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Perturbations of slowly-rotating black holes

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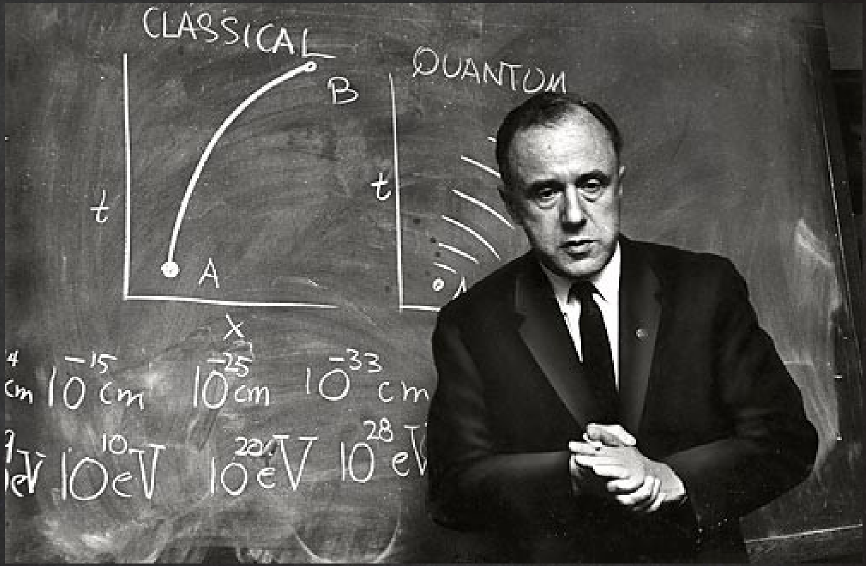
http://blackholes.ist.utl.pt

PP, Cardoso, Gualitieri, Berti, Ishibashi to appear soon

Outline

- **Motivation:** the reach of perturbation theory
- Black hole perturbation theory in a nutshell
- Open problems
- **Kojima's method** for slowly-rotating stars
- **Proca fields on a Kerr background**
  - Equations
  - Superradiant instabilities
  - Astrophysical implications
- **Extensions**
  - Second order
  - QNMs of Kerr-Newman BHs

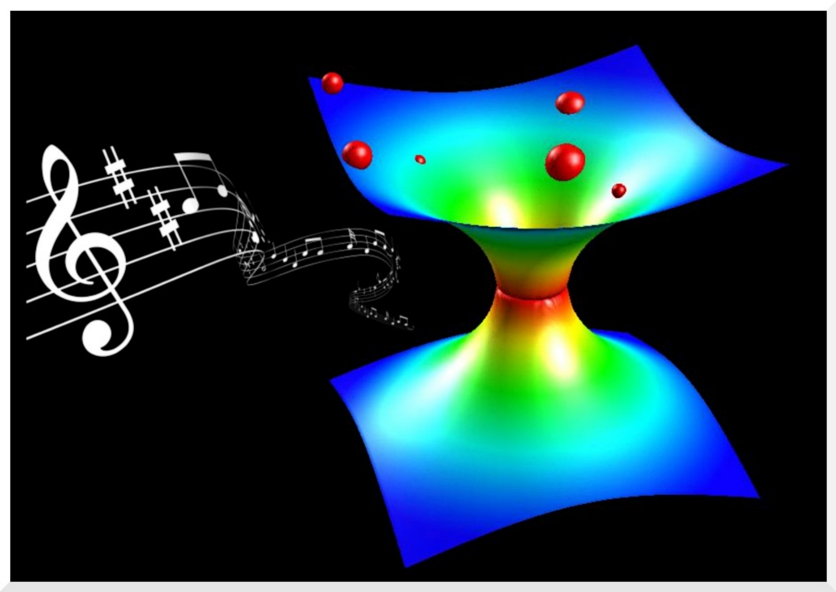


"Black holes teach us that space can be crumpled like a piece of paper into an infinitesimal dot, that time can be extinguished like a blown-out flame, and that the laws of physics that we regard as 'sacred', as immutable, are anything but".

John Archibald Wheeler's Autobiography, 1998

Motivation

- **Perturbation theory is ubiquitous in physics:**
  - Epicycles in Ptolemaic astronomy
  - Stark and Zeeman effects
  - Feynman diagrams
- **Particularly useful in GR:**
  - BH and stellar perturbations
  - gravitational waves, cosmology, PN theory, ...
- **PDEs → ODEs**
- **The reach of PT is given by its ability in simplifying the problem**



"Hearing" the spacetime shape

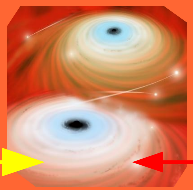
Approx. methods VS Hard numerics

Idealized situations

Physical insights

"Easy" to perform

Approximate methods

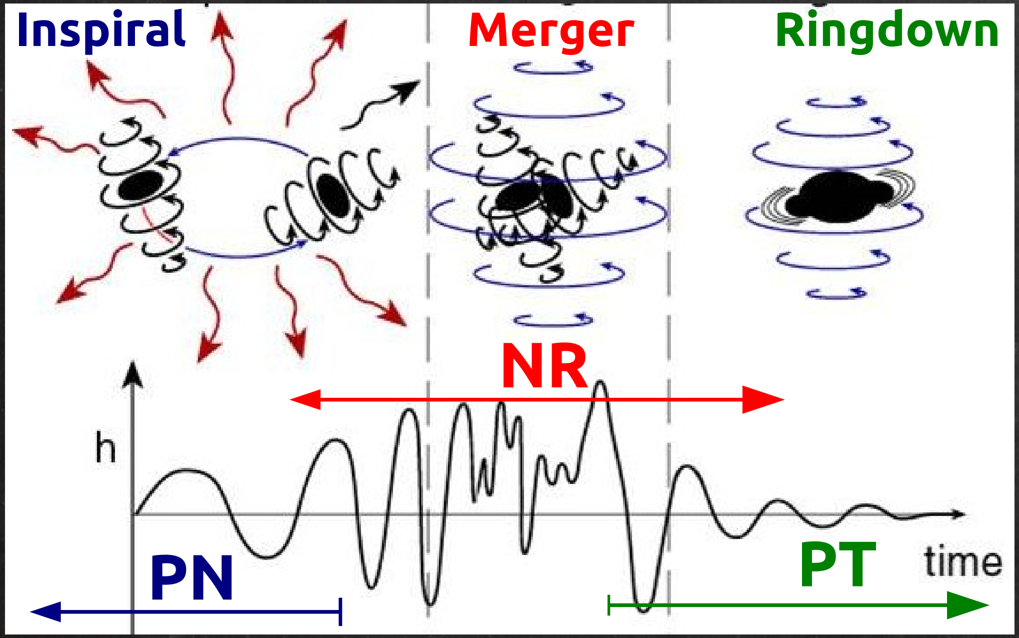


Numerical methods

Realistic situations

Numerics → Physics

Supercomputers



Adapted from Thorne

- Approximate doesn't mean worst!
- Complementary approach
- Synergy between semianalytical and fully numerical methods

Part I
BH perturbations in a nutshell

BH perturbations. *Spherical symmetry*

[Kokkotas & Schmidt 1998]
[Berti et al. 2009]
[Konoplya & Zhidenko 2011]

$$ds^2 = \underbrace{-f(r)dt^2 + h(r)^{-1}dr^2 + r^2 d\Omega_2}_{\text{background}} + \underbrace{(\delta_{\rm RW} g_{\mu\nu})e^{-i\omega t}dx^\mu dx^\nu}_{\text{perturbations}}$$

1) Regge-Wheeler formalism: **Polar** **Axial**

$$||\delta_{\rm RW} g_{\mu\nu}|| = \begin{bmatrix} f(r)H_0(r)Y_{lm} & H_1(r)Y_{lm} & -h_0(r)\dfrac{1}{\sin\theta}\dfrac{\partial Y_{lm}}{\partial\varphi} & h_0(r)\sin\theta\dfrac{\partial Y_{lm}}{\partial\theta} \\ * & \dfrac{H_2(r)Y_{lm}}{h(r)} & -h_1(r)\dfrac{1}{\sin\theta}\dfrac{\partial Y_{lm}}{\partial\varphi} & h_1(r)\sin\theta\dfrac{\partial Y_{lm}}{\partial\theta} \\ * & * & r^2K(r)Y_{lm} & 0 \\ * & * & * & r^2\sin^2\theta K(r)Y_{lm} \end{bmatrix}$$

2) Insert into Einstein eqs: 10 linearized coupled eqs (Mathematica helps!)

3) Fields redefinition and new "tortoise" coordinates: system of linear equations:

$$\frac{d^2\vec{\Psi}}{dr_*^2} + \left[\omega^2 - V(r)\right]\vec{\Psi} = 0$$

4) Solved with suitable boundary conditions (quasinormal modes, instabilities)

5) Any spherically symmetric background, any theory, any field

BH perturbations. *Symmetries matter*

- In spherically symmetry the field eqs. can be always separated
- If the background is rotating, separability is not guaranteed!
- Teukolsky formalism
  - Newman-Penrose tetrad formalism
  - Weyl scalars
  - Separability in Kerr is almost a miracle! (Petrov Type D)

[Teukolsky ~ 1973]
[Teukolsky and Press]
[Chandra's book]

- Perturbations of generic rotating BHs are important:
  - Astrophysical BHs are spinning
  - Stability (e.g. superradiance, r-modes in stars, no-hair theorem)

Non-separable (?) problems

- **Four dimensions**
  - Massive vector (Proca) fields on a Kerr background
  - Gravito-EM perturbations of Kerr-Newman BHs
  - Rotating objects in alternative theories
- **Higher dimensions**
  - Myers-Perry BHs with generic spins
  - Rotating solutions
- **Stability, greybody factors, quasinormal modes?**

Superradiance and BH bomb

[Press and Teukolsky '70]

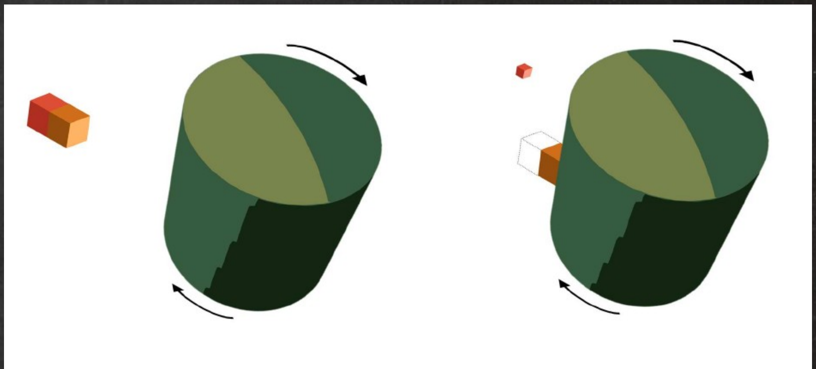
- Amplified scattering of waves
- Requires <u>dissipation</u> → needs an event horizon

[Thorne, Price, Macdonald's book]

[Richartz et al. 2008]

[Cardoso & Pani, 2012]

- Waves scattered off a Kerr BH are amplified if $\omega < m\Omega_H$



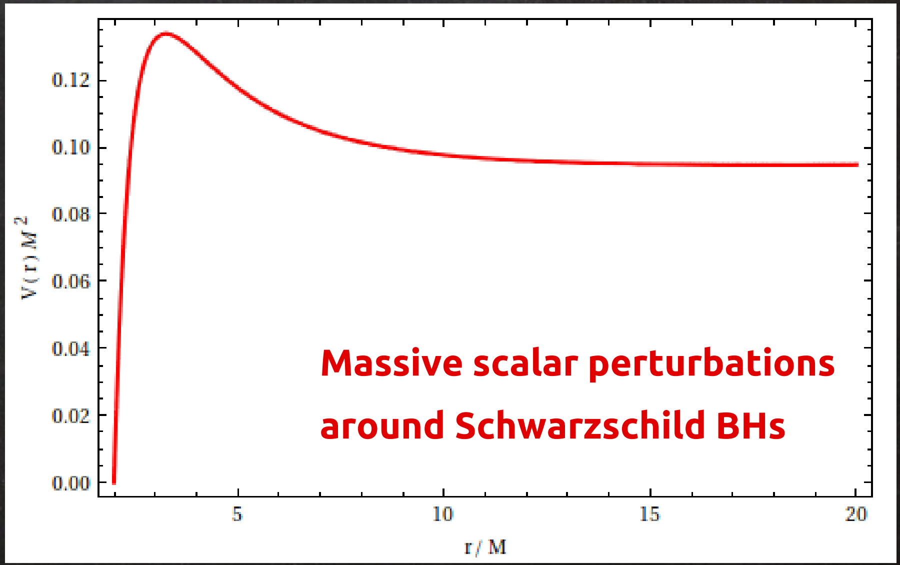
Zel'dovich effect. [Credit: Ana Sousa]

- Reflecting boundaries → BH bomb!

- "Nature may provide its own mirrors"

[Cardoso & Dias, 2004]

    - AdS boundaries
    - Massive fields



Massive fields & superradiance

- Massive fields around spinning BHs are unstable
- Instability is well-studied in the scalar case
  - Strongest instability when μM ~1
  - Astrophysically relevant only for
    - Primordial BHs (10^{14} - 10^{23} kg) and SM particles
    - Ultra-light particles (m ~ 10^{-21} - 10^{-9} eV) and massive BHs
  - Axiverse scenario (QCD axions, Peccei-Quinn mechanism, etc...)
    [Arvanitaki et al. 2010-2011]
  - Bosenova (numerical simulations are challenging)
    [Kodama & Yoshino 2011-2012]
- The massive spin-1 case is still uncharted territory... (stronger instability?)
- Rosa and Dolan (2011) studied the non-rotating case

[Damour et al. 1976]
[Detweiler, 1980]
[Earley & Zouros]
[Cardoso & Yoshida 2005]
[Dolan 2007]

Part II

Perturbations of slowly-rotating BHs

Method. *Perturbations of slowly rotating spacetimes*

[Kojima 1992, 1993, 1997]

[Pani et al., to appear]

- **Slowly-rotating background metric:**

$$ds_0^2 = -F(r)dt^2 + B(r)^{-1}dr^2 + r^2 d^2\Omega - 2\varpi(r)\sin^2\theta d\varphi dt$$

- **Expand any equation (scalar, vector, tensor…) in spherical harmonics**

$$\delta X_{\mu_1\dots}(t,r,\vartheta,\varphi) = \delta X^{(i)}_{\ell m}(r)\mathcal{Y}^{\ell m\,(i)}_{\mu_1\dots}e^{-i\omega t}$$

- **For any metric, any theory and any perturbations: system of radial ODEs:**

$$\mathcal{A}_{\ell m} + \tilde{a}m\bar{\mathcal{A}}_{\ell m} + \tilde{a}(\mathcal{Q}_{\ell m}\tilde{\mathcal{P}}_{\ell-1m} + \mathcal{Q}_{\ell+1m}\tilde{\mathcal{P}}_{\ell+1m}) = 0$$

$$\mathcal{P}_{\ell m} + \tilde{a}m\bar{\mathcal{P}}_{\ell m} + \tilde{a}(\mathcal{Q}_{\ell m}\tilde{\mathcal{A}}_{\ell-1m} + \mathcal{Q}_{\ell+1m}\tilde{\mathcal{A}}_{\ell+1m}) = 0$$

- **Zeeman splitting**
- **Laporte-like selection rule**
- **Propensity rule**

$$\mathcal{Q}_{\ell m} = \sqrt{\frac{\ell^2 - m^2}{4\ell^2 - 1}}$$

$\mathcal{A}, \mathcal{P} \rightarrow$ **Linear combinations of axial and polar perturbations**

Method. *Perturbations of slowly rotating BHs*

- At first order in the rotation, the couplings can be neglected:

$$\mathcal{A}_{\ell m} + \tilde{a} m \bar{\mathcal{A}}_{\ell m} + \tilde{a}(\mathcal{Q}_{\ell m}\tilde{\mathcal{P}}_{\ell-1m} + \mathcal{Q}_{\ell+1m}\tilde{\mathcal{P}}_{\ell+1m}) = 0$$

$$\mathcal{P}_{\ell m} + \tilde{a} m \bar{\mathcal{P}}_{\ell m} + \tilde{a}(\mathcal{Q}_{\ell m}\tilde{\mathcal{A}}_{\ell-1m} + \mathcal{Q}_{\ell+1m}\tilde{\mathcal{A}}_{\ell+1m}) = 0$$

$$\mathcal{Q}_{\ell m} = \sqrt{\frac{\ell^2 - m^2}{4\ell^2 - 1}}$$

- Symmetry of the equations

$$a_{\ell m} \to \mp a_{\ell -m}\,, \qquad p_{\ell m} \to \pm p_{\ell -m}\,, \qquad \tilde{a} \to -\tilde{a}\,, \qquad m \to -m$$

- Eigenfrequency

$$\omega = \omega_0 + \tilde{a} m\, \omega_1 + \mathcal{O}(\tilde{a}^2)$$

- "Decoupled" equations:

$$\mathcal{A}_{\ell m} + \tilde{a} m \bar{\mathcal{A}}_{\ell m} = 0 \qquad \mathcal{P}_{\ell m} + \tilde{a} m \bar{\mathcal{P}}_{\ell m} = 0$$

- Generic: any metric, any perturbation, any theory

Proca equation

$m = \hbar\mu/c$
Mass

$$\nabla_\sigma F^{\sigma\nu} - \mu^2 A^\nu = 0$$

$$\Longrightarrow \quad \nabla_\sigma A^\sigma = 0\,, \qquad \Box A^\nu - \mu^2 A^\nu = 0$$

- Massive hidden U(1) vector fields are quite generic features of extensions of standard model

[Goodsel et al. 2009]
[Jaekel et al. 2010]
[Goldhaber and Nieto 2008]

- (apparently) nonseparable in a Kerr background
- Note that EM (massless) perturbations in Kerr-(A)dS are separable!

$$\nabla_\sigma F^{\sigma\nu} = 0 \quad \Longrightarrow \quad \Box A^\nu - \nabla^\nu(\nabla_\sigma A^\sigma) + \Lambda A^\nu = 0$$

- However → role of the gauge freedom → massless fields propage 2 DOF
- Proca eq. implies Lorenz condition → no more freedom → 3 DOF

Proca in slowly-rotating Kerr

[Pani et al., to appear]

- The Proca problem becomes tractable in the slow-rotation approximation
- Let us decompose the vector field in vector spherical harmonics:

$$Y_a^{\ell m} = \left(\partial_\vartheta Y^{\ell m}, \partial_\varphi Y^{\ell m}\right) \qquad S_a^{\ell m} = \left(\frac{1}{\sin\vartheta}\partial_\varphi Y^{\ell m}, -\sin\vartheta\partial_\vartheta Y^{\ell m}\right)$$

$$\delta A_\mu(t,r,\vartheta,\varphi) = \underbrace{\sum_{l,m}\begin{bmatrix} 0 \\ 0 \\ u_{(4)}^{\ell m}(t,r) S_a^{\ell m}\end{bmatrix}}_{\text{Axial parity}} + \underbrace{\sum_{l,m}\begin{bmatrix} u_{(1)}^{\ell m}(t,r) Y^{\ell m} \\ u_{(2)}^{\ell m}(t,r) Y^{\ell m} \\ u_{(3)}^{\ell m}(t,r) Y_a^{\ell m}\end{bmatrix}}_{\text{Polar parity}}$$

Proca in slowly-rotating Kerr

- **Proca equations can be written as**

$$\delta\Pi_t \equiv (A^{(0)}_{\ell m} + \tilde{A}^{(0)}_{\ell m}\cos\vartheta)Y^{\ell m} + B^{(0)}_{\ell m}\sin\vartheta\partial_\vartheta Y^{\ell m} = 0$$

$$\delta\Pi_r \equiv (A^{(1)}_{\ell m} + \tilde{A}^{(1)\,\ell m}\cos\vartheta)Y^{\ell m} + B^{(1)}_{\ell m}\sin\vartheta\partial_\vartheta Y^{\ell m} = 0$$

$$\delta\Pi_\vartheta \equiv \alpha_{\ell m}\partial_\vartheta Y^{\ell m} - im\beta_{\ell m}\frac{Y^{\ell m}}{\sin\vartheta} + \eta_{\ell m}\sin\vartheta Y^{\ell m} = 0$$

$$\frac{\delta\Pi_\varphi}{\sin\vartheta} \equiv \beta_{\ell m}\partial_\vartheta Y^{\ell m} + im\alpha_{\ell m}\frac{Y^{\ell m}}{\sin\vartheta} + \zeta_{\ell m}\sin\vartheta Y^{\ell m} = 0$$

- **Lorenz condition can be written in the same form as {t} or {r} components**
- **All coefficients can be divided in two sets:**

$$A^{(I)}_{\ell m}\,,\quad \alpha_{\ell m}\,,\quad \zeta_{\ell m} \qquad\qquad \tilde{A}^{(I)}_{\ell m}\,,\quad B^{(I)}_{\ell m}\,,\quad \beta_{\ell m}\,,\quad \eta_{\ell m}$$

Axial coefficients   **Polar coefficients**

Proca in slowly-rotating Kerr

- The angular part can be eliminated using the orthogonality properties of the spherical harmonics. E.g. :

$$\delta\Pi_t \equiv (A^{(0)}_{\ell m} + \tilde{A}^{(0)}_{\ell m}\cos\vartheta)Y^{\ell m} + B^{(0)}_{\ell m}\sin\vartheta\partial_\vartheta Y^{\ell m} = 0$$

- We compute the following integral:

$$\int \delta\Pi_I Y^{*\,\ell m} d\Omega\,, \quad (I = t, r, L)$$

- Useful properties of spherical harmonics:

$$\cos\vartheta Y^{\ell m} = \mathcal{Q}_{\ell+1 m} Y^{\ell+1 m} + \mathcal{Q}_{\ell m} Y^{\ell-1 m}$$

$$\mathcal{Q}_{\ell m} = \sqrt{\frac{\ell^2 - m^2}{4\ell^2 - 1}}$$

$$\sin\vartheta\partial_\vartheta Y^{\ell m} = \mathcal{Q}_{\ell+1 m}\ell Y^{\ell+1 m} - \mathcal{Q}_{\ell m}(\ell+1) Y^{\ell-1 m}$$

Proca in slowly-rotating Kerr

- **Radial equations:**

$$A^{(I)}_{\ell m} + \mathcal{Q}_{\ell m}\left[\tilde{A}^{(I)}_{\ell-1m} + (\ell-1)B^{(I)}_{\ell-1m}\right] + \mathcal{Q}_{\ell+1m}\left[\tilde{A}^{(I)}_{\ell+1m} - (\ell+2)B^{(I)}_{\ell+1m}\right] = 0$$

$$\Lambda\alpha_{\ell m} - im\zeta_{\ell m} - \mathcal{Q}_{\ell m}(\ell+1)\eta_{\ell-1m} + \mathcal{Q}_{\ell+1m}\ell\eta_{\ell+1m} = 0$$

$$\Lambda\beta_{\ell m} + im\eta_{\ell m} - \mathcal{Q}_{\ell m}(\ell+1)\zeta_{\ell-1m} + \mathcal{Q}_{\ell+1m}\ell\zeta_{\ell+1m} = 0$$

- **Same form as the general equations:**

$$\mathcal{A}_{\ell m} + \tilde{a}m\bar{\mathcal{A}}_{\ell m} + \tilde{a}(\mathcal{Q}_{\ell m}\tilde{\mathcal{P}}_{\ell-1m} + \mathcal{Q}_{\ell+1m}\tilde{\mathcal{P}}_{\ell+1m}) = 0$$

$$\mathcal{P}_{\ell m} + \tilde{a}m\bar{\mathcal{P}}_{\ell m} + \tilde{a}(\mathcal{Q}_{\ell m}\tilde{\mathcal{A}}_{\ell-1m} + \mathcal{Q}_{\ell+1m}\tilde{\mathcal{A}}_{\ell+1m}) = 0$$

Proca in SR Kerr. Field equations

- Polar and axial sector are coupled:

$$
\begin{cases}
\hat{\mathcal{D}}_2 u^{\ell}_{(2)} - \frac{2F}{r^2}\left(1-\frac{3M}{r}\right)\left[u^{\ell}_{(2)} - u^{\ell}_{(3)}\right] = \\
\quad = \frac{2\tilde{a}M^2 m}{\Lambda r^5 \omega}\left[\Lambda\left(2r^2\omega^2 + 3F^2\right)u^{\ell}_{(2)} + 3F\left(r\Lambda F u'^{\ell}_{(2)} - \left(r^2\omega^2 + \Lambda F\right)u^{\ell}_{(3)}\right)\right] \\
\quad - \frac{6i\tilde{a}M^2 F\omega}{\Lambda r^3}\left[(\ell+1)\mathcal{Q}_{\ell m} u^{\ell-1}_{(4)} - \ell \mathcal{Q}_{\ell+1 m} u^{\ell+1}_{(4)}\right] \\
\hat{\mathcal{D}}_2 u^{\ell}_{(3)} + \frac{2F\Lambda}{r^2} u^{\ell}_{(2)} = \frac{2\tilde{a}M^2 m}{r^5\omega}\left[2r^2\omega^2 u^{\ell}_{(3)} + 3rF^2 u'^{\ell}_{(3)} - 3\left(\Lambda + r^2\mu^2\right)F u^{\ell}_{(2)}\right] \\
\hat{\mathcal{D}}_2 u^{\ell}_{(4)} - \frac{4\tilde{a}M^2 m\omega}{r^3} u^{\ell}_{(4)} = -\frac{6i\tilde{a}M^2 F}{r^5\omega}\left[(\ell+1)\mathcal{Q}_{\ell m}\,\psi^{\ell-1} - \ell\mathcal{Q}_{\ell+1 m}\,\psi^{\ell+1}\right]
\end{cases}
$$

- Where we have used the Lorenz condition and defined:

$$
\hat{\mathcal{D}}_2 = \frac{d^2}{dr_*^2} + \omega^2 - F\left[\frac{\ell(\ell+1)}{r^2} + \mu^2\right], \qquad \psi^{\ell} = \left(\Lambda + r^2\mu^2\right)u^{\ell}_{(2)} - (r-2M)u'^{\ell}_{(3)}
$$

Proca in SR Kerr. Boundary conditions

- **Near-horizon behavior**

$$u_{(i)} \sim e^{-ik_H r_*} \qquad k_H = \sqrt{\omega\left(\omega - \frac{m\tilde{a}}{2M}\right)} \sim \underbrace{\omega - m\Omega_H \simeq \omega - \frac{m\tilde{a}}{2r_+}}$$

Superradiance (?)

- **Caution: in principle at first order the method works only if** $\omega M \gg \tilde{a}$

- **Behavior at infinity**

$$u_{(i)} \sim B_{(i)} e^{-k_\infty r} r^{-\frac{M(\mu^2-2\omega^2)}{k_\infty}} + C_{(i)} e^{k_\infty r} r^{\frac{M(\mu^2-2\omega^2)}{k_\infty}} \qquad k_\infty = \sqrt{\mu^2 - \omega^2}$$

B=0 → quasinormal modes (purely outgoing waves at infinity)

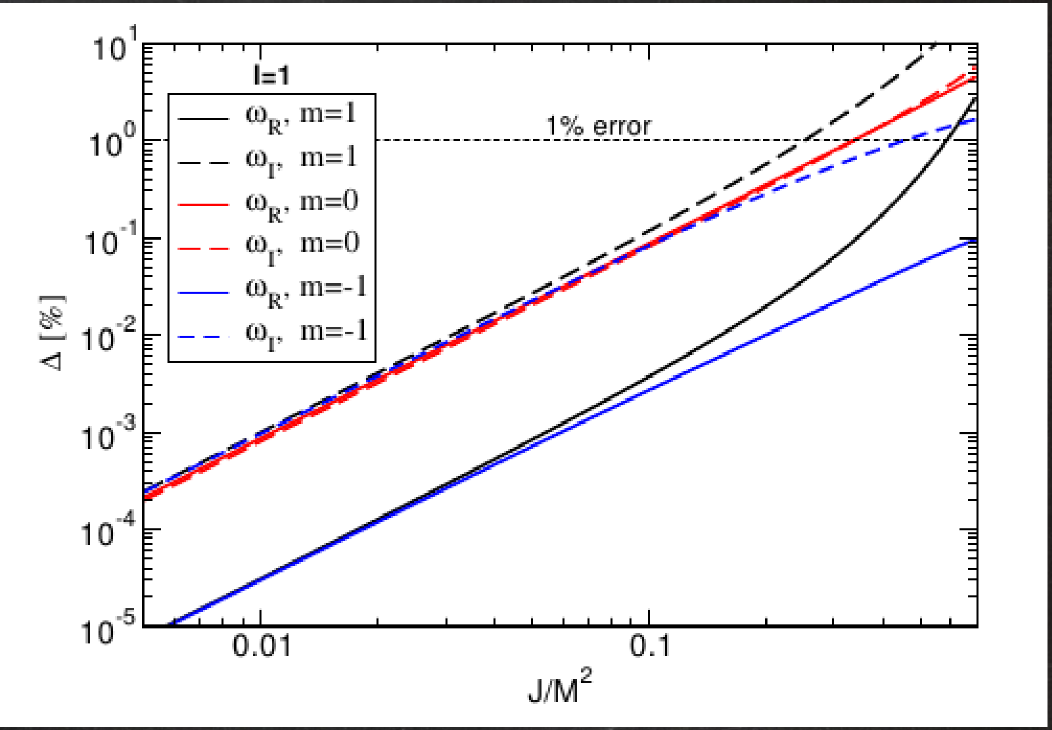
C=0 → bound states (exponential decay, spacially localized near the BH)

Proca in SR Kerr. Results

Numerical calculations in the slow rotation approximation are

not any more complicated than in the nonrotating case-horizon behavior

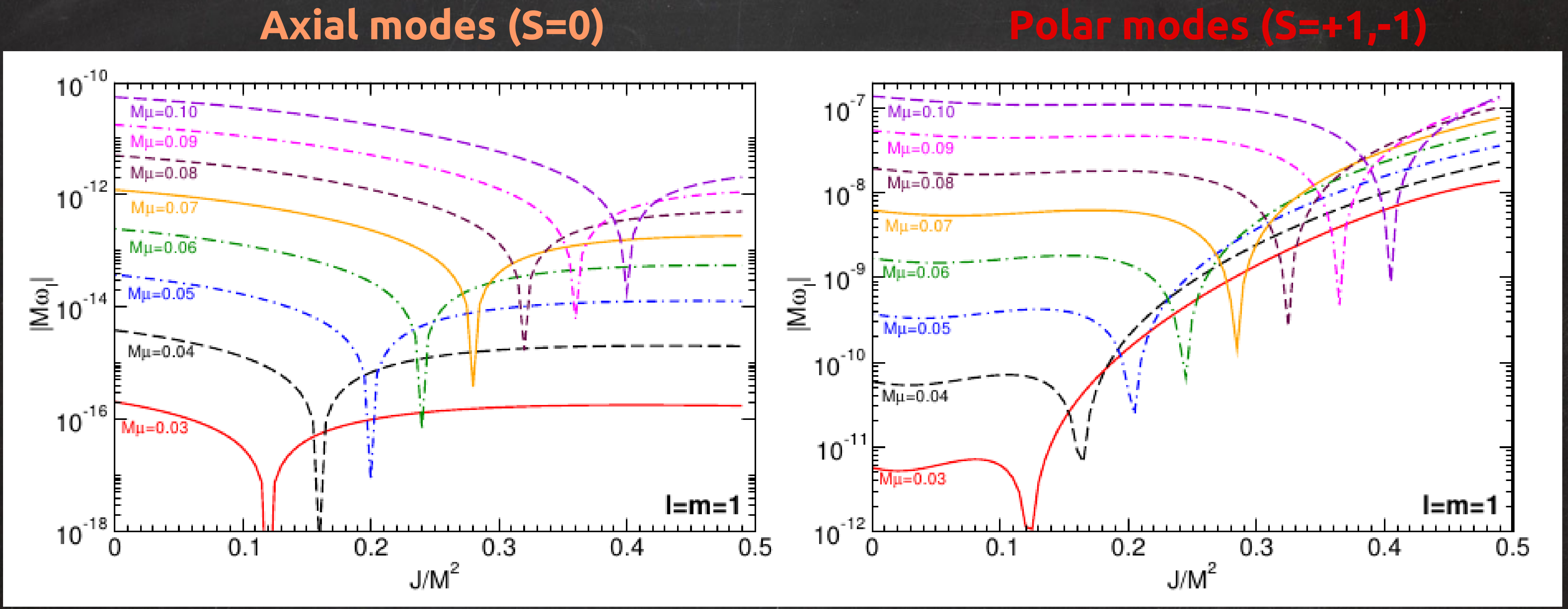
- Standard techniques:
  - direct integration (bound states)
  - continued fractions (QNMs, BS)
  - Breit-Wigner method (QNMs, BS)



Test of the method: EM (massless) QNMs of Kerr

- Good results even for moderately large spin

Proca in SR Kerr. Results

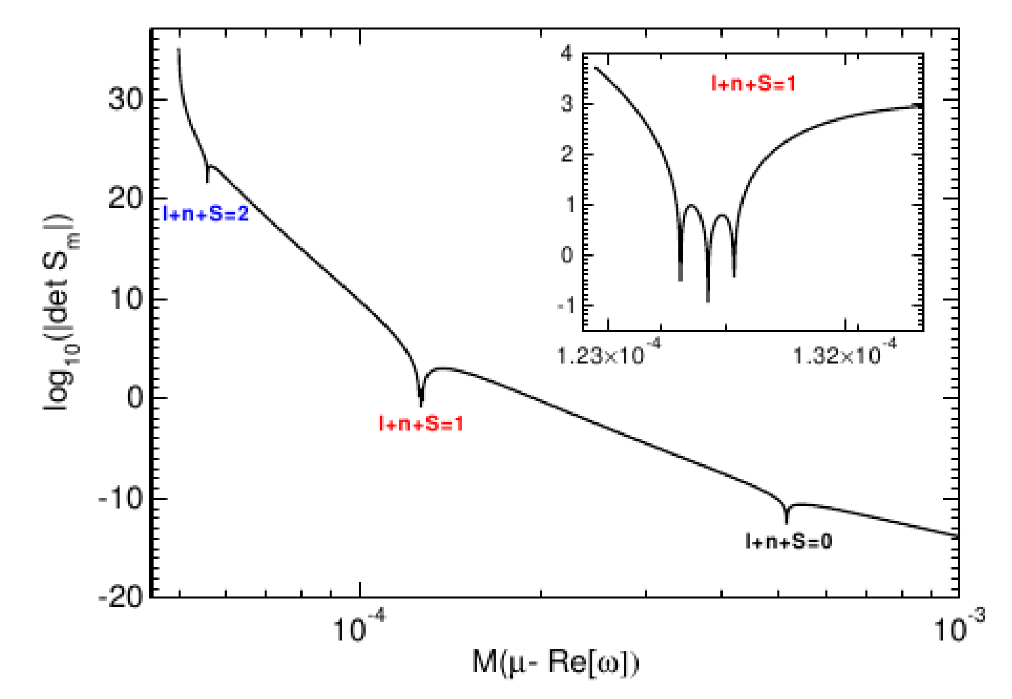


- **Small mass limit:**

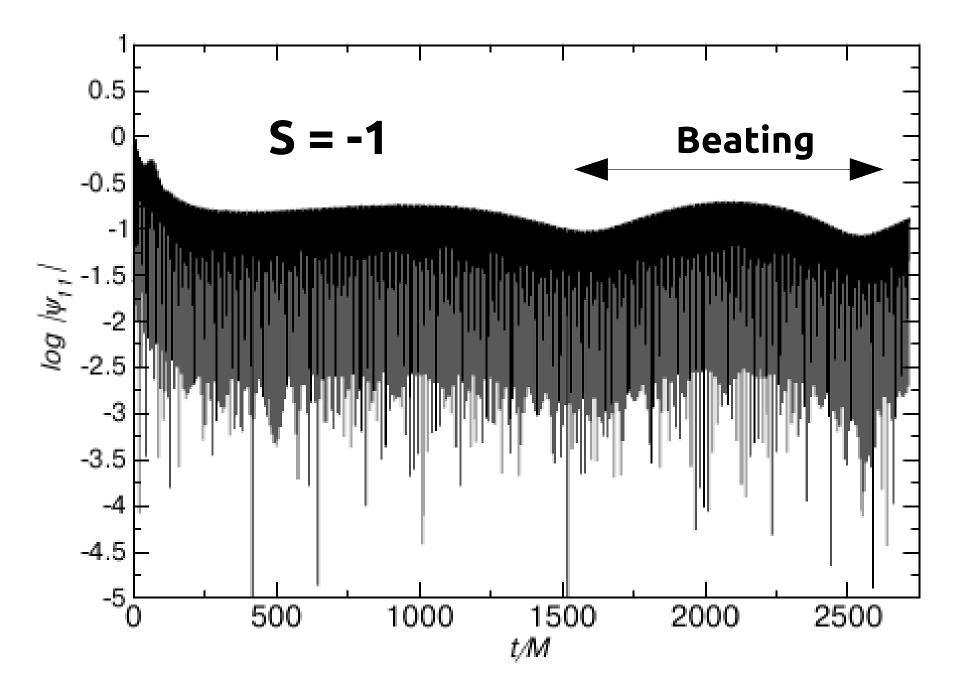
$$\omega_R \sim \mu - \frac{\mu(M\mu)^2}{2(\ell+n+S+1)} \qquad M\omega_I \sim \gamma_{S\ell}\left(\tilde{a}m - 2r_+\mu\right)(M\mu)^{4\ell+5+2S}$$

Proca in SR Kerr. *Fully coupled system*

$$\omega_R \sim \mu - \frac{\mu(M\mu)^2}{2(\ell+n+S+1)} \qquad M\omega_I \sim \gamma_{S\ell}\left(\tilde{a}m - 2r_+\mu\right)(M\mu)^{4\ell+5+2S}$$



Breit-Wigner resonances



Confirmed by numerical simulations

[Witek et al., work in progress]

Proca in SR Kerr. *Analytical results*

- In the axial case → **master equation** (scalar → **s=0** , axial vector → **s=1**)

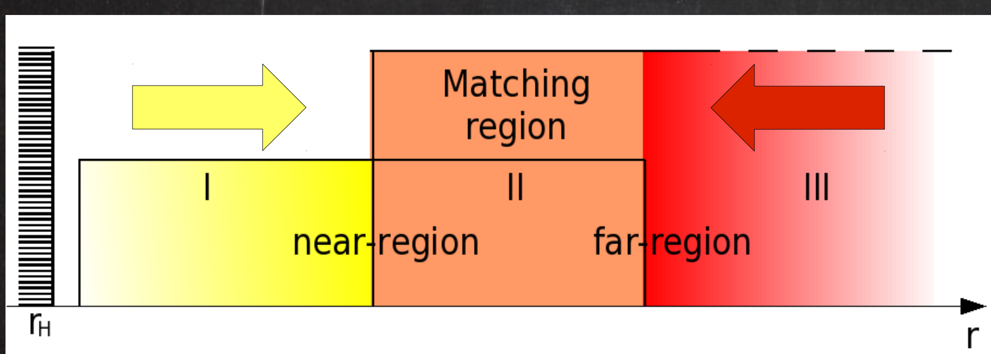
$$\frac{d^2\Psi}{dr_*^2}+\left[\omega^2-\frac{2m\varpi(r)\omega}{r^2}-F\left(\frac{\Lambda}{r^2}+\mu^2+(1-s^2)\left\{\frac{B'}{2r}+\frac{BF'}{2rF}\right\}\right)\right]\Psi=0$$

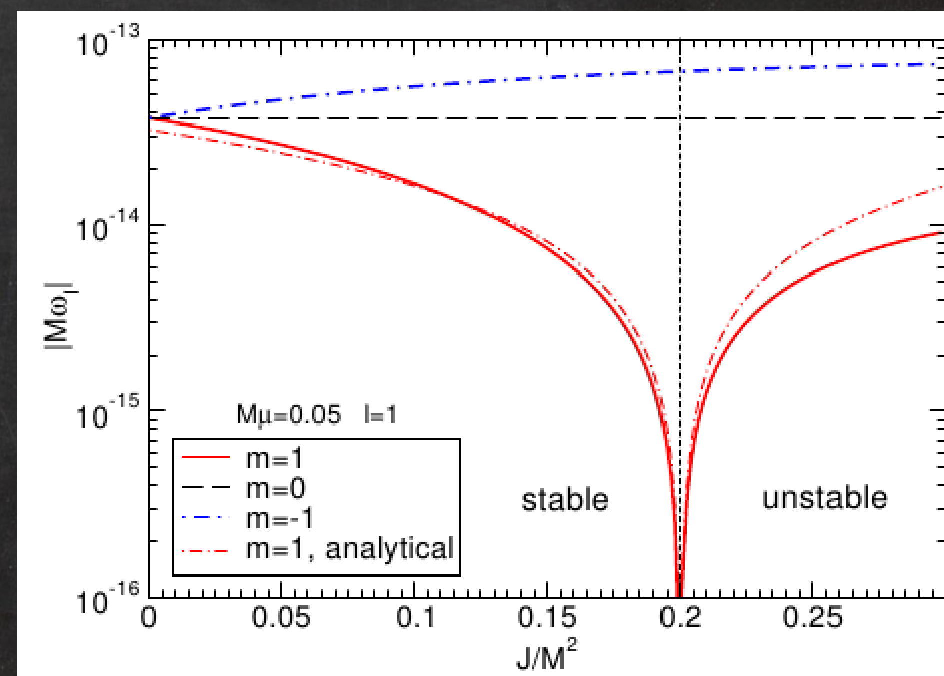
$$ds_0^2=g_{\mu\nu}^{(0)}dx^\mu dx^\nu=-F(r)dt^2+B(r)^{-1}dr^2+r^2d^2\Omega-2\varpi(r)\sin^2\theta d\varphi dt$$

- Suitable for analytical methods
- Matching asymptotics

[Starobisky 1973]

[Detweiler 1980]





$$M\omega_I\sim\gamma_{s\ell}\left(\tilde{a}m-2r_+\mu\right)(M\mu)^{4\ell+5}$$

Part III

Astrophysical consequences of the Proca instability

Proca instability

- Can we extrapolate these results to high rotation?
- Scalar case (l=1) $$M\omega_I \sim \frac{1}{48}\left(\tilde{a}m - 2r_+\mu\right)(M\mu)^9$$

Maximum at →
$$\begin{cases} a = M \\ M\mu^{\max} = 0.45 \\ M\omega_I^{\max} = 1.6\times 10^{-6} \end{cases}$$

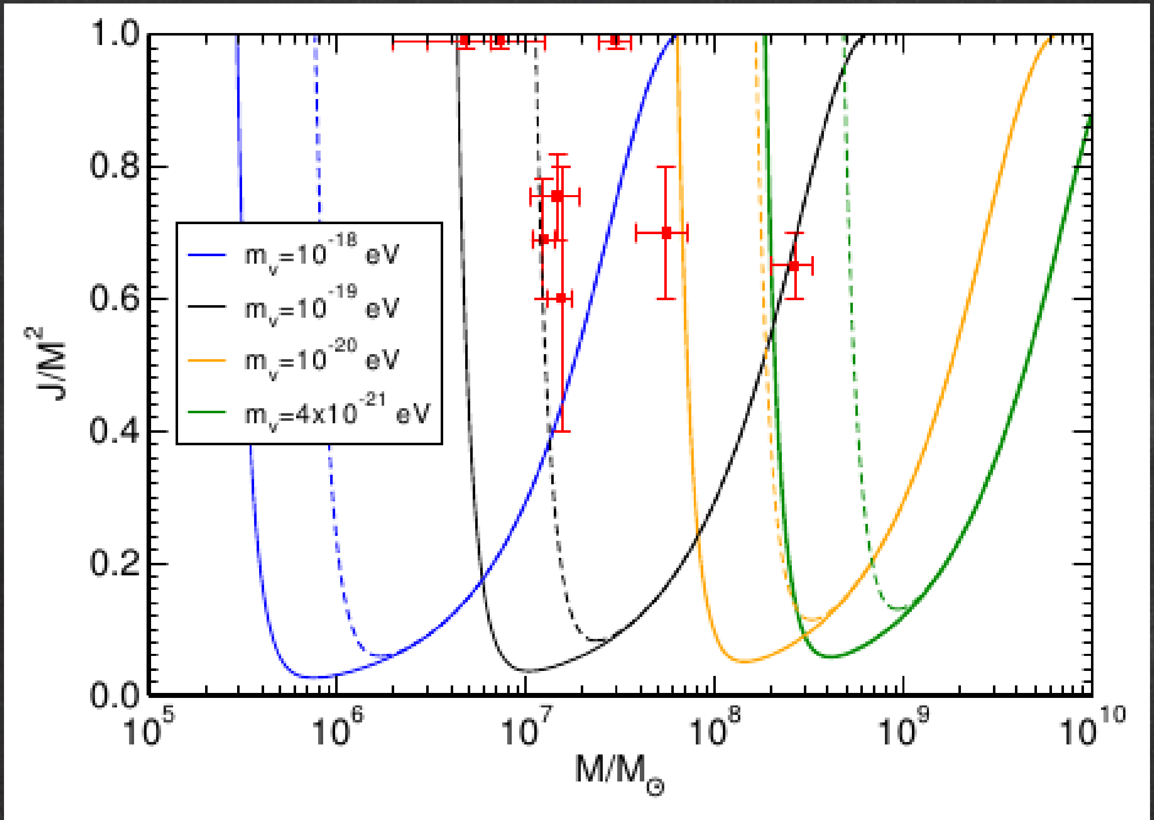
Numerically →
[Cardoso Yoshida 2005]
[Dolan 2007]
$$\begin{cases} a \sim M \\ M\mu^{\max} \sim 0.42 \\ M\omega_I^{\max} = 1.5\times 10^{-6} \end{cases}$$

- Extrapolation should provide an order of magnitude for the instability
- Proca case: $$M\omega_I \sim \gamma_{S\ell}\left(\tilde{a}m - 2r_+\mu\right)(M\mu)^{4\ell+5+2S}$$
- Stronger instability when S = -1 and l=1:

$$\tau_{\rm vector} = \omega_I^{-1} \sim \frac{M(M\mu)^{-7}}{\gamma_{-11}(\tilde{a} - 2\mu r_+)}$$

Proca instability. *Regge plane*

- Instability is effective roughly for any non-vanishing spin!

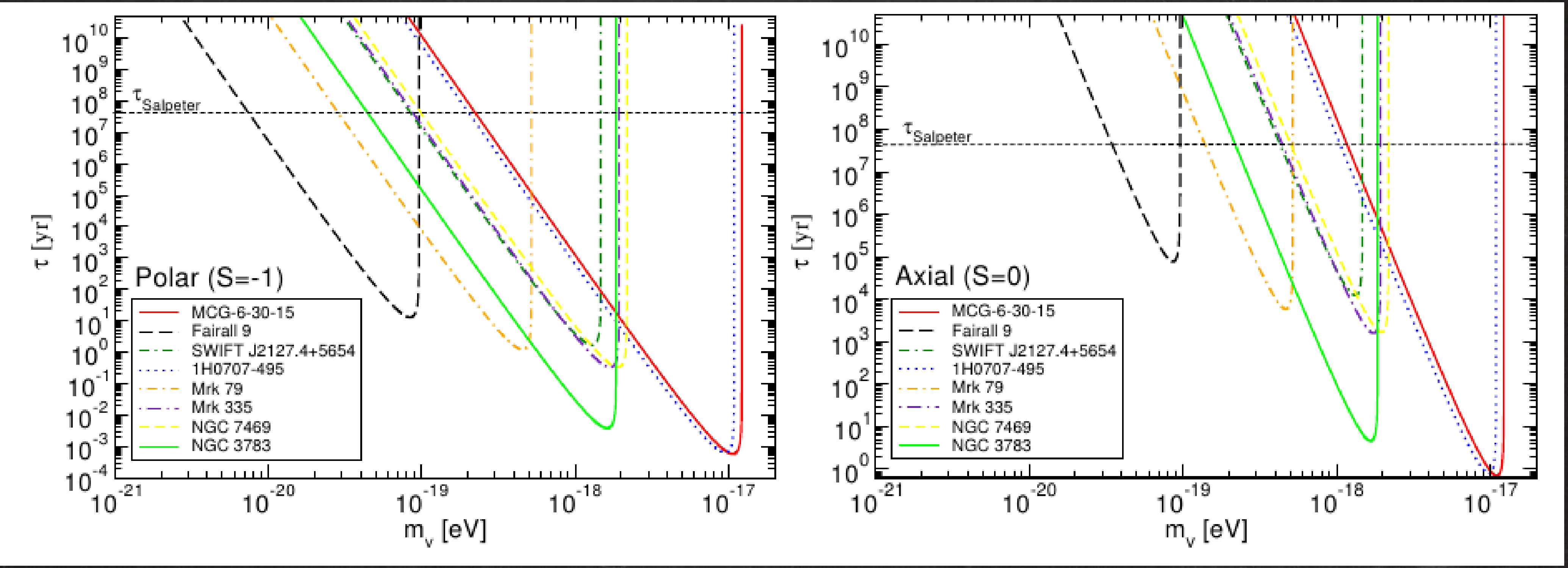


[Data taken from Brenneman et. al 2011]

- Current bound on the photon mass [from PDG] → $m_\gamma < 10^{-18}\mathrm{eV}$
- Depend very mildly on the fit coefficient and on the threshold
- τ_{Salpater} → timescale for accretion at the Eddington limit

Proca instability

- **Not strongly dependent on the timescale nor on type of mode**



Part IV
Further applications

Second order

[even more in preparation]

- **Particularly advantageous:**
  - Cauchy horizon, even horizons, ergosphere

$$r_+ = 2M\left(1-\frac{\tilde{a}^2}{4}\right) \qquad r_- = \frac{M\tilde{a}^2}{2} \qquad r_{\rm ER} = 2M\left(1-\cos^2\vartheta\,\frac{\tilde{a}^2}{4}\right)$$

  - The superradiance regime is now consistent

$$\omega = \omega_0 + \tilde{a}m\omega_1 + \tilde{a}^2\omega_2 + \mathcal{O}(\tilde{a}^3)$$

- **Caution:** different expansion!
  - Spheroidal harmonics VS spherical harmonics

$$S_{\ell m} = Y_{\ell m} + \mathcal{O}(\tilde{a})$$

  - Cannot recover Teukolsky → superposition of modes

Second order

[even more in preparation]

$$0 = \mathcal{A}_\ell$$

Zeroth order

$$0 = \mathcal{P}_\ell$$

$\mathcal{P}_{L+3}$

$\mathcal{A}_{L+2}$

$\mathcal{P}_{L+1}$

$\mathcal{A}_L$

$\mathcal{P}_{L-1}$

$\mathcal{A}_{L-2}$

$\mathcal{P}_{L-3}$

Second order

[even more in preparation]

$$0 = \mathcal{A}_\ell$$

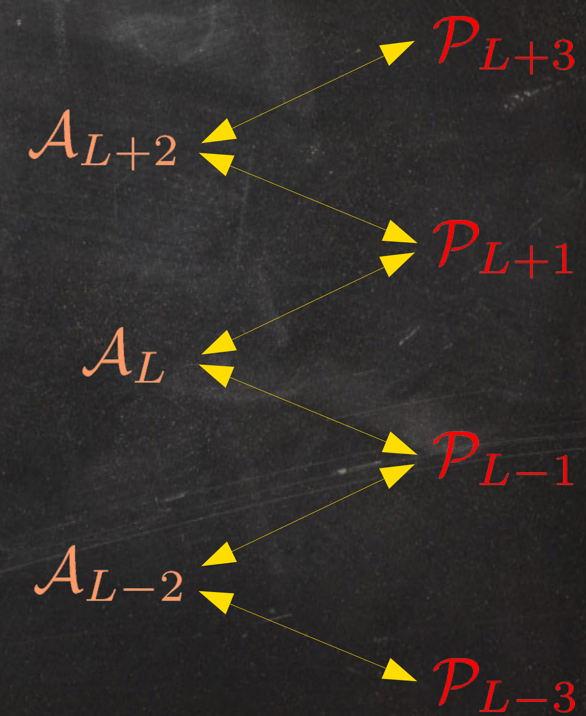
Zeroth order

$$+\tilde{a}m\bar{\mathcal{A}}_\ell + \tilde{a}(\mathcal{Q}_\ell\tilde{\mathcal{P}}_{\ell-1} + \mathcal{Q}_{\ell+1}\tilde{\mathcal{P}}_{\ell+1})$$

First order

$$0 = \mathcal{P}_\ell$$

$$+\tilde{a}m\bar{\mathcal{P}}_\ell + \tilde{a}(\mathcal{Q}_\ell\tilde{\mathcal{A}}_{\ell-1} + \mathcal{Q}_{\ell+1}\tilde{\mathcal{A}}_{\ell+1})$$



Second order

[even more in preparation]

$$0 = \mathcal{A}_\ell$$

Zeroth order

$$+\tilde{a}m\bar{\mathcal{A}}_\ell + \tilde{a}(\mathcal{Q}_\ell\tilde{\mathcal{P}}_{\ell-1} + \mathcal{Q}_{\ell+1}\tilde{\mathcal{P}}_{\ell+1})$$

First order

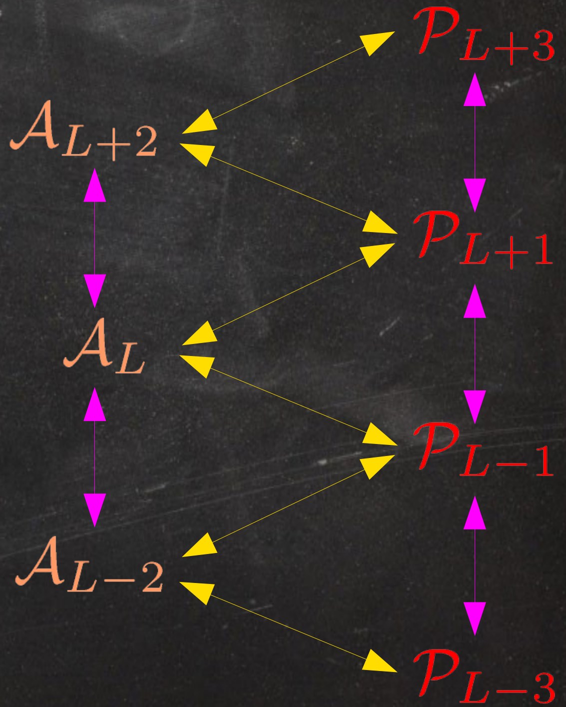
$$+\tilde{a}^2\left[\hat{\mathcal{A}}_\ell + \mathcal{Q}_{\ell-1}\mathcal{Q}_\ell\breve{\mathcal{A}}_{\ell-2} + \mathcal{Q}_{\ell+2}\mathcal{Q}_{\ell+1}\breve{\mathcal{A}}_{\ell+2}\right]$$

Second order

$$0 = \mathcal{P}_\ell$$

$$+\tilde{a}m\bar{\mathcal{P}}_\ell + \tilde{a}(\mathcal{Q}_\ell\tilde{\mathcal{A}}_{\ell-1} + \mathcal{Q}_{\ell+1}\tilde{\mathcal{A}}_{\ell+1})$$

$$+\tilde{a}^2\left[\hat{\mathcal{P}}_\ell + \mathcal{Q}_{\ell-1}\mathcal{Q}_\ell\breve{\mathcal{P}}_{\ell-2} + \mathcal{Q}_{\ell+2}\mathcal{Q}_{\ell+1}\breve{\mathcal{P}}_{\ell+2}\right]$$



Kerr-Newman BHs

- Most general rotating solution in GR
- Gravitational and EM perturbations are coupled → not separable?
- Apply the method to slowly-rotating Reissner-Nordstrom:

Kerr-Newman BHs

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  - **Axial sector: (isospectrality?)**

$$\hat{\mathcal{D}} Z_i = V_0^{(i)} Z_i$$

Zeroth order (i=1,2)

$$\hat{\mathcal{D}} = \frac{d^2}{dr_*^2} + \omega^2 - F\frac{\ell(\ell+1)}{r^2}$$

$$F(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

Kerr-Newman BHs

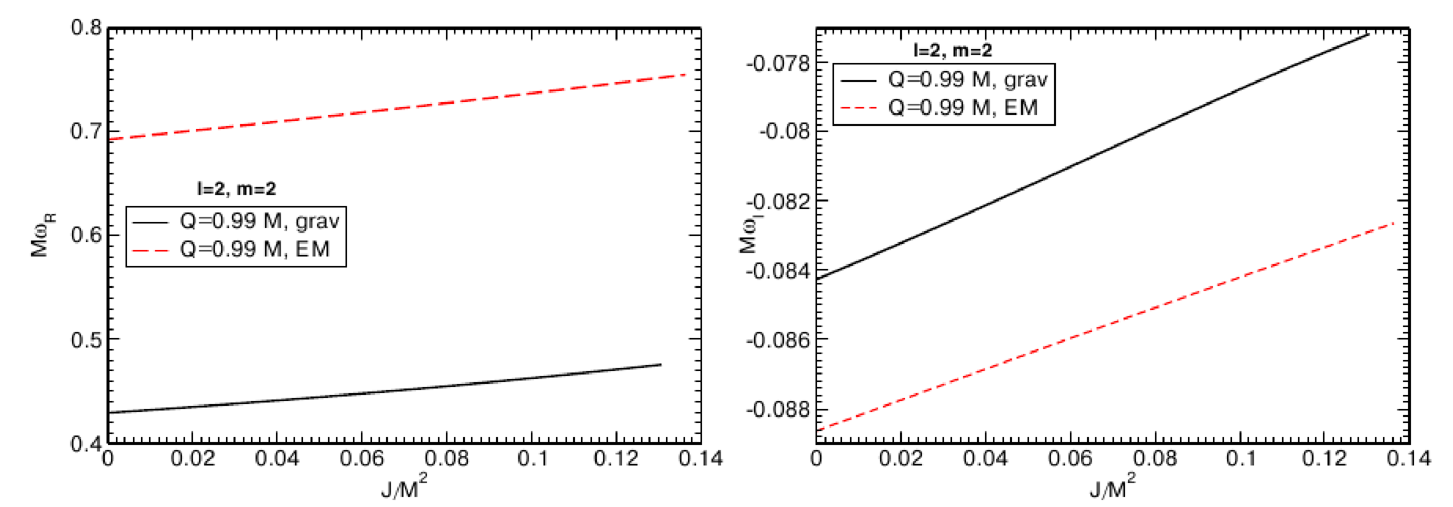
- Most general rotating solution in GR
- Gravitational and EM perturbations are coupled → not separable?
  [Berti & Kokkotas 2004]
- Apply the method to slowly-rotating Reissner-Nordstrom:
  - Axial sector: (isospectrality?)

$$\hat{\mathcal{D}} Z_i = V_0^{(i)} Z_i + m\tilde{a}\left[V_1^{(i)} Z_i + V_2^{(i)} Z_i'\right] + m\tilde{a}Q^2\left[W_1^{(i)} Z_j + W_2^{(i)} Z_j'\right]$$

Zeroth order (i=1,2)   First order: coupliing between i and j)

$$\hat{\mathcal{D}} = \frac{d^2}{dr_*^2} + \omega^2 - F\frac{\ell(\ell+1)}{r^2}$$

$$F(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$



Conclusion & Extensions

- Linear pertubations of BHs are important in a variety of situations
  - Stability, GWs, synergy with numerical simulations
- Perturbation theory of rotating solutions is challenging
- Slowly-rotating approximation: general method
  - Superradiance, BHs in alternative theories
- #1 Application: Proca perturbations of Kerr BHs in GR
  - Stronger instability than for scalars, bounds on the photon mass, Hidden U(1) sector
- #2 Application: gravito-EM pertubations of Kerr-Newman BHs in GR
- Second order formalism
- BHs in alternative theories (Chern-Simons, Gauss-Bonnet) [Yunes & Pretorius 2009] [Pani et al. 2011]
- Higher dimensions

$T^{\text{he}}_{\mu\nu} G^{\text{ravity}}_{\mu\nu} R^{\text{oom}}_{\mu\nu}$

thegravityroom.blogspot.com

Calls for bloggers are now open!

Thanks!

Backup slides

“Nothing is More Necessary than the Unnecessary”

- **Curiosity: similar bounds for the graviton? → probably not! (S= -2, l=2)**

$$M\omega_I \sim \gamma_{S\ell} \left(\tilde{a}m - 2r_+\mu\right) (M\mu)^{4\ell+5+2S}$$

Proca in SR Kerr. *Field equations*

- In Proca theory, the monopole (l=0,m=0) is dynamical:

$$\left[\frac{d^2}{dr_*^2}+\omega^2-F\left(\frac{2(r-3M)}{r^3}+\mu^2\right)\right]u^{00}_{(2)}=\underbrace{\frac{2i\sqrt{3}\tilde{a}M^2\omega F}{r^3}u^{10}_{(4)}}$$

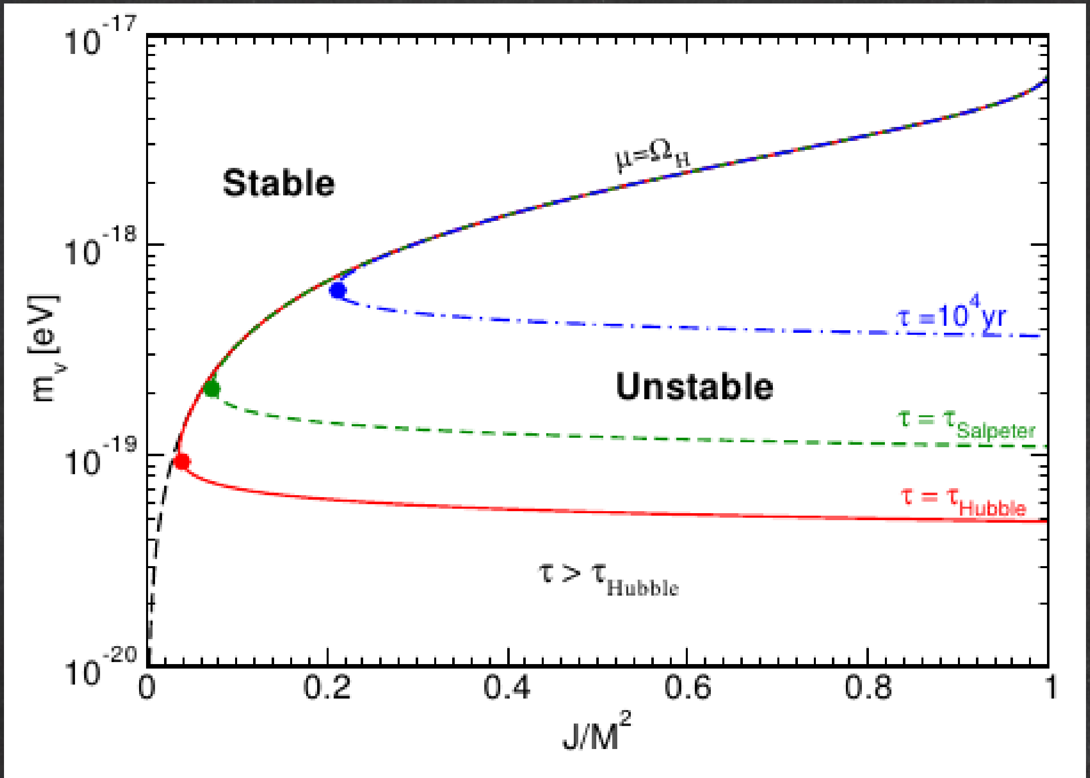
Propensity rule $(\mathcal{Q}_{00}=0)$

- m=0 → no corrections at first order! Same modes as in Schwarzschild

[Rosa & Dolan 2011]

- Modes can be labelled by the total angular momentum → j=l+S
  - Axial → S=0
  - Polar → S=+1 , S= -1
  - Monopole → S=+1

Proca instability



$$m_v^{(c)} = \hbar\mu^{(c)} \sim \frac{7.055\times 10^{-20}}{\gamma_{-11}^{1/7}}\left[\frac{10^7 M_\odot}{M}\right]^{6/7}\text{eV}$$

- Depend very mildly on the fit coefficient and on the threshold
- $\tau_{Salpeter}$ → timescale for accretion at the Eddington limit

$\mathcal{O}(\nu^2)$ $\mathcal{O}(\nu^3)$

AAA AAA