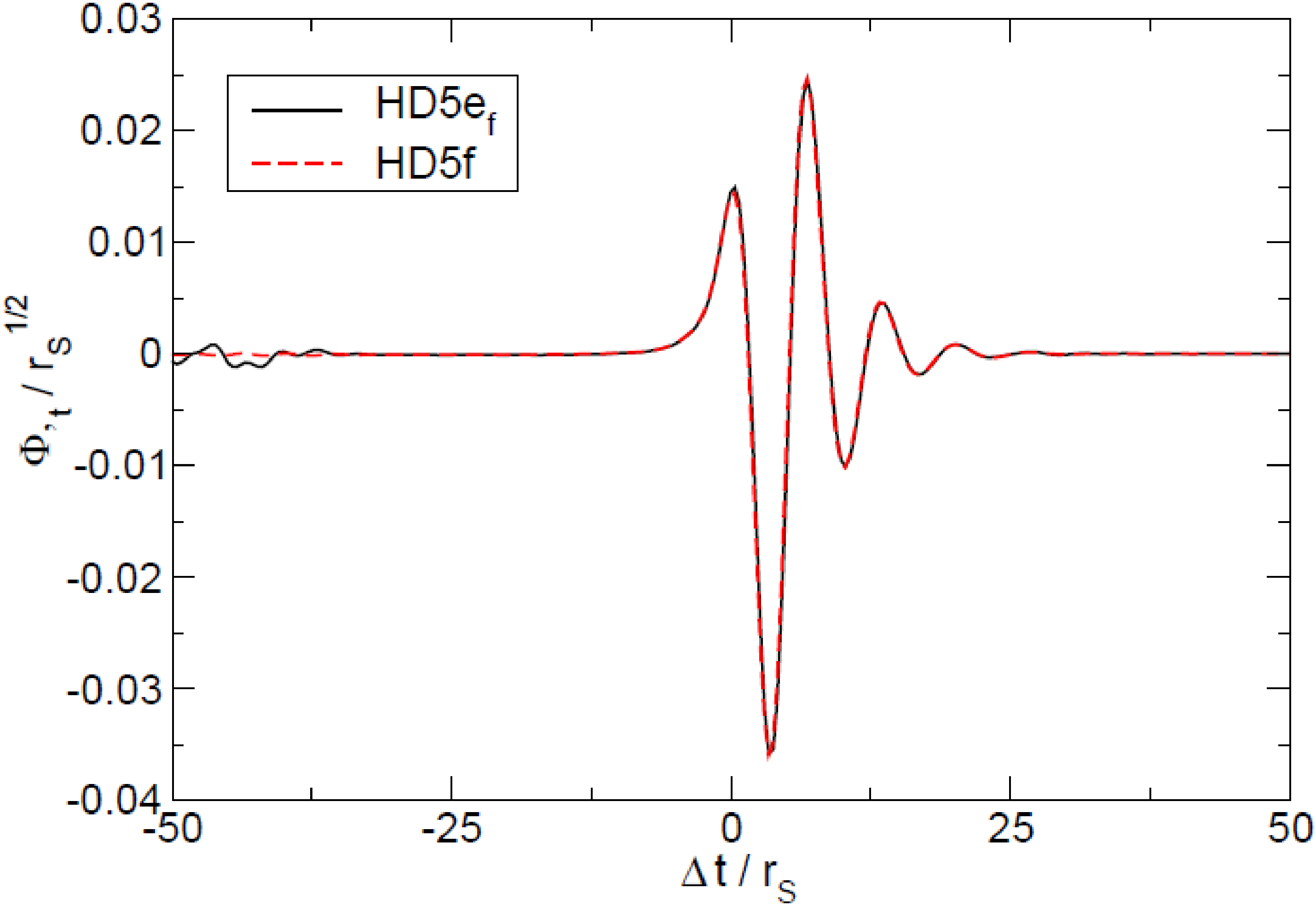
$$(\partial_t - \mathcal{L}_\beta)\,\phi = -\alpha K_\phi$$

$$(\partial_t - \mathcal{L}_\beta)\,K_\phi = \alpha\left[(D-5)e^{-2\phi} - (D-4)\partial_i\phi\partial^i\phi\right]$$

$$+\alpha\left[(D-4)K_\phi^2 + KK_\phi - D^i\partial_i\phi\right] - \partial^i\alpha\partial_i\phi$$



250
200
150
100
50
0
0
2.5
5
7.5
10
12.5
15
t / r_S
L / r_S
t_CAH / r_S
t_traj / r_S
t_free-fall / r_S



HD5e_f
HD5f
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-50
-25
0
25
50
Φ,t / r_S^1/2
Δt / r_S

D	$E/M(\%)$	$E^{\text{area}}/M(\%)$	$E^{\text{PP}}_{\text{ext}}M/\mu^2$	$P^{\text{PP}}_{\text{ext}}M/\mu^2$
4	0.055	29.3	0.0102	0.00083
5	0.089	20.6	0.0160	0.0024

BH collisions are a fascinating topic in GR

Cosmic Censorship preserved

Much remains to be done:

Understand initial data, add charge, go to higher boosts,

higher dimensional spacetimes, compactified EDs, anti-de Sitter

Thank you

Numerical simulations

LEAN code (*Sperhake '07)*

Based on the Cactus computational toolkit

BSSN formulation (ADM-like, but strongly hyperbolic)

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\phi = \tfrac{1}{12}\ln\gamma \qquad \hat{\gamma}_{ij} = e^{-4\phi}\gamma_{ij}$$

$$K = \gamma_{ij}K^{ij} \qquad \hat{A}_{ij} = e^{-4\phi}\left(K_{ij} - \tfrac{1}{3}\gamma_{ij}K\right)$$

$$\hat{\Gamma}^i = \gamma^{ij}\hat{\Gamma}^i_{jk} = -\partial_j\hat{\gamma}^{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\hat{\gamma}_{ij} = -2\alpha\hat{A}_{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\phi = -\tfrac{1}{6}\alpha K$$

$$(\partial_t - \mathcal{L}_\beta)\hat{A}_{ij} = e^{-4\phi}\left(-D_iD_j\alpha + \alpha R_{ij}\right)^{\mathrm{TF}} + \alpha(K\hat{A}_{ij} - 2\hat{A}_{ik}\hat{A}^k{}_j)$$

$$(\partial_t - \mathcal{L}_\beta)K = -D^iD_i\alpha + \alpha\left(\hat{A}_{ij}\hat{A}^{ij} + \tfrac{1}{3}K^2\right)$$

$$\partial_t\hat{\Gamma}^i = 2\alpha\left(\hat{\Gamma}^i_{jk}\hat{A}^{jk} + 6\hat{A}^{ij}\partial_j\phi - \tfrac{2}{3}\hat{\gamma}^{ij}\partial_jK\right) - 2\hat{A}^{ij}\partial_j\alpha + \hat{\gamma}^{jk}\partial_j\partial_k\beta^i$$
$$+\tfrac{1}{3}\hat{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\partial_j\hat{\Gamma}^i + \tfrac{2}{3}\hat{\Gamma}^i\partial_j\beta^j \quad \underbrace{-\left(\chi + \tfrac{2}{3}\right)\left(\hat{\Gamma}^i - \hat{\gamma}^{jk}\hat{\Gamma}^i_{jk}\right)\partial_l\beta^l}_{\text{Yo et al. (2002)}}$$